Competing for Talents

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Abstract: Two organizations compete for high quality agents from a fixed population of heterogeneous qualities by designing how to distribute their resources among members according to their quality ranking. The peer effect induces both organizations to spend the bulk of their resources on higher ranks in an attempt to attract top talents that benefit the rest of their membership. Equilibrium is asymmetric, with the organization with a lower average quality offering steeper increases in resources per rank. High quality agents are present in both organizations, while low quality agents receive no resources from either organization and are segregated by quality into the two organizations. A stronger peer effect increases the competition for high quality agents, resulting in both organizations concentrating their resources on fewer ranks with steeper increases in resources per rank, and yields a greater equilibrium difference in average quality between the two organizations.

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1. Introduction

Consider an academic department trying to improve its standing by hiring a new faculty member. Several economic forces influence such a decision. First, if the potential appointee is of high quality, the presence of such a colleague in the department will make the department more attractive to other faculty members due to the peer effect and may therefore help the department's other recruiting efforts. Second, the new recruit can upset the department's existing hierarchical structure and bring about implications for the internal distribution of departmental resources. "Salary inversion" is often seen as a potential problem in academia (Lamb and Moates, 1999; Siegfried and Stock, 2004). More generally, conventional wisdom in personnel management emphasizes the importance of "internal relativity" in the reward structure of any organization. In other words, the decision to make a job offer cannot be viewed in isolation; instead the entire reward structure of the organization has to be taken into account. Third, in a thin market with relatively few employers, the recruitment efforts of one department will affect the availability of the talent pool for another department. Hiring decisions in one department therefore have implications for the sorting of talents across all departments that need to be considered in an analysis of strategic competition for talents.

In this paper we develop a model of the competition for talents which incorporates all these economic forces. While the concern for the quality of one's peers, or the "peer effect," is widely acknowledged in the education literature (e.g., Coleman et al., 1966; Summers and Wolfe, 1977; Lazear, 2001; Sacerdote, 2001), and modeled extensively in the literature on locational choice (De Bartolome, 1990; Epple and Romano, 1998), the implications for organization design and especially organization competition, have received little attention.[1] We take the first step with a stylized model to study organizational strategies to attract

[1] In De Bartolome (1990), two communities decide their public service output and tax rate by majority voting, while in Epple and Roman (1998), private schools choose admission and tuition policies to maximize profits in a competitive equilibrium with free entry. Neither paper addresses the issue of strategic competition between organizations in a non-cooperative game. The existing economic literature on the competition for talents typically focuses on either the informational spillovers resulting from offers and counter-offers (Bernhardt and Scoones, 1993; Lazear, 1996), or the implications of raiding for firms' incentive to offer training (Moen and Rosen, 2004). Tranaes (2001) studies the impact of raiding opportunities on unemployment in a search environment.

talents in the presence of the peer effect, and analyze the resulting equilibrium pattern of sorting of talents.

Section 2 introduces a game between two organizations, A and B. Talents have one-dimensional types distributed uniformly, and a utility function linear in the average type of the organization they join (the peer effect) and the resource they receive in the organization. Each organization faces a fixed capacity constraint that allows it to accept half of an exogenously given talent pool, and a fixed total budget of resources that can be allocated among its ranks. There are three stages of the game. In the first stage of resource distribution, the two organizations each simultaneously choose a budget-balanced schedule that associates the rank of each agent by type with an amount of resource the agent receives. In the second stage of talent sorting, after observing the pair of resource distribution schedules, all agents simultaneously choose one organization to apply to. In the third and final stage of admissions, each organization admits a subset of its applicants no larger than the capacity. The payoff to each organization is zero if the capacity is not filled, and is given by the average type otherwise. The payoff to any agent that is not admitted by an organization is zero.

Finding a subgame perfect equilibrium in the above game of organizational competition is difficult, because the space of resource distribution schedules is large, and because little can be said in general about the continuation equilibrium given an arbitrary pair of resource distribution schedules. In section 3 we develop an indirect approach based on the quantile-quantile plot of any given sorting of talents between the two organizations, defined as follows.[2] For each rank r, the quantile-quantile plot identifies the type that has this rank in organization B, and gives the fraction of higher types in organization A. Since the type distribution is uniform, the difference in the average types between A and B, referred to as "quality difference," has a one-to-one relation with the integral of the quantile-quantile plot. Thus, to characterize a continuation equilibrium for a given pair of resource distribution schedules, instead of using two type distribution functions for the two organizations, we use the associated quantile-quantile plot.

[2] The quantile-quantile plot, also known as QQ-plot, is widely used in graphical data analysis for comparing distributions (e.g., Wilk and Gnanadesikan, 1968; Chambers et al., 1983).

There are typically multiple continuation equilibria for an arbitrary pair of resource distribution schedules. We focus on subgame perfect equilibria with the largest quality difference between the two organizations by adopting the "A-dominant" criterion that selects the continuation equilibrium with the largest difference in average types in favor of A for any pair of schedules. This selection criterion allows us to provide sharp predictions for both equilibrium resource distribution schedules and equilibrium sorting of talents in section 4. In our equilibrium the targets of competition are the top talents; only these types receive positive shares of resources from either organization. Furthermore, equilibrium resource distribution schedules are systematically different between the high quality organization A and the low quality organization B, even if the two organizations have the same resource budget, provided the peer effect is sufficiently strong. Organization A has a more egalitarian distribution of resources than B, as the latter is disadvantaged by the peer effect and must concentrate its resources on a smaller set of top talents. The equilibrium sorting of talents exhibits mixing of top talents, with a greater share of them going to the high quality organization, while segregation occurs for all types that receive no resources in equilibrium, with the better types going to the high quality organization. If the resource advantage of the dominant organization becomes greater, or if the peer effect becomes more important in the talents' utility function, the equilibrium quality difference between the two organizations is greater. The weaker organization finds it more difficult to compete against the dominant organization, and as a result must focus its resources on fewer ranks at the top. This leads to steeper resource distribution schedules for both organizations.

In section 5, we establish that the equilibrium constructed in section 4 is unique under the A-dominant criterion. The key result the "Raiding Lemma," which characterizes the minimum resource budget for organization A to achieve any quality difference in a continuation equilibrium, against any a resource distribution schedule of B. This minimum budget is achieved by targeting the ranks in B which have the lowest resource-to-rank ratio (suitably adjusted to incorporate the equilibrium quality difference). We obtain this characterization by converting this minimization problem into linear programming in quantile-quantile plot. The objective is linear because the required budget for A's resource distribution schedule to yield the quantile-quantile plot as a continuation is a weighted integral of the quantile-quantile plot; the constraint is linear because the quantile-quantile plot has to integrate to yield the given quality difference.

A critical implication of the Raiding Lemma is that the minimum resource budget for A to achieve any quality difference in a continuation equilibrium depends on B's resource schedule only through the amount of resource received by the ranks with the lowest resource-to-rank ratio. Thus, if B wants to maximize the resource budget needed for A to achieve any quality difference in a continuation equilibrium, it should offer a resource distribution schedule that is linear in rank. An immediate implication is that for any quality difference, there is a unique budget-balanced resource distribution schedule for B that maximizes the minimum budget required for A's resource distribution schedule to be consistent with the quality difference in a continuation equilibrium. We refer to the resulting budget requirement for A as the "resource budget function," which is a function of quality difference.

Under the selection criterion, the unique equilibrium quality difference is the largest value of the difference in average types for which the budget function is below the exogenous resource budget available to A. This is quality difference that we have used to construct the equilibrium; this immediately leads to the equilibrium resource distribution schedule for B constructed earlier. However, as suggested in the Raiding Lemma, there are many best responses of organization A to B's equilibrium schedule. In the last step of section 5, we complete the argument for the uniqueness of the equilibrium by showing that A's equilibrium schedule constructed earlier is the unique schedule to which B's equilibrium schedule is a best response. This again relies on the linearity of A's equilibrium schedule, which makes B indifferent among all paid ranks in A as they have the same resource-to-rank ratio.

2. The Model

There are two organizations, A and B. Each $i = A, B$ is endowed with a measure 1 of positions and a fixed resource budget Y_i to be allocated among its members. We assume that $Y_A \geq Y_B$. There is a continuum of agents, of measure 2. Agents differ with respect to a one-dimensional characteristic, called "type" and denoted by θ, which may be interpreted as ability or productivity. The distribution of θ is uniform on $[0, 1]$.

We consider a complete information extensive form game with three stages. In the first stage (resource distribution stage), each organization $i = A, B$, chooses a "resource distribution schedule," which is a function $S_i : [0, 1] \rightarrow \mathbb{R}_+$ stipulating how Y_i is allocated among i's members according to their rank by type: For each $r \in [0, 1]$, $S_i(r)$ denotes the amount of resources received by an agent of type θ when a fraction r of the organization's members are of type smaller than θ.[3] We assume that each S_i is weakly increasing, that is, organizations can only adopt "meritocratic" resource distribution schedules in which members of higher ranks receive at least as much resources as lower ranks. The monotonicity of S_i ensures integrability, and we require it to satisfy the resource constraint, i.e., $\int_{r=0}^{r=1} S_i(r)\, \mathrm{d}r \leq Y_i$. We make two technical assumptions: S_i is left-continuous,[4] and there is a countable partition of the interval $[0, 1]$ such that the derivative S_i' is continuous and monotone in the interior of each element of the partition.[5] In the second stage (application stage) of the game, after observing the pair of distribution schedules chosen by the two organizations, all agents simultaneously apply to join either A or B. In the third and final stage (admissions stage), after observing the pool of applicants, each organization chooses the lowest type that will be admitted subject to the capacity constraint.

The payoff to each organization is zero if the capacity is not filled, and is given by the average type otherwise. The payoff to any agent that is not admitted by an organization is normalized to 0. If admitted by organization i, $i = A, B$, an agent of type θ receives a payoff given by

$$V_i(\theta) = \alpha S_i(r_i(\theta)) + m_i, \tag{1}$$

[3] We implicitly assume that the resource distribution schedules cannot directly depend on the type of agents. This and other assumptions are briefly discussed in section 6.

[4] This is needed because in our continuous-type model two different types can have the same rank. If S_i is not left-continuous at some rank r, each type θ in some open interval (θ', θ'') may strictly prefer joining i when no type higher than θ in the interval joins i because θ's rank in i would be r, but the preference is reversed when any positive mass of types above θ joins i. This may lead to non-existence of an equilibrium.

[5] By monotonicity $S_i'(r)$ exists for almost all r. The second technical assumption rules out the case where S_i' is continuous but nowhere monotone on some open interval in $[0, 1]$.

where m_i is the average type of agents in organization i, $r_i(\theta)$ is the quantile rank of the agent in i, and α is a positive constant that represents the weight on the concern for the resource he receives relative to the concern for the average type (the peer effect).

We have made two simplifying assumptions in setting up the game of organization competition. First, the type distribution is uniform; second, resources and peer average type are perfect substitutes in the payoff function of the agents. Neither assumption is conceptually necessary for a model of organization competition. However they are critical to make our approach in the analysis successful; this will become clear in section 3 when we introduce the quantile-quantile plot.

3. Preliminary Analysis

3.1. Continuation equilibrium

We begin the analysis with a characterization of the continuation equilibrium for a given pair of resource distribution schedules. In the continuation game following (S_A, S_B), if the outcome is a pair of type distribution functions (H_A, H_B) for A and B, then for any type θ the rank in organization i, $i = A, B$, is given by $r_i(\theta) = H_i(\theta)$, and the average type in organization i is $m_i = \int \theta \ \mathrm{d}H_i(\theta)$. The payoffs V_A and V_B to type θ from joining A and B are defined as in (1), with $H_i(\theta)$ replacing $r_i(\theta)$. Thus, a continuation equilibrium is characterized by (H_A, H_B) that satisfy two conditions: (i) $H_A(\theta) + H_B(\theta) = 2\theta$ for all $\theta \in [0, 1]$; and (ii) if H_i is strictly increasing at θ and $H_j(\theta) > 0$, then $V_i(\theta) \geq V_j(\theta)$, for $i, j = A, B$ and $i \neq j$. Condition (i) says that all agents join, in equilibrium, one of the two organizations and is necessary because each agent's outside option is zero and each organization prefers to admit any type to leaving positions unfilled. If condition (ii) does not hold, then because H_i is strictly increasing at θ some type just above θ is applying to organization i in the continuation equilibrium, and such type could instead profitably deviate by successfully applying to j. It is immediate that for any (H_A, H_B) that satisfies (i) and (ii), there is a continuation equilibrium in application and admission strategies.

In a continuation equilibrium an agent will join organization i whenever he prefers organization i and his type is higher than the lowest type in that organization. The types lower than the maximum of the lowest types in the two organizations do not have a choice as they are only acceptable to one organization. The other types are free to choose which organization to join, and are the targets of the organization competition. Because of the linearity in the payoff functions (1), which organization these types choose depends on the comparison of the resources they get from A and B, adjusted by the same factor determined by the quality difference $m_A - m_B$. Existence of a continuation equilibrium for any (S_A, S_B) will be established, using a fixed point argument, after introducing the quantile-quantile plot approach. In the formal argument, for any given difference $m_A - m_B$ we characterize the pair of type distribution functions (H_A, H_B) that satisfy conditions (i) and (ii) above. Each pair then implies a new quality difference between the two organization, and a continuation equilibrium is a fixed point in this mapping.

Because of both the direct "peer effect" externality and the indirect externality through rank dependent resource distributions, agents face a coordination problem in their decision of which organization to join. This will typically lead to multiple continuation equilibria for a given pair of resource distribution schedules. For example, if the two resource distribution schedules are identical, there is a continuation equilibrium with zero quality difference. For a sufficiently large peer effect (sufficiently small α), there is another continuation equilibrium with perfect segregation, generating the maximum possible quality difference.[6] In this paper we construct a subgame perfect equilibrium in which, following the choice of any pair of resource distribution schedules, the continuation equilibrium with the largest quality difference in favor of A is played. We refer to this selection criterion as "A-dominant." It captures the idea that there is a "focal" organization expected to attract the best distribution of agents, and it is similar to the notion of favorable expectations used in the literature on competition in two-sided markets (Caillaud and Jullien, 2003; Hagiu, 2006; and Jullien, 2008). Further, the continuation equilibrium with maximal quality difference is always "stable" to small perturbations in distribution functions (H_A, H_B) while other equilibria might be unstable.

[6] With identical resource distribution schedules, the larger the difference between the highest and lowest paid rank the larger the peer effect needed for perfect segregation to be an equilibrium. For any positive α an organization can prevent perfect segregation in favor of the rival organization by concentrating a sufficiently large amount of resources on a small number of ranks.

3.2. Quantile-quantile plot approach

Definition 1. Given a pair of type distribution functions (H_A, H_B), the associated quantile-quantile plot $t : [0,1] \to [0,1]$, is given by

$$t(r) \equiv 1 - H_A\left(\inf\{\theta : H_B(\theta) = r\}\right).$$

The above definition associates to each pair of type distributions a unique non-increasing function on the unit interval. The variable $t(r)$ is the fraction of agents in organization A of type higher than the type with rank r in organization B.[7] For example, if the distribution of talents is perfectly segregated with the higher types exclusively in organization A, then $t(r) = 1$ for all r; and if there is even mixing so that the distribution of types is identical across the two organizations, then $t(r) = 1 - r$ for every r. The infimum operator in the definition of t is a convention adopted to handle the case where H_B is flat over some interval, that is, when there is local segregation with all types in the interval going to organization A.

Conversely, each non-increasing function $t : [0,1] \to [0,1]$ identifies a unique pair of type distribution functions (H_A, H_B), with $H_A(\theta) = 2\theta - H_B(\theta)$ for all θ, where H_B is given by

$$H_B(\theta) = \begin{cases} 0 & \text{if } 2\theta \leq 1 - t(0), \\ \sup\{r : 2\theta \geq r + 1 - t(r)\} & \text{if } 1 - t(0) < 2\theta < 2 - t(1), \\ 1 & \text{if } 2\theta \geq 2 - t(1). \end{cases}$$

For any type θ around which H_B is increasing, which means that types around θ are not joining organization A exclusively, $H_B(\theta)$ is uniquely defined by r such that $r+1-t(r) = 2\theta$; if there is no solution to the equation, then $H_B(\theta)$ is either 0 or 1. For any type θ such that H_B is flat just above θ, then $H_B(\theta)$ is given by 1 minus the rank of the highest type that does not exclusively join A.

Thus, there is a one-to-one mapping from a pair of type distribution functions to a non-increasing function on the unit interval. The convenience of working with quantile-quantile plot is made explicit by the following lemma, where we show that, for any pair of

[7] The quantile-quantile plot is usually defined as $1 - t(r)$. We use the non-standard definition for convenience of notation.

type distribution functions, the difference in average type between the two organizations only depends on the integral of the associated quantile-quantile plot.[8]

LEMMA 1. *Let (H_A, H_B) be a pair of type distribution functions for A and B such that $H_A(\theta) + H_B(\theta) = 2\theta$, and t the associated quantile-quantile plot. Then*

$$m_A - m_B = -\frac{1}{2} + \int_0^1 t(r)\ dr.$$

PROOF. Using the definition of $t(r)$ and a change of variable $\theta = \inf\{\theta' : H_B(\theta') = r\}$, we can write

$$-\frac{1}{2} + \int_0^1 t(r)\ \mathrm{d}r = \frac{1}{2} - \int_{\underline{\theta}_B}^{\overline{\theta}_B} H_A(\theta)\ \mathrm{d}H_B(\theta),$$

where $\underline{\theta}_B = \sup\{\theta : H_B(\theta) = 0\}$ and $\overline{\theta}_B = \inf\{\theta : H_B(\theta) = 1\}$. Since $H_A(\theta) = 2\theta - H_B(\theta)$, the right-hand-side of the above equation is equal to

$$\frac{1}{2} - 2m_B + \int_{\underline{\theta}_B}^{\overline{\theta}_B} H_B(\theta)\ \mathrm{d}H_B(\theta).$$

The claim then follows immediately from the fact that $m_A + m_B = 1$. *Q.E.D.*

By Lemma 1, the difference in average types is the integral of the quantile-quantile plot minus $\frac{1}{2}$. For convenience, we will refer to the integral

$$T = \int_0^1 t(r)\ \mathrm{d}r$$

as the "quality difference." Since the agents' payoff function is linear, the function

$$P(T) = \frac{1}{\alpha}\left(T - \frac{1}{2}\right)$$

[8] The definition of quantile-quantile plot does not rely on the assumption that θ is distributed uniformly on $[0, 1]$. However, the representation of a pair of type distribution functions through its associated quantile-quantile plot is not generally useful because the difference in average type cannot be written as an integral of the quantile-quantile plot alone. If the distribution of types in the population is given by the function $F(\theta)$ and the population average type is $\overline{m}$, using the same argument as in Lemma 1 we have that

$$m_A - m_B = 2\left(\overline{m} - \frac{1}{2}\right) - \frac{1}{2} + \int_0^1 \left(t(r) - 2H_A^{-1}(1 - t(r)) + F(H_A^{-1}(1 - t(r)))\right) \mathrm{d}r.$$

Compared to the formula in Lemma 1, the constant term has changed to reflect the difference between the population mean and the mean of the uniform distribution, and the additional terms inside the integral sign correct for the difference between the type distribution F and the uniform distribution.

can be interpreted as the peer premium of A over B, in that any agent would be just indifferent between the two if the agent receives from B an amount of resource greater than what he receives from A by that premium.

Now we provide a characterization of a continuation equilibrium for given S_A and S_B in terms of a fixed point in the quality difference T. For any quality difference $T \in [0,1]$, let $\underline{t}^T$ and $\bar{t}^T$ be the quantile-quantile plots defined as

$$\underline{t}^T(r) = \begin{cases} 1 & \text{if } S_A(0) + P(T) > S_B(r), \\ 1 - \sup\{\tilde{r} \in [0,1] : S_A(\tilde{r}) + P(T) \leq S_B(r)\} & \text{otherwise;} \end{cases}$$

$$\bar{t}^T(r) = \begin{cases} 0 & \text{if } S_A(1) + P(T) < S_B(r), \\ 1 - \inf\{\tilde{r} \in [0,1] : S_A(\tilde{r}) + P(T) \geq S_B(r)\} & \text{otherwise.} \end{cases} \tag{2}$$

The functions $\underline{t}^T(r)$ and $\bar{t}^T(r)$, are the lower bound and the upper bound on the quantile-quantile plot consistent with a continuation equilibrium with quality difference T. The agent who has rank r in B must have rank at most $1 - \underline{t}^T(r)$ in A or he would prefer to switch; he must also have rank at least $1 - \bar{t}^T(r)$ in A or otherwise some agent from A would want to switch. Given (S_A, S_B), a quantile-quantile plot t corresponds to a continuation equilibrium if and only if it satisfies[9]

$$\underline{t}^T(r) \leq t^T(r) \leq \bar{t}^T(r) \text{ for all } r \in [0,1], \text{ and } T = \int_0^1 t(r)\, \mathrm{d}r.$$

Thus, a continuation equilibrium for any (S_A, S_B) is given by a quality difference $T \in [0,1]$ that is a fixed point of the correspondence

$$D(T) = \left[\int_0^1 \underline{t}^T(r)\, \mathrm{d}r, \int_0^1 \bar{t}^T(r)\, \mathrm{d}r\right]. \tag{3}$$

Existence of a fixed point follows from an application of Tarski's fixed point theorem.

To study the game in which the two organizations compete by choosing resource distribution schedules, we now assume that organization A is dominant in that the continuation equilibrium with the largest quality difference T is played in the sorting stage. Given any

[9] The "only if" part follows immediately by construction. The "if" part uses the assumption that each S_i is left-continuous. This characterization remains valid in a discrete model. Of course the assumption that each S_i is left-continuous would be unnecessary in such a model.

pair of resource distribution schedules, the largest quality difference T corresponds to the largest fixed point of the mapping

$$\overline{D}(T) = \int_0^1 \bar{t}^T(r)\ \mathrm{d}r, \tag{4}$$

and the equilibrium quantile-quantile plot is given by $\bar{t}^T$. We will refer to the continuation equilibrium with the largest T as the A-dominant continuation equilibrium, the selection criterion as the A-dominant criterion, and the resulting subgame perfect equilibrium as the A-dominant equilibrium.

4. Main Result: Equilibrium

In this section we demonstrate the main result of the paper that there exists an A-dominant equilibrium in which the resource-distribution schedules of both A and B are piece-wise linear. Taking as given a characterization of the equilibrium quality difference, we construct a pair of resource distribution schedules and verify that they are mutual best responses. Discussion and comparative statics analysis of the equilibrium follows the construction. In section 5 we provide a derivation of the equilibrium quality difference and show that it is unique, establishing that the equilibrium constructed in this section is the unique A-dominant equilibrium.

4.1. Equilibrium construction

The main result of this paper is that there is an A-dominant equilibrium in which the resource-distribution schedules of both A and B are piece-wise linear. Because of organization A's advantage, it is natural to expect that the equilibrium quality difference $m_A^* - m_B^*$ will be non-negative, or equivalently, $T^* \geq \frac{1}{2}$. The equilibrium quality difference T^* is defined by the largest T such that

$$Y_A = \frac{(Y_B - P(T)(1-T))^2}{2Y_B(1-T)}; \tag{5}$$

a derivation of this equilibrium value will be provided in section 5.2. It is straightforward to verify that $T^* = \frac{1}{2}$ if $Y_A = Y_B \geq 1/(2\alpha)$ and otherwise

$$T^* > 1 - \frac{2\alpha Y_B}{2\alpha Y_A + 1}. \tag{6}$$

The equilibrium resource distribution schedule of B is given by

$$S_B^*(r) = \begin{cases} 0 & \text{if } r \leq r_B^*; \\ P(T^*) + 2[Y_B - P(T^*)(1 - r_B^*)](r - r_B^*)/(1 - r_B^*)^2 & \text{if } r > r_B^* \end{cases} \tag{7}$$

where

$$r_B^* = 1 - \frac{2Y_B(1 - T^*)}{Y_B + P(T^*)(1 - T^*)}. \tag{8}$$

The equilibrium resource distribution schedule of A is given by

$$S_A^*(r) = \begin{cases} 0 & \text{if } r \leq r_A^*; \\ (r - r_A^*)S_B^*(1) & \text{if } r > r_A^*. \end{cases} \tag{9}$$

where

$$r_A^* = \frac{P(T^*)}{S_B^*(1)}. \tag{10}$$

See panel (a) of Figure 1 for an illustration of S_A^* and S_B^*. For convenience we use different origins O_A and O_B for the two schedules. Note that, by construction, S_A^* and S_B^* satisfy the resource constraint of organization A and B respectively. The following additional properties are immediate from the construction:

(i) For each $i = A, B$, there is a threshold rank r_i^* such that agents below that rank in i receive no resource.

(ii) Adjusted for the peer premium, the top rank agents receive the same resources and the threshold rank agents receive no resources from the two organizations, i.e., $S_A^*(1) = S_B^*(1) - P(T^*)$ and $S_A^*(r_A^*) = S_B^*(r_B^*) - P(T^*) = 0$.

(iii) For each $i = A, B$, the slope of S_i^* is constant for all ranks above r_i^* and further $S_A^{*'} = S_B^*(1)$.

Given S_A^* and S_B^*, the quantile-quantile plot $\bar{t}^{T^*}$ is obtained from (2) as

$$\bar{t}^{T^*} = \begin{cases} 1 & \text{if } r \leq r_B^*; \\ (1 - r)(1 - r_A^*)/(1 - r_B^*) & \text{if } r > r_B^*. \end{cases}$$

It is immediate to verify that the integral of $\bar{t}^{T^*}$ is exactly T^*, thus T^* is a continuation equilibrium given S_A^* and S_B^*. Panel (b) of figure 1 gives the equilibrium sorting of types, which is represented by a pair of distribution functions (H_A^*, H_B^*). The distribution of talents across the two organization takes a simple form. Types above a critical threshold

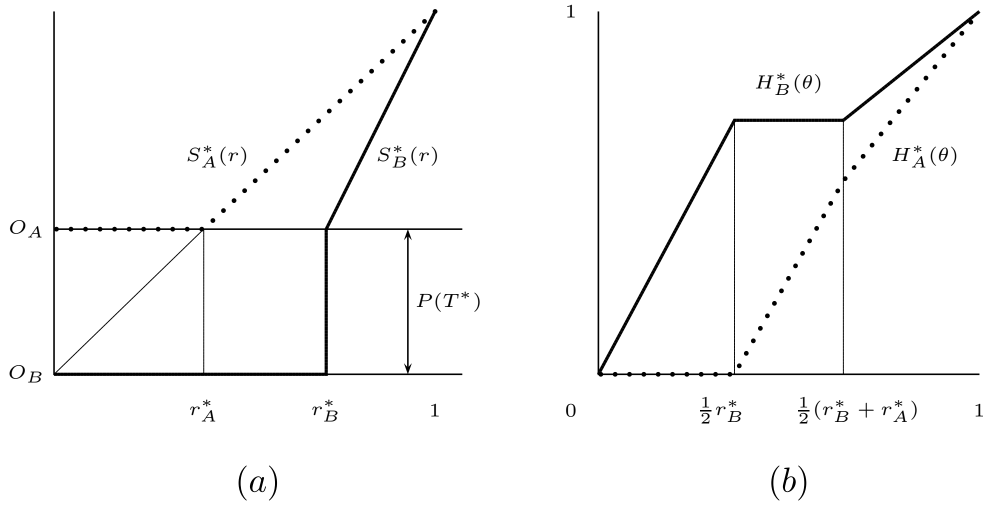

Figure 1

are indifferent and mix between the two organizations, with a larger share going to the dominant organization. Types below the critical threshold all prefer A to B and are completely segregated with the relatively higher types joining the dominant organization A and the remaining forced to join B. The critical threshold dividing the mixing and segregation regions depends on both the threshold ranks who receive resources in the two organizations and is given by $\frac{1}{2}(r_B^* + r_A^*)$. In the segregation region, the threshold that divides types joining A from those joining B is $\frac{1}{2}r_B^*$.

Proposition 1. *The strategy profile (S_A^*, S_B^*) forms an A-dominant equilibrium.*

Proof. Under the A-dominant criterion, to prove that S_B^* is a best response to S_A^*, it suffices to verify that, given S_A^*, for any S_B there is a continuation equilibrium with quality difference $T \geq T^*$. Given S_A^* and T^*, for any S_B we have, from the definition (equation 2),

$$\int_0^1 \bar{t}^{T^*}(r)\,\mathrm{d}r = 1 - \int_{r^0}^{r^1} S_A^{*-1}(S_B(r) - P(T^*))\,\mathrm{d}r,$$

where r^0 is the lowest rank in B that receives more resources than $P(T^*)$, and r^1 is the highest rank that receives less resources than $S_A^*(1) + P(T^*)$. By the construction of S_A^*,

$$S_A^{*-1}(S_B(r) - P(T^*)) = \frac{S_B(r)}{S_B^*(1)}.$$

Together with the resource constraint for B, the above expression implies

$$\int_0^1 \bar{t}^{T^*}(r)\,\mathrm{d}r \geq 1 - \frac{Y_B}{S_B^*(1)}.$$

Using equation (7) for the schedule S_B^* and eliminating r_B^* through equation (8), we can verify that

$$S_B^*(1) = \frac{2Y_B}{1 - r_B^*} - P(T^*) = \frac{Y_B}{1 - T^*}.$$

Therefore, the mapping (4) has one fixed point greater than or equal to T^*.

To show that S_A^* is a best response to S_B^*, fix any $T \geq T^*$. Suppose that some resource distribution schedule S_A achieves T against S_B^*. Then, we can assume $S_A(1) \leq S_B^*(1) - P(T)$. Let $\underline{r}$ be the infimum of ranks who receive a strictly positive wage in A, and $\overline{r}$ solve $S_A(1) = S_B^*(\overline{r}) - P(T)$. In the case when $S_A(r)$ is continuous and strictly increasing between $\underline{r}$ and 1, by definition we have that

$$t^T(r) = \begin{cases} 1 & \text{if } r < \underline{r}; \\ 1 - S_A^{-1}(S_B^*(r) - P(T)) & \text{if } r \in [\underline{r}, \overline{r}]; \\ 0 & \text{if } r > \overline{r}. \end{cases}$$

Integrating $t^T(r)$, we have that the quality difference at the resulting allocation is

$$\overline{r} - \int_{\underline{r}}^{\overline{r}} S_A^{-1}(S_B^*(r) - P(T))\,\mathrm{d}r.$$

After a change in variable $\tilde{r} = S_A^{-1}(S_B^*(r) - P(T))$ and integration by part we can rewrite the above as

$$\overline{r} - \frac{S_A(1) - Y_A}{S_B^{*\prime}} = r_B^* + \frac{Y_A}{S_B^{*\prime}} + \frac{P(T) - P(T^*)}{S_B^{*\prime}}.$$

When $S_A^{-1}(S_B^*(r) - P(T))$ is not defined for some $r \in (\underline{r}, \overline{r})$, the domain of the quantile-quantile plot $t^T(r)$ can be partitioned into intervals such that, in each interval, either $S_A^{-1}(S_B^*(r) - P(T))$ is defined for all r, or $t^T(r)$ is constant. The same manipulations as above lead to the exact same expression for the quality difference.

To conclude the proof that S_A^* is a best response to S_B^*, it suffices to show that no continuation equilibrium $T > T^*$ exists for the pair of schedules S_A, S_B^*. This follows if we show that the derivative with respect to T of the expression on the right-hand-side of the inequality above is strictly smaller than 1, or equivalently $\alpha S_B^{*'} > 1$. Using (7) and (8), we can show that $\alpha S_B^{*'} > 1$ is equivalent to

$$(\alpha Y_B)^2 - 2\alpha Y_B(1-T^*)^2 > \left(T^* - \frac{1}{2}\right)^2 (1-T^*)^2,$$

which follows from (6). *Q.E.D.*

The linearity of the constructed equilibrium schedules plays a crucial role in establishing that they are mutual best responses. The linearity property implies that for any distribution of types across the two organizations that is consistent with any given expected quality difference $T \geq T^*$, any given choice of S_A yields the same quality difference against the equilibrium schedule S_B^*.[10] An analogous result holds for any S_B given S_A^* and $T = T^*$, but requires in addition that the equilibrium construction satisfy $S_A^{*'} = S_B^*(1)$. These results are a consequences of the Raiding Lemma in section 5.1 which establishes the most effective way to raid a rival organization given its resource distribution schedule. Roughly speaking, it is optimal to target a rank in the rival organization if it has the smallest ratio of the resources needed to raid it to the difference between the rank and the highest rank requiring no resources to raid. Conversely, to effectively counter raiding by the rival, each organization must make the rival indifferent in targeting any paid rank. By construction, for the quality difference T^*, both S_A^* and S_B^* exhibit this indifference property, which allows us to establish that they are best responses to each other.

4.2. Comparative statics

When $Y_A = Y_B \geq 1/(2\alpha)$, the quality difference T^* given in equation (5) is equal to $\frac{1}{2}$, implying that the peer premium $P(T^*)$ is 0. From equations (8) and (10) we have that $r_B^* = r_A^* = 0$, and thus the two equilibrium schedules coincide. The equilibrium given in Proposition 1 is symmetric, even though our selection criterion has given the dominant organization A the greatest advantage allowed by the peer effect. This happens because the peer effect is too weak.

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[10] The only requirement is that S_A does not "waste resources," i.e., $S_A(1) \leq S_B^*(1) - P(T)$. An analogous requirement applies to S_B against S_A^*.

As long as the peer effect is sufficiently strong, or $Y_A > Y_B$, the equilibrium constructed in Proposition 1 is asymmetric, as illustrated in Figure 1. First, $r_A^* < r_B^*$, so that more ranks receive positive resources in organization A than in B. Second, $S_A^{*'} < S_B^{*'}$, so that S_A^* is flatter than S_B^* for ranks that receive positive resources in their respective organizations. Our model therefore suggests that organization resources are less concentrated at the top ranks in the dominant organization than in the weaker organization. This is perhaps not surprising, as the weaker organization must compensate for the peer premium $P(T^*)$ that results from the weaker peer effect compared to the dominant organization. The asymmetry in the resource distribution schedules between the two organizations in equilibrium is not due to the disparity in the resource budget. Even if $Y_B = Y_A$ and thus organization B faces no resource disadvantage, if the peer effect is strong relative to the resource budget, that is, if $\alpha Y_B < \frac{1}{2}$, under our selection criterion the dominant organization enjoys a peer premium and employs a schedule less concentrated in the top ranks.

Our model predicts both segregation and mixing in the A-dominant equilibrium. Low and intermediate types receive no resources in either organization, and thus segregate completely, with the low types left to join B and the intermediate types going to A and receiving the peer premium. In contrast, high types receive the same resources from A and B, adjusted for the quality difference, and they mix between A and B in a constant proportion determined by the slopes of the two equilibrium resource distribution schedules. The dominant organization A gets a greater share of the high types than B because S_A^* is flatter than S_B^*: starting from type $\frac{1}{2}(r_B^* + r_A^*)$ that is indifferent between joining A at rank r_A^* and joining B at rank r_B^*, a flatter schedule S_A^* in A means that the increase in the amount of resource received by each higher rank is smaller in A than in B, and thus the rank must grow faster with type in A than in B to maintain the indifference of each higher type between the two organizations. As a result, the dominant organization gets a greater share of the types that it competes for against the weaker organization.

For comparative statics analysis, the following proposition summarizes the effects on the equilibrium outcome of an increase in the relative importance of the peer effect (i.e., a decrease in α), or an increase in the resource gap between the two organizations[11] (i.e., a decrease in Y_B or an increase in Y_A). A proof is provided in the appendix.

[11] For the effects of a change in α, the proposition implicitly assumes that the equilibrium is asymmetric.

Proposition 2. *A decrease in α or Y_B, or an increase in Y_A, leads to: (i) a larger T^*; (ii) a higher r_B^*; (iii) steeper S_A^* and S_B^*; and (iv) a larger fraction of the types present in both organizations joining A. Furthermore, a decrease in α or Y_B leads to a higher r_A^*, and an increase in Y_A leads to a higher r_A^* if $T^* < \frac{3}{4}$ and a lower r_A^* if $T^* \geq \frac{3}{4}$.*

The proposition illustrates that an increase in the magnitude of the peer effects has qualitatively the same effects as an increase in the resources available to organization A, or a reduction in the resources available to B. For fixed resource distributions, a stronger peer effect induces more segregation, which favors the dominant organization. A widening of the resource gap between the two organizations further advantages the dominant organization. The larger equilibrium quality difference is accompanied by a reduction in the top ranks that receive positive resources as organization B must now focus its resources on fewer ranks to compensate for the loss in competitiveness. Competition for top talents becomes more fierce, resulting in steeper schedules. Finally, organization A, with its enhanced competitive position, captures a larger share of high types.

5. Uniqueness of Equilibrium

In this section we demonstrate that the equilibrium of Proposition 1 is the unique A-dominant equilibrium in our model of competition in resource-distribution schedules. Instrumental to establishing this result is the Raiding Lemma of section 5.1, which characterizes the minimum cost for organization A to obtain a continuation equilibrium with a given quality difference. This result is then used in section 5.2 to solve the problem of organization B choosing a resource distribution schedule to maximize that minimum cost, from which we show that in any equilibrium the quality difference is T^* given by (5), and B's resource distribution schedule is S_B^* given by (7) and (8). Section 5.3 completes the uniqueness of equilibrium claim by showing that S_A^* given by (9) and (10) is the only one distribution schedule for A to which S_B^* is a best response.

A small change in α would have no effects on the equilibrium when $Y_A = Y_B = Y$ and $\alpha Y > \frac{1}{2}$.

5.1. Optimal raiding

In this subsection, we put aside the resource constraint and solve the problem of minimizing the amount of resources needed by organization A to have a continuation equilibrium with some quality difference $T \geq \frac{1}{2}$ against a fixed resource distribution schedule S_B for organization B. The solution provides a characterization of the cheapest way for A to raid organization B for its talents. As a by-product, we also obtain a formula for the minimum amount of resources $C(T; S_B)$ that A needs to raid B and achieve T. For notational convenience, for fixed T and S_B define

$$\Delta(r) \equiv \max\{S_B(r) - P(T), 0\}$$

as the premium-adjusted resource distribution schedule of B. There are two cases of A's optimal raiding strategy $S_A(r)$.

(i) Organization A targets a single rank $\hat{r} = T$ in B. It does so by offering to all ranks in A an amount of resources $S_A(r) = \Delta(T)$, so that S_A is flat and all ranks in A are just indifferent between staying in A and switching to B for rank $\hat{r} = T$. The resulting continuation equilibrium exhibits segregation with types between $\frac{1}{2}T$ and $\frac{1}{2}(1+T)$ joining A and all other types joining B. This solves the resource minimization problem for A when rank T in B has the lowest resource-to-rank ratio $\Delta(r)/r$.

(ii) A targets two ranks r^1 and r^0 in B satisfying $r^1 < T < r^0$ with a schedule

$$S_A(r) = \begin{cases} \Delta(r^1) & \text{if } r \leq 1 - (T - r^1)/(r^0 - r^1); \\ \Delta(r^0) & \text{if } r > 1 - (T - r^1)/(r^0 - r^1). \end{cases} \tag{11}$$

That is, S_A is a step function with two levels, such that the lowest rank in A is indifferent between staying and joining B at rank r^1, and the lowest rank receiving the higher level of resources in A is indifferent between staying and joining B at rank r^0. In the resulting continuation equilibrium, types between $\frac{1}{2}r^1$ and $\frac{1}{2}(r^1 + (r^0 - T)/(r^0 - r^1))$, and types between $\frac{1}{2}(r^0 + (r^0 - T)/(r^0 - r^1))$ and $\frac{1}{2}(1 + r^0)$ join organization A, and all other types join B. Organization A achieves quality difference T by targeting two ranks in B, r^1 and r^0, and giving the minimum resources to its own ranks between 0 and $(r^0 - T)/(r^0 - r^1)$ so that they are indifferent between staying and joining B at rank r^1, and to all higher ranks the minimum resources so that they are indifferent between staying and joining B at rank r^0. This solves the resource minimization problem for A when ranks r^1 and r^0 in B have the lowest resource-to-rank ratios.

To establish the optimality of the above raiding strategies, we work with quantile-quantile plots. This transforms the problem of finding the cheapest S_A against S_B to achieve some T into a problem of finding the corresponding quantile-quantile plot. Note that if $S_B(T) < P(T)$, then by the definition we have that $\underline{t}^T(r) = 1$ for all $r \leq T$. That is, regardless of the resources received, all types prefer joining A to joining B at a rank lower than T. In this case, there might be no continuation equilibrium with quality difference T, and even if A gives no resources to all of its ranks, there is a continuation equilibrium with quality difference at least as large as T. We write $C(T; S_B) = 0$.

For the remainder of this subsection, we assume that $S_B(T) \geq P(T)$. Fix any quantile-quantile plot $t : [0,1] \to [0,1]$ and non-increasing, with $\int_0^1 t(r)\ \mathrm{d}r = T$ and $t(r) = 1$ for any r such that $S_B(r) < P(T)$. Next, we construct a function $S_A^t : [0,1] \to \mathbb{R}_+$ in two steps. First, we take the the point-wise smallest non-decreasing function that satisfies

$$S_A^t(1 - t(r)) \geq \Delta(r);$$

and then, we change the value of S_A^t at every discontinuity point to make the function left-continuous. Then given (S_A^t, S_B), we have $\underline{t}^T(r) \leq t(r) \leq \bar{t}^T(r)$ for all $r \in [0,1]$. It follows that T is a fixed point of the mapping (3) and hence there exists a continuation equilibrium with quality difference T for the pair of schedules (S_A^t, S_B). By construction, S_A^t is the resource distribution schedule with the minimum integral for which there is a continuation equilibrium with quantile-quantile plot t. It follows that

$$\begin{aligned} &C(T; S_B) = \min_t \int_0^1 S_A^t(r)\ \mathrm{d}r \quad \text{subject to} \\ &\quad t : [0,1] \to [0,1] \text{ non-increasing; } \int_0^1 t(r)\ \mathrm{d}r = T;\ t(r) = 1 \text{ if } S_B(r) < P(T). \end{aligned} \tag{12}$$

The following lemma, established in the appendix, characterizes a step function solution to (12).[12] This is achieved through a change of variables, so that the objective

[12] If S_B is a convex function, after the change of variables the solution to (12) can be obtained by dropping the monotonicity constraint on $t(r)$, using the "Bathtub principle" (see Lieb and Loss 2001) and then verifying that the solution satisfies the monotonicity constraint. In general the Bathtub principle cannot be applied directly due to the monotonicity constraint.

function in (12) becomes linear in the choice variable t.[13] By studying the resulting linear programming problem, we establish that there is always a step function that solves (12). In particular, if the quality-adjusted schedule Δ of B is locally concave in the neighborhood of some rank r, then a quantile-quantile plot that is flat around r does better than any decreasing plot; and if Δ is locally convex around r, then a t that is a step function with two values in the neighborhood of r does better. We then show that there exists a step function t that solves the linear programming problem with at most one value strictly between 0 and 1. This simple characterization is why we have chosen to deal with the quantile-quantile plot instead of a pair of type distribution functions directly. The linear programming nature of the minimization problem is a result of the two assumptions: the type distribution is uniform, and the agents' payoff function is linear. Without either assumption our quantile-quantile plot approach would not be analytically advantageous.

LEMMA 2. *There exists a solution t to (12) which assumes at most one value strictly between 0 and 1.*

Now we use the above lemma to provide an explicit characterization of the solution to (12) and a value for $C(T; S_B)$. The lemma implies that we can restrict the search for a solution to (12) to two cases. In the first case, consider a quantile-quantile plot t that has just one positive value. In this case t is entirely characterized by its only discontinuity point, say $\hat{r}$, because the constraint $\int_0^1 t(r)\ \mathrm{d}r = T$ and the assumption of $t(r) = 0$ for $r > \hat{r}$, imply that $t(r) = T/\hat{r}$ for all $r \leq \hat{r}$. Then, $S_A^t(r)$ equals $\Delta(r)$ for r strictly greater than $1 - T/\hat{r}$ and 0 otherwise. Note that $\hat{r} \geq T$ must hold in this case. In the second case, consider a quantile-quantile plot t that has one value strictly between 0 and 1, with two discontinuity points defined as $r^1 = \sup\{r : t(r) = 1\}$ and $r^0 = \sup\{r :\ t(r) > 0\}$. Using the constraint $\int_0^1 t(r)\ \mathrm{d}r = T$, we have $t(r) = (T - r^1)/(r^0 - r^1)$ for $r \in (r^1, r^0]$. Then, $S_A^t(r)$ is given by (11). Note that $r^1 \leq T \leq r^0$ in this case. Thus,

$$C(T; S_B) = \min\left\{\min_{T \leq \hat{r} \leq 1} \frac{T}{\hat{r}}\Delta(\hat{r}),\ \min_{0 \leq r^1 \leq T \leq r^0 \leq 1} \frac{r^0 - T}{r^0 - r^1}\Delta(r^1) + \frac{T - r^1}{r^0 - r^1}\Delta(r^0)\right\}. \quad (13)$$

[13] For a general distribution of types, the objective function in (12) remains unchanged, and can be rewritten so that it is linear in t. However, as remarked in Footnote 6, the constraint that the difference in mean type between the two organization be equal to T is no longer linear in t.

Let $\underline{\Delta}$ be the largest convex function with $\underline{\Delta}(0) = 0$ which is point-wise smaller than or equal to Δ. In other words, $\underline{\Delta}$ is obtained as the lower contour of the convex hull of the function Δ and the origin:

$$\underline{\Delta}(r) = \min\left\{y : \ (r, y) \in co\left(\{(\tilde{r}, \tilde{y}) : 0 \leq \tilde{r} \leq 1; \tilde{y} \geq \Delta(\tilde{x})\} \cup (0,0)\right)\right\}.$$

The next lemma provides a simple characterization of the discontinuity points of a solution to (12) which depends only on the functions Δ and $\underline{\Delta}$. In particular, if $\Delta(T) = \underline{\Delta}(T)$, then there is a solution to (12) with only one discontinuity point at exactly T. The solution t is a step function equal to 1 for $r \leq T$ and equal to 0 for $r > T$. When $\Delta(T) > \underline{\Delta}(T)$ instead, there is a solution t to (12) such that t has two discontinuity points $r^1 < T$ and $r^0 > T$, corresponding to the largest rank below T and the smallest above T at which Δ coincides with $\underline{\Delta}$. The solution t equals 1 up to r^1 and 0 after r^0.

Lemma 3. (The Raiding Lemma) *Let $Q = \{r : \Delta(r) = \underline{\Delta}(r)\} \cup \{0, 1\}$, and denote as $\overline{Q}$ the closure of Q. (i) If $T \in \overline{Q}$, then the following quantile-quantile plot with one discontinuity point at $\hat{r} = T$ solves (12):*

$$t(r) = \begin{cases} 1 & \text{if } r \leq T; \\ 0 & \text{otherwise.} \end{cases}$$

(ii) If $T \notin \overline{Q}$, the following quantile-quantile plot with two discontinuity points given by $r^1 = \sup\{r \in \overline{Q} : \ r < T\}$ and $r^0 = \inf\{r \in \overline{Q} : \ r > T\}$ solves (12):

$$t(r) = \begin{cases} 1 & \text{if } r \leq r^1; \\ (T - r^1)/(r^0 - r^1) & \text{if } r^1 < r \leq r^0; \\ 0 & \text{if } r > r^0. \end{cases}$$

In both cases, the value of the objective function is $C(T; S_B) = \underline{\Delta}(T)$.

Proof. First we show that the objective function assumes the value $\underline{\Delta}(T)$ at the claimed solution in both cases. In the first case, at the claimed solution, the value of the objective function is $\Delta(T)$, which is equal to $\underline{\Delta}(T)$ because $T \in \overline{Q}$. In the second case, when $T \notin \overline{Q}$, at the claimed solution the value of the objective function is given by

$$\frac{r^0 - T}{r^0 - r^1}\Delta(r^1) + \frac{T - r^1}{r^0 - r^1}\Delta(r^0) = \frac{r^0 - T}{r^0 - r^1}\underline{\Delta}(r^1) + \frac{T - r^1}{r^0 - r^1}\underline{\Delta}(r^0) = \underline{\Delta}(T),$$

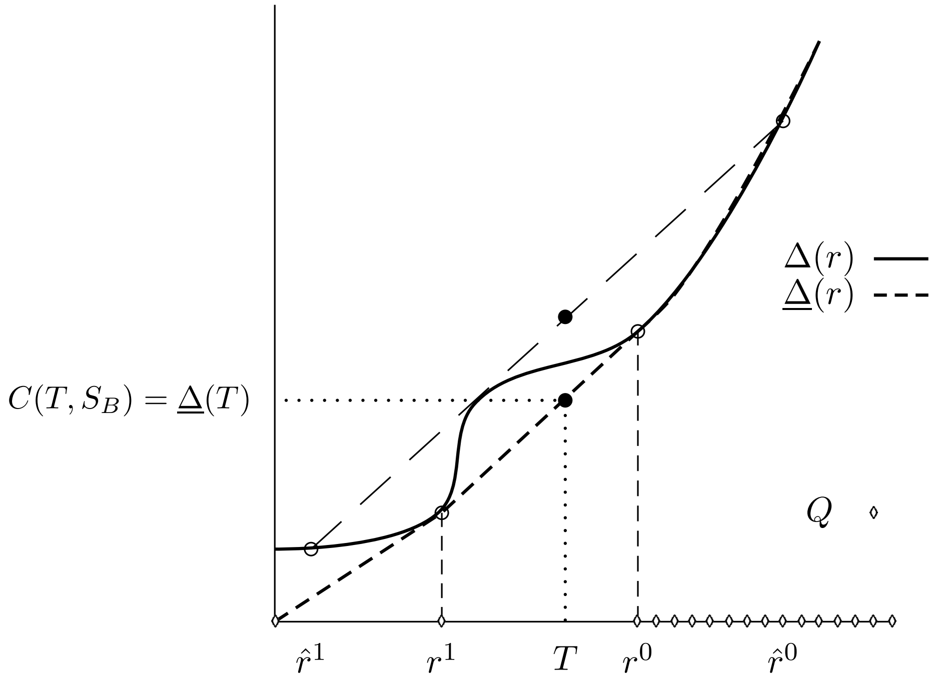

Figure 2

where the second equality holds because $\underline{\Delta}$ is linear between r^1 and r^0.

Next, we show that (13) is never smaller than $\underline{\Delta}(T)$. For all $r \geq T$, we have

$$\underline{\Delta}(T) \leq \frac{r-T}{r}\underline{\Delta}(0) + \frac{T}{r}\underline{\Delta}(r) \leq \frac{T}{r}\Delta(r),$$

where the first inequality follows from the fact that $\underline{\Delta}$ is convex and the second from $\underline{\Delta}(r) \leq \Delta(r)$ for all r. Similarly, for all $r^1 \leq T$ and $r^0 \geq T$, we have

$$\underline{\Delta}(T) \leq \frac{r^0-T}{r^0-r^1}\underline{\Delta}(r^1) + \frac{T-r^1}{r^0-r^1}\underline{\Delta}(r^0) \leq \frac{r^0-T}{r^0-r^1}\Delta(r^1) + \frac{T-r^1}{r^0-r^1}\Delta(r^0).$$

Thus the claimed solution minimizes (13). *Q.E.D.*

Figure 2 illustrates the second case of the above lemma. The quality difference to be achieved for organization A is T. The discontinuity points of the quantile-quantile plot in the statement of the lemma, r^0 and r^1, are identified in the diagram. For any other quantile-quantile plot with discontinuity points $\tilde{r}^0$ and $\tilde{r}^1$, the resource requirement for the plot to be a continuation equilibrium is greater. This is because from (13) the resource requirement is equal to a weighted average of the quality-adjusted resource schedule of B

at $\tilde{r}^1$ and $\tilde{r}^0$, with the weights such that the average of $\tilde{r}^1$ and $\tilde{r}^0$ equals T. Similarly, for any quantile-quantile plot with a single discontinuity point, say $\hat{r}$, the resource requirement for the plot to be a continuation equilibrium is also greater, because it is an average of zero and the quality-adjusted resource schedule of B at $\hat{r}$, with the weights such that the average of zero and $\hat{r}$ equals T.

5.2. Equilibrium quality difference

In this subsection, we use the Raiding Lemma to show that in any A-dominant equilibrium the quality difference is uniquely given by T^* specified as in (5). For each $T \geq \frac{1}{2}$, define the budget function that gives the minimum total resources that A needs to achieve a given quality difference target T as the value of the following optimization problem:

$$E(T) \equiv \max_{S_B} C(T; S_B) \quad \text{subject to} \quad \int_0^1 S_B(r) \, \mathrm{d}r \leq Y_B. \tag{14}$$

The result in this subsection is that T^* given as in (5) satisfies

$$T^* = \max \left\{ T \in \left[\frac{1}{2}, 1\right] \,:\, E(T) = Y_A \right\}, \tag{15}$$

and it is the quality difference in any A-dominant equilibrium. As a by product, we also show that in any A-dominant equilibrium B's resource distribution schedule is uniquely given by S_B^* specified in (7) and (8).

First, we characterize the resource distribution schedule S_B that maximizes the minimum amount of resources needed by organization A to have a continuation equilibrium with quality difference T. The Raiding Lemma suggests that organization B would be wasting its resources if it chooses a schedule S_B such that $\Delta(r) > \underline{\Delta}(r)$ for some r. Our next result shows that there is a solution S_B to (14) such that $S_B(r)$ is 0 for all r up to some threshold $\tilde{r}$, and takes a linear form with intercept $P(T)$ between $\tilde{r}$ and 1.

Lemma 4. *Given any resource distribution schedule S_B that satisfies the resource constraint, there exists another resource distribution schedule $\tilde{S}_B$ given by*

$$\tilde{S}_B(r) = \begin{cases} 0 & \text{if } r \leq \tilde{r}, \\ P(T) + \beta(r - \tilde{r}) & \text{if } r > \tilde{r}; \end{cases}$$

for some $\tilde{r} \in [0, 1]$ and $\beta \geq 0$, such that $\int_0^1 \tilde{S}_B(r) \, \mathrm{d}r = Y_B$ and $C(T; \tilde{S}_B) \geq C(T; S_B)$.

PROOF. Let $\Delta_{S_B}(r) = \max\{S_B(r) - P(T), 0\}$. First, since $C(T; S_B)$ only depends on $\Delta_{S_B}(r)$, by changing into zero the value of $S_B(r)$ whenever $S_B(r) < P(T)$, the value of $C(T; S_B)$ is unchanged. Second, by the Raiding Lemma, $C(T; S_B) = \underline{\Delta}_{S_B}(T)$ for any resource distribution schedule S_B, implying that $C(T; S_B) = C(T; \tilde{S}_B)$ for any $\tilde{S}_B$ such that $\Delta_{\tilde{S}_B} = \underline{\Delta}_{S_B}$. Thus for any S_B, there is a resource distribution schedule $\tilde{S}_B$ which is convex whenever positive and $\Delta_{\tilde{S}_B}(0) = 0$ such that $C(T; \tilde{S}_B) \geq C(T; S_B)$. Finally, for any S_B that is convex whenever positive and $\Delta_{S_B}(0) = 0$, there is a $\tilde{S}_B$ which is linear when positive such that $C(T; \tilde{S}_B) \geq C(T; S_B)$. The lemma then immediately follows from the resource constraint because binding the constraint increases the resource requirement for A. *Q.E.D.*

The result of Lemma 4 greatly simplifies the solution to (14). First, resource distribution schedules of the form in the lemma are entirely characterized by their discontinuity point $\tilde{r}$, which determines the slope of $S_B(r)$ to be

$$\beta = \frac{2}{(1-\tilde{r})^2}[Y_B - P(T)(1-\tilde{r})].$$

Further, the quality-adjusted resource distribution schedule of B is 0 for all $r \leq \tilde{r}$ and equal to $\beta(r - \tilde{r})$ for all $r > \tilde{r}$, thus from the Raiding Lemma $C(T; S_B) = \beta(T - \tilde{r})$ for all $\tilde{r} < T$, and $C(T; S_B) = 0$ otherwise. The value of any solution to (14), $E(T)$, is simply obtained by choosing the discontinuity point $\tilde{r} = r(T)$ that maximizes $\beta(T - \tilde{r})$, which is easily shown to be given by

$$r(T) = \begin{cases} 1 - 2Y_B(1-T)/[Y_B + P(T)(1-T)] & \text{if } Y_B > P(T)(1-T); \\ T & \text{otherwise.} \end{cases} \tag{16}$$

If organization B's resource budget is not enough to cover the peer premium $P(T)$ for all ranks above T, then B is not competitive against A at quality difference T.[14] Provided that this is not the case, $r(T)$ is increasing in T, with $r\left(\frac{1}{2}\right) = 0$ and $r(1) = 1$. Thus, the optimal way for B to deter A from achieving a greater quality difference in A's favor is to concentrate more of its resources to reward its higher-rank members.

[14] In this case, there is a continuation equilibrium with a quality difference at least as large as T even if A pays nothing to all its ranks. We write $E(T) = 0$.

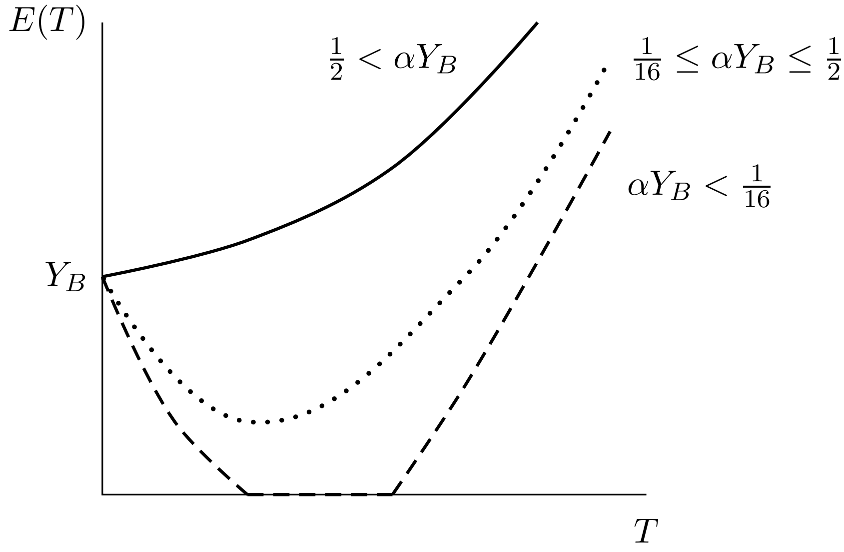

Figure 3

From the characterization of $r(T)$ it is immediate to obtain an explicit form for the resource budget function as

$$E(T) = \begin{cases} [Y_B - P(T)(1-T)]^2/[2Y_B(1-T)] & \text{if } Y_B > P(T)(1-T); \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

The following result characterizes the properties of this resource budget function; the proof is in the appendix.

Lemma 5. *The resource budget function $E(T)$ satisfies $\lim_{T\to 1} E(T) = \infty$ and $E\left(\frac{1}{2}\right) = Y_B$. Moreover, (i) if $\alpha Y_B > \frac{1}{2}$, then $E'(T) > 0$ for all $T \ge \frac{1}{2}$; (ii) if $\alpha Y_B \in \left[\frac{1}{16}, \frac{1}{2}\right]$, then there exists a $\hat{T}$ such that $E'(T) < 0$ for $T \in \left(\frac{1}{2}, \hat{T}\right)$ and $E'(T) > 0$ for $T \in (\hat{T}, 1)$; and (iii) if $\alpha Y_B < \frac{1}{16}$, then there exist T_- and T_+ such that $E'(T) < 0$ for $T \in \left(\frac{1}{2}, T_-\right)$, $E(T) = 0$ for $T \in [T_-, T_+]$ and $E'(T) > 0$ for $T \in (T_+, 1)$.*

See Figure 3 for the three different cases of $E(T)$. An increase in the target quality difference T has two opposite effects on the resource budget $E(T)$ required for organization A to achieve the target in a continuation equilibrium. On one hand, to achieve a greater quality difference T organization A must be competitive with more ranks in B and this requires a larger budget. On the other hand, a greater T also increases the peer premium that A enjoys over B and this reduces the resource requirement. The first effect dominates when the peer effect is relatively small, which happens when either α or Y_B is large. This explains why $E(T)$ is monotonically increasing in T when αY_B is greater than $\frac{1}{2}$. In contrast, the peer effect is strong and $E(T)$ may decrease when αY_B is small. Indeed, the resource requirement to achieve some intermediate values of T can be zero because organization B's resource budget Y_B is not even sufficient to cover the peer premium $P(T)$ for all its ranks above T. Note that for $T = \frac{1}{2}$, the quality difference is zero, and by adopting a linear resource distribution schedule for all its ranks, B can make sure that A needs at least the same amount of resources. Finally, for sufficiently large T, the resource budget function must be increasing. This is because by concentrating its resources on a few top ranks organization B can make it increasingly costly for A to achieve large quality differences. Indeed, it is impossible for A to induce the perfect segregation of types in the sorting stage regardless of the resource advantage it has, because B could give all its limited resource to an arbitrarily small number of its top ranks.

As an immediate implication of Lemma 5, equation (17) implies that T^* given in (5) satisfies (15). We claim that under the A-dominant criterion, T^* is the lower bound on the equilibrium quality difference in the game of organization competition. To see this, note that $C(T^*; S_B) \leq E(T^*) = Y_A$ for any S_B. Thus there is always a resource distribution schedule S_A that satisfies A's resource constraint and such that under (S_A, S_B) there is a continuation equilibrium with quality difference $T \geq T^*$. Further, from $Y_A \geq Y_B$ and the characterization of $E(T)$ in Lemma 5, we have that $T^* \geq \frac{1}{2}$, with equality if and only if $Y_A = Y_B \geq 1/(2\alpha)$. Finally, since $Y_B > P(T^*)(1-T^*)$, we immediately have that r_B^* given in equation (8) satisfies $r_B^* = r(T^*)$ where the function $r(\cdot)$ is given by (16). It follows that the wage schedule $S_B^*(r)$ given by (7) achieves $E(T^*)$. The following proposition verifies that T^* is also the upper bound of the equilibrium quality difference, hence the quality difference in any A-dominant equilibrium.[15]

Proposition 3. *In any A-dominant equilibrium of the organization competition game, the quality difference is T^* given by (5).*

[15] Alternatively we can define a zero-sum "resource distribution game" in which the two organizations simultaneously choose their resource distribution schedules. For any (S_A, S_B), the payoff to organization A is the quality difference $\overline{T}(S_A, S_B)$ in the A-dominant continuation equilibrium. Then T^* corresponds to the minmax value, or $\min_{S_B} \max_{S_A} \overline{T}(S_A, S_B)$, and S_B^* corresponds to the solution.

Proof. From the Raiding Lemma, we have $C(T; S_B^*) = S_B^*(T) - P(T)$. Since S_B^* is linear, $C(T; S_B^*)$ is also linear in T. Since $C(T^*; S_B^*) = Y_A$ by definition, we have $C(T; S_B^*) > Y_A$ for all $T > T^*$ if and only if $C(T; S_B^*)$ has a positive slope in T. Since S_B^* solves the maximization problem (14), by the envelope theorem, the derivative of $C(T; S_B^*)$ with respect to T at T^* is equal to $E'(T)$ at $T = T^*$, which is positive by Lemma 5 and (5).

Q.E.D.

When organization B chooses resource distribution schedule S_B^*, in order to achieve the quality difference T^* organization A must expend all its available resources. However, this may fail to guarantee that a larger quality difference is out of reach for A, because a larger T increases the peer premium and frees some resources for A. The above proposition establishes that given S_B^*, this peer effect is dominated by the additional resource requirement for obtaining a greater quality difference than T^*. This is because, by the Raiding Lemma, the minimum resource budget $C(T; S_B^*)$ required for A to achieve any quality difference T is linear in T, with a slope equal to $E'(T)$ at $T = T^*$ by the envelope theorem. Even though the resource budget function $E(T)$ may be decreasing, from Figure 3, it is necessarily increasing at $T = T^*$. Since $C(T^*; S_B^*) = E(T^*) = Y_A$, the result that $C(T; S_B^*)$ is a positively-sloped linear function in T implies that by choosing S_B^*, organization B ensures that the minimum resource budget required to achieve any quality difference greater than T^* is strictly greater than Y_A, and hence T^* is an upper bound on the quality difference in any A-dominant equilibrium.[16]

5.3. Equilibrium distribution schedules

In this subsection, we establish that S_A^* and S_B^* are the unique A-dominant equilibrium resource distribution reschedules for A and B respectively. By Proposition 3, in any A-dominant equilibrium, B's resource distribution schedule is given by S_B^*. Our first step is to establish that the condition for A not to overpay its top rank, or $S_A(1) \leq S_B^*(1) - P(T^*)$, is necessary for S_A to be an equilibrium resource distribution schedule.

[16] The value of T^* also provides an upper bound to the quality difference between the two organizations in any subgame perfect equilibrium (without the A-dominance selection criterion). From the characterization of Lemma 5 we have that T^* is always strictly smaller than 1, thus perfect segregation of types is never an equilibrium outcome.

LEMMA 6. *If S_A is an A-dominant equilibrium resource distribution schedule for A, then $S_A(r) \le S_B^*(1) - P(T^*)$ for all $r \le 1$.*

PROOF. Suppose

$$\underline{r} \equiv \inf\{r \in [0,1] : S_A(r) > S_B^*(1) + P(T^*)\} < 1.$$

Consider a modified resource distribution schedule $\tilde{S}_A$ that coincides with $S_A(r)$ except $\tilde{S}_A(r) = S_B^*(1) + P(T^*)$ for all $r \in (\underline{r}, 1]$. Then, for any $r, \tilde{r} \in [0,1]$, we have that $\tilde{S}_A(r) \ge S_B^*(\tilde{r}) - P(T^*)$ whenever $S_A(r) \ge S_B^*(\tilde{r}) - P(T^*)$. By the definition of $\bar{t}^{T^*}$ (equation 2), T^* is still a fixed point of the mapping $\overline{D}(T)$ under $(\tilde{S}_A, S_B^*)$. Since by construction $\int_0^1 \tilde{S}_A(r)\ \mathrm{d}r < \int_0^1 S_A(r)\ \mathrm{d}r$, there exists some other resource distribution schedule of A which satisfies the resource constraint and, against S_B^*, yields a fixed point $\tilde{T}$ of $\overline{D}(T)$ that is strictly greater than T^*. Thus S_A is not a best response to S_B^*.

Q.E.D.

The next step in establishing the uniqueness of (S_A^*, S_B^*) is to show that any equilibrium schedule S_A must be linear when positive. Otherwise, if $S_A(r)$ is locally convex around some rank $\hat{r}$ with $S_A(\hat{r}) > 0$, then organization B can modify S_B^* by identifying a rank $\tilde{r}$ that competes with $\hat{r}$, i.e., $\tilde{r}$ satisfying $S_B^*(\tilde{r}) = S_A(\hat{r}) + P(T^*)$, and flattening S_B^* around $\tilde{r}$ in a way that still satisfies the resource budget. Analogous to the argument used in establishing Lemma 2, this modification lowers the mapping $\overline{D}(T)$ at T^*, and thus T^* is no longer a continuation equilibrium quality difference.[17] However, in contrast to Lemma 2, under the A-dominant criterion, we also need to show that the modification does not create a larger fixed point than T^*. This additional step is accomplished by making the modification around $\tilde{r}$ sufficiently small. Similarly, if $S_A(r)$ is locally concave at $\hat{r}$, B can identify a rank $\tilde{r}$ that competes with $\hat{r}$ and replace S_B^* with a step function of two values in a sufficiently small neighborhood of $\tilde{r}$. The proof of Lemma 7 is in the appendix.

[17] While Lemma 2 looks at the solution in quantile-quantile plot to (12) that minimizes the budget required for A to achieve a given quality difference T, the next lemma modifies the resource distribution schedule to reduce the integral of the quantile-quantile plot while satisfying the resource constraint for B. A step function solution in t in Lemma 2 corresponds to a step function in schedule S_B.

Lemma 7. *If S_A is an A-dominant equilibrium resource distribution schedule for A, then S_A is a linear function in the range of r where $S_A(r)$ is positive.*

The equilibrium resource distribution schedule S_A^* constructed in Proposition 1 is linear above rank r_A^*, which is uniquely identified by connecting the starting point and the end point of S_B^*. As we have seen, S_A^* satisfies the resource constraint of A. By the above two lemmas, any candidate equilibrium schedule for A must not overpay its top rank, and is linear whenever it is positively valued. Since a candidate equilibrium schedule for A must satisfy the resource constraint, the uniqueness of S_A^* as an equilibrium schedule for A follows immediately once we show that if S_A is an A-dominant equilibrium resource distribution schedule for A, then (i) $S_A(1) \geq S_B^*(1) - P(T^*)$ so that B is not overpaying its top rank, and (ii) S_A is continuous at the threshold $\hat{r}$ defined as $\inf\{r \in [0,1] : S_A(r) > 0\}$ so that there is no jump at the lowest rank that receives a positive amount of resources. For the first property, if $S_A(1) < S_B^*(1) - P(T^*)$, then B can modify S_B^* by reducing the amount of resources for ranks around the very top rank 1, and redistributing the resources to ranks just below r such that $S_B^*(r) - P(T^*) = S_A(1)$. With a similar argument as in the proof of Lemma 7, we can show that, for a sufficiently small modification, B can decrease the largest fixed point of the mapping $\overline{D}(T)$. For the second property, if S_A is discontinuous at $\hat{r}$, then B can modify S_B^* by reducing the amount of resources for ranks just above r_B^*, and redistributing the resources to ranks around the very top rank 1. This modification reduces the the mapping $\overline{D}(T)$ for all $T \geq T^*$. Thus, if either of the two properties is not satisfied, S_B^* is not a best response for B. The following proposition immediately follows.

Proposition 4. *In the unique A-dominant equilibrium, the resource distribution schedules for A and B are S_A^* and S_B^*.*

The equilibrium resource distribution schedule S_A^*, as well as S_B^*, has the property that the premium-adjusted resource-to-rank ratio is constant for ranks that receive positive resources. As demonstrated in the proof of Lemma 7, the linear feature of our model, embodied in the uniform type distribution and in the linear payoff function for the agents, plays a critical role in the uniqueness result. Proposition 4 strengthens the result from

Proposition 3 that the equilibrium quality difference is unique. Thus, the resource distributions in the two organizations and the sorting of types described in section 4.2 are necessary implications of the A-dominant equilibrium.

6. Discussion

For a fixed pair of increasing resource distribution schedules, the payoff specification in (1) captures the trade-off facing the agent between relative rank and average type when choosing between the two organizations. In Damiano, Li and Suen (2010), we study a more general model of this trade-off with exogenously fixed concern for relative ranking, with different focuses on competitive equilibrium implementation and welfare implications. In terms of the model presented here, the benchmark model of Damiano, Li and Suen (2010) assumes that the resource distribution schedules are fixed at $S_A(r) = S_B(r) = r$ with $Y_A = Y_B = \frac{1}{2}$. The concern of an agent for relative ranking in the organization he joins, referred to as "pecking order effect," may be motivated by self-esteem (Frank, 1985) or competition for mates or resources (Cole, Mailath and Postlewaite, 1992; Postlewaite, 1998). The equilibrium pattern of mixing and segregation characterized in Damiano, Li and Suen (2010) is of an "overlapping interval" structure, where the very talented are captives in the high quality organization and the least talented are left to the low quality organization, while the intermediate talents are present in both. In the present model instead, agents do not directly care about their relative ranking in the organization. The concern for relative ranking is generated endogenously because the organizations choose how to distribute resources according to ranks. The equilibrium sorting pattern, given in Proposition 1, is qualitatively different from an overlapping interval structure, with top talents mixing between the two organizations and lower types perfectly segregated. Thus, mixing occurs for the intermediate types and high types are captives when the trade-off between the peer effect and the pecking order effect does not respond to organizational choices as in Damiano, Li and Suen (2010), while under strategic competition between organization, the high types are the only ones receiving resources and mix between organizations.

We have restricted organization strategies to resource distributions that do not depend on type directly. This is a simple way of ensuring that the resource constraint is satisfied. To relax this assumption and allow the organizations to post type-dependent resource distribution schedules, we must make additional assumptions about how resources are rationed when the distribution of types that joins an organization is such that the total amount of resources promised exceeds the organization's exogenous constraint. A natural assumption is that the resource constraint is satisfied by rationing the lowest types, that is, by giving no resources to sufficiently many low types to satisfy the organization's resource constraint.[18] This implies that the amount of resources received by a type θ joining organization $i = A, B$ will depend on both the type-dependent distribution schedule posted by i as well as the entire distribution of types joining the same organization. Precisely, if σ_i is the type-dependent schedule posted by i and H_i the distribution of types joining organization i, the amount of resources received by type θ in i, $\tilde{\sigma}_i(\theta)$, equals the promised resources $\sigma_i(\theta)$ if $\int_\theta^1 \sigma_i(\theta)\mathrm{d}H_i(\theta) \leq Y_i$ and is zero otherwise. The definition of continuation equilibrium remains unchanged after redefining the payoff to a type θ who joins organization i as $V_i(\theta) = \alpha\tilde{\sigma}_i(\theta) + m_i$. In this model, we can show that our equilibrium is robust to deviations in type-dependent schedules. That is, at the equilibrium quality difference and against the equilibrium resource distribution schedule of the rival organization, each organization cannot improve its quality by deviating to a resource distribution schedule that depends on types. To start, observe that any continuation equilibrium following a deviation in type-dependent schedule is also a continuation equilibrium for some deviating schedule that depends on ranks only.[19] Under our selection criterion, for the dominant organization A we immediately have the robustness result that it cannot profit from using any type-dependent schedule instead of its equilibrium rank-dependent schedule S_A^*. For B, the argument is more involved due to the general multiplicity of continuation equilibria, so we need a direct argument to show that there is no profitable deviation for B.[20]

[18] We thank an anonymous referee for this suggestion.

[19] This follows because if σ_i is a type-dependent deviation by organization i and (H_A, H_B) is a continuation equilibrium following such deviation, then it is also a continuation equilibrium if i posts a rank-dependent schedules S_i such that $S_i(H_i(\theta)) = \tilde{\sigma}_i(\theta)$ for all θ.

[20] Even though any continuation equilibrium after a supposedly profitable deviation to some type-dependent schedule σ_B can be replicated by a rank-dependent schedule S_B, there may be another continuation equilibrium with a greater quality difference under (S_A^*, S_B) that makes it unprofitable for B to deviate from S_B^* to S_B.

Consider the counterpart of the optimal raiding problem (12) in section 5.1 of minimizing the total amount of resources needed for B to use a type-dependent schedule σ_B to achieve the equilibrium quality difference T^*:

$$\begin{aligned} &\min_t \int_0^1 \sigma_B^t(\theta)\ \mathrm{d}H_B^t(\theta) \\ &\text{subject to } t:[0,1]\to[0,1] \text{ non-increasing; and } \int_0^1 t(r)\ \mathrm{d}r = T^*, \end{aligned} \tag{18}$$

where H_B^t is the type distribution function in organization B that is implied by the quantile-quantile plot t, and σ_B^t is the point-wise smallest non-decreasing function that satisfies $\sigma_B^t({H_B^t}^{-1}(r)) \geq S_A^*(1-t(r)) + P(T^*)$ for all r. Following the same steps of Lemma 1 and the Raiding Lemma, and using the equilibrium construction of Proposition 1, we can show that the value of the above minimization problem is equal to Y_B. Thus, there is no profitable type-dependent deviations for B either. The key intuition is that the choice variable is a quantile-quantile plot in the cost minimization problem (18). The amount of resources needed to attract a given quality difference is the same whether a rank-dependent or type-dependent schedule is used, and thus the value of the minimization problem (18) is the same as when B uses a rank-dependent schedule.

Another assumption we have made about organization strategies is that they are meritocratic, that is, resource distribution schedules are non-decreasing in rank. This is a natural assumption given how we model the sorting of talents after organizations choose their schedules, since non-meritocratic resource distribution schedules would create incentives for talented agents to "dispose of" their talent. More importantly, as in the case of type-dependent schedules, we can show that the equilibrium in Proposition 1 is robust to deviations of either organization to schedules that are decreasing for some ranks. Again, the definition of continuation equilibrium remains intact even when we allow non-monotone schedules. Furthermore, in a similar cost minimization problem as (18), with the constraint of $\sigma_B^t(r) \geq S_A^*(1-t(r)) + P(T^*)$ for all r, the choice variable is a quantile-quantile plot. Given the equilibrium schedule S_A^* and the quality difference T^*, the value of this minimization problem is the same as when B is restricted to using monotone schedules.

Organizations in our model have a fixed capacity of half of the talent pool and must fill all positions. In particular, an organization cannot try to improve its average talent by rejecting low types even though the capacity is not filled. We have made this assumption in order to circumvent the issue of size effect, and focus on implications of sorting of talents. We may justify the assumption of fixed capacity if the peer effect enters the preferences of talents in the form of total output (measured by the sum of individual types) as opposed to the average type, and the objective of the organization is to maximize the total output. Since all agents contribute positively to the total output, in this alternative model all positions will be filled.

Our main results of linear resource distribution schedules rely on the assumption of uniform type distribution. This assumption implies that the impact on the quality difference of an exchange of one interval of types for another interval between the two organizations depends only on the difference in the average types of the two intervals. Together with the assumption that the payoff functions of agents are linear, this property allows us to transform the problem of finding the optimal response in resource distribution schedules to a linear programming problem in quantile-quantile plots, and characterize the solution in the Raiding Lemma. We leave the question of whether the method we develop here is applicable to more general type distributions and payoff functions to future research.

Appendix

Proof of Proposition 2. In the proof of Proposition 1, we have established that

$$Y_A = (S_B^*(1) - P(T^*))\frac{T^* - r_B^*}{1 - r_B^*}, \tag{A.1}$$

and

$$S_B^*(1) = \frac{2Y_B}{1 - r_B^*} - P(T^*) = \frac{Y_B}{1 - T^*}. \tag{A.2}$$

Then, combining equations (10) and (A.2), we can write the threshold rank r_A^* in organization A as

$$r_A^* = \frac{P(T^*)}{S_B^*(1)} = \frac{P(T^*)(1 - T^*)}{Y_B}. \tag{A.3}$$

For B, the threshold r_B^* satisfies (8). Using equation (A.2), we can write the slope of S_A^* for ranks above r_A^* as

$$S_A^{*\prime} = S_B^*(1) = \frac{Y_B}{1 - T^*}. \tag{A.4}$$

For B, the slope of S_B^* for ranks above r_B^* satisfies

$$S_B^{*\prime} = \frac{S_B^*(1) - P(T^*)}{1 - r_B^*} = \frac{Y_A}{T^* - r_B^*} = \frac{1 - r_A^*}{1 - r_B^*} \frac{Y_B}{1 - T^*}, \tag{A.5}$$

where the second equality follows from (A.1), and the third from (A.2).

First consider the effect of a change in α. (i) From the resource budget function (17), a decrease in α lowers the peer premium $P(T)$ and hence shifts down $E(T)$. Since equilibrium T^* is defined by $E(T^*) = Y_A$, this means that equilibrium quality difference between the two organizations falls as agents put less weight on the peer effect. (ii) Since r_B^* is increasing in T^* and decreasing in α in equation (8), a rise in α lowers r_B^*. For organization A, using the budget function (17) to express the condition $E(T^*) = Y_A$ and substituting the result in (A.3), we get

$$(1 - r_A^*)^2 = 2(1 - T^*) \frac{Y_A}{Y_B}.$$

Hence a fall in T^* also implies a fall in r_A^*. (iii) As α increases, since r_A^* decreases, the resource constraint for A implies that $S_A^*(1)$ decreases and S_A^* becomes flatter. Moreover, S_B^* becomes flatter as well, because $S_B^*(1) - P(T^*)$ is equal to $S_A^*(1)$ and thus decreases, while r_B^* also decreases. (iv) Using (A.3) and (8), and noting that Y_A equals $E(T^*)$, which is given by (17), we have

$$\frac{1 - r_A^*}{1 - r_B^*} = \frac{1 + r_A^*}{1 - r_A^*} \frac{Y_A}{Y_B}. \tag{A.6}$$

The right-hand-side of the above relation is decreasing in α because r_A^* decreases with α.

Next, consider an increase in Y_B. Using equation (17) and the definition of T^*, we have

$$Y_A = \frac{[Y_B - P(T^*)(1 - T^*)]^2}{2Y_B(1 - T^*)} = \left[1 - \frac{P(T^*)(1 - T^*)}{Y_B}\right]^2 \frac{Y_B}{2(1 - T^*)}. \tag{A.7}$$

(i) The first part of the above equation implies that T^* decreases as Y_B increases. (ii) From the second part of equation (A.7), since T^* decreases, both $Y_B/(1 - T^*)$ and $P(T^*)(1 -$

$T^*)/Y_B$ decrease as Y_B increases. It then follows from equation (8) that the threshold r_B^* in B decreases, and from equation (A.3) that the threshold r_A^* in A also decreases. (iii) From (A.4), S_A^* becomes flatter because $Y_B/(1-T^*)$ decreases as Y_B increases. Moreover, for organization B, from equation (8) we have

$$T^* - r_B^* = (1 - T^*)\,\frac{1 - r_A^*}{1 + r_A^*}, \tag{A.8}$$

which increases because r_A^* decreases and T^* decreases. From (A.5), $S_B^{*'}$ decreases. (iv) From (A.6) the fraction $(1 - r_A^*)/(1 - r_B^*)$ decreases because r_A^* decreases as Y_B increases.

Finally, consider the effects of an increase in the resource budget of the dominant organization. (i) An increase in Y_A raises T^* because the budget function is upward sloping at T^*. (ii) Since r_B^* is increasing in T^* from equation (8), organization B responds by raising the threshold r_B^*. Thus, the effects of an increase in Y_A on T^* and r_B^* are opposite of the effects of an increase in Y_B. However, the effect on r_A^* is generally non-monotone. By equation (A.3), an increase in Y_A raises the threshold r_A^* if and only if $T^* < \frac{3}{4}$. This is due to two opposing effects: the increase in r_B^* induces A to devote more resources to the top ranks to stay competitive with B, but with a greater resource budget, this may not lead to a reduction in the range of ranks that receive positive resources. The first effect dominates when the competition is tougher because T^* is relatively small. (iii) By equation (A.4), an increase in Y_A always makes the schedule S_A^* steeper for ranks above r_A^* because T^* increases. To study the effect of an increase in Y_A on the slope of S_B^*, we consider two cases. If $T^* \geq \frac{3}{4}$, then $S_B^{*'}$ increases by equation (A.5), because both r_A^* and $1-r_B^*$ decrease with T^*. If $T^* < \frac{3}{4}$, then from equation (A.8) $T^* - r_B^*$ decreases because r_A^* increases, and thus by (A.5), $S_B^{*'}$ again increases with Y_A. (iv) Finally, to study the effect of an increase in Y_A on the mixing of high types between A and B, again we distinguish two cases. If $T^* \geq \frac{3}{4}$, the fraction $(1 - r_A^*)/(1 - r_B^*)$ increases, because r_A^* decreases while r_B^* increases as Y_A increases. If $T^* < \frac{3}{4}$, from equation (A.6), the fraction $(1-r_A^*)/(1-r_B^*)$ again increases, because r_A^* increases with Y_A.

Proof of Lemma 2. Suppose that there is an interval $(r_-, r_+]$ such that t is continuous and strictly decreasing and Δ' is monotone. Let $r^0 \in (r_-, r_+]$ and $r^1 \in (r_-, r_+]$ solve

$$(r_+ - r_-)t(r^0) = (r^1 - r_-)t(r_-) + (r_+ - r^1)t(r_+) = \int_{r_-}^{r_+} t(r)\ \mathrm{d}r.$$

We construct a new quantile-quantile plot $\tilde{t}$ that is identical to t outside the interval $(r_-, r_+]$, and such that $\tilde{t}(r) = t(r^0)$ for all $r \in (r_-, r_+]$ if Δ' is decreasing, and $\tilde{t}(r) = t(r_-)$ for all $r \in (r_-, r^1]$ and $\tilde{t}(r) = t(r_+)$ for all $r \in (r^1, r_+]$ if Δ' is increasing. By construction $\tilde{t}$ is non-increasing with $\int_0^1 \tilde{t}(r)\ \mathrm{d}r = \int_0^1 t(r)\ \mathrm{d}r$.

Since t and $\tilde{t}$ only differ in the interval $(r_-, r_+]$, by definition S_A^t and $S_A^{\tilde{t}}$ only differ in the interval $(1-t(r_-), 1-t(r_+)]$. By a change of variable $r = 1 - t(\tilde{r})$ and then integration by parts, we have

$$\int_{1-t(r_-)}^{1-t(r_+)} S_A^t(r)\ \mathrm{d}r = \int_{r_-}^{r_+} t(\tilde{r})\Delta'(\tilde{r})\ \mathrm{d}\tilde{r} - (\Delta(r_+)t(r_+) - \Delta(r_-)t(r_-)).$$

If Δ' is decreasing, by construction we have $S_A^{\tilde{t}}(r) = \Delta(r_-)$ for all $r \in (1-t(r_-), 1-t(r^0)]$ and $S_A^{\tilde{t}}(r) = \Delta(r_+)$ for all $r \in (1-t(r^0), 1-t(r_+)]$, and thus

$$\int_{1-t(r_-)}^{1-t(r_+)} S_A^{\tilde{t}}(r)\ \mathrm{d}r = \Delta(r_-)(t(r_-) - t(r^0)) + \Delta(r_+)(t(r^0) - t(r_+)).$$

If Δ' is increasing, we have $S_A^{\tilde{t}}(r) = \Delta(r^1)$ for all $r \in (1-t(r_-), 1-t(r_+)]$, and thus

$$\int_{1-t(r_-)}^{1-t(r_+)} S_A^{\tilde{t}}(r)\ \mathrm{d}r = \Delta(r^1)(t(r_-) - t(r_+)).$$

In either case,[21]

$$\int_0^1 (S_A^{\tilde{t}}(r) - S_A^t(r))\ \mathrm{d}r = \int_{1-t(r_-)}^{1-t(r_+)} (S_A^{\tilde{t}}(r) - S_A^t(r))\ \mathrm{d}r = \int_{r_-}^{r_+} \big(\tilde{t}(r) - t(r)\big)\,\Delta'(r)\ \mathrm{d}r.$$

If Δ' is decreasing on (r_-, r_+), then

$$\begin{aligned}\int_{r_-}^{r_+} \big(\tilde{t}(r) - t(r)\big)\,\Delta'(r)\ \mathrm{d}r &= \int_{r_-}^{r^0} \big(t(r^0) - t(r)\big)\,\Delta'(r)\ \mathrm{d}r + \int_{r^0}^{r_+} \big(t(r^0) - t(r)\big)\,\Delta'(r)\ \mathrm{d}r \\ &\le \Delta'(r^0)\int_{r_-}^{r^0} \big(t(r^0) - t(r)\big)\ \mathrm{d}r + \Delta'(r^0)\int_{r^0}^{r_+} \big(t(r^0) - t(r)\big)\ \mathrm{d}r,\end{aligned}$$

which is equal to 0. If instead Δ' is increasing, then

$$\begin{aligned}\int_{r_-}^{r_+} \big(\tilde{t}(r) - t(r)\big)\,\Delta'(r)\ \mathrm{d}r &= \int_{r_-}^{r^1} (t(r_-) - t(r))\,\Delta'(r)\ \mathrm{d}r + \int_{r^1}^{r_+} (t(r_+) - t(r))\,\Delta'(r)\ \mathrm{d}r \\ &\le \Delta'(r^1)\int_{r_-}^{r^1} (t(r_-) - t(r))\ \mathrm{d}r + \Delta'(r^1)\int_{r^1}^{r_+} (t(r_+) - t(r))\ \mathrm{d}r,\end{aligned}$$

[21] The same expression below can also be obtained using the integration by parts formula for the Lebesgue-Stieltjes integral.

which is equal to 0. Thus, if there is an interval on which t is strictly decreasing and Δ' is monotone, we can replace t with a $\tilde{t}$ that assumes at most two values on that interval without increasing the value of the objective function. Since t is monotone and Δ' is monotone on each element of a countable partition of $[0,1]$, there are countably many intervals $I \subset [0,1]$ such that: (i) t is continuous and strictly decreasing on I; (ii) Δ' is monotone on I; and (iii) there is no open interval $I' \supset I$ that satisfies (i) and (ii). Further, t assumes a countable number of different values outside the union of all such intervals. Thus, there is a solution to (12) which assumes countably many values.

Next, let t be a quantile-quantile plot that assumes a countable number of values, and suppose that there are two consecutive intervals such that $t(r) = t^j$ for $r \in (r^{j-1}, r^j]$ and $t(r) = t^{j+1}$ for $r \in (r^j, r^{j+1}]$ for some $1 > t^j > t^{j+1} > 0$. Consider a new quantile-quantile plot $\tilde{t}_\epsilon$ defined as follows:

$$\tilde{t}_\epsilon(r) = \begin{cases} t^j + \epsilon/(r^j - r^{j-1}) & \text{if } r \in (r^{j-1}, r^j]; \\ t^{j+1} - \epsilon/(r^{j+1} - r^j) & \text{if } r \in (r^j, r^{j+1}]; \\ t(r) & \text{otherwise.} \end{cases}$$

For ϵ small, $\tilde{t}_\epsilon$ is a decreasing function. Moreover, $\int_0^1 \tilde{t}_\epsilon(r)\ \mathrm{d}r = \int_0^1 t(r)\ \mathrm{d}r$ and

$$\begin{aligned} &\int_0^1 (S_A^{\tilde{t}_\epsilon}(r) - S_A^t(r))\ \mathrm{d}r \\ &= -\frac{\epsilon}{r^j - r^{j-1}}\Delta(r_{j-1}) + \left(\frac{\epsilon}{r^j - r^{j-1}} + \frac{\epsilon}{r^{j+1} - r^j}\right)\Delta(r_j) - \frac{\epsilon}{r^{j+1} - r^j}\Delta(r^{j+1}). \end{aligned}$$

Since $\int_0^1 (S_A^{\tilde{t}_\epsilon}(r) - S_A^t(r))\ \mathrm{d}r$ is linear in ϵ, there is some ϵ for which $\tilde{t}_\epsilon$ assumes one fewer value than t and does at least as well as t for (12).

Proof of Lemma 5. Substituting $T = \frac{1}{2}$ and $T = 1$ into the resource budget function (17), we immediately have $E\left(\frac{1}{2}\right) = Y_B$, and $\lim_{T \to 1} E(T) = \infty$.

Next, when $E(T) > 0$, its derivative is positive if and only if

$$\alpha Y_B + (1 - T)\left(3T - \frac{5}{2}\right) > 0.$$

The above holds for all $T \geq \frac{1}{2}$ if $\alpha Y_B > \frac{1}{2}$, thus establishing (i).

When $\alpha Y_B \leq \frac{1}{2}$, there exists a unique $\hat{T} \in \left[\frac{1}{2}, 1\right]$ such that $E'(T) > 0$ for $T > \hat{T}$ while the opposite holds for $T < \hat{T}$. Finally, from (17) we have that $E(T) = 0$ when $Y_B \leq P(T)(1-T)$. The quadratic equation $Y_B = P(T)(1-T)$ has two real roots T_- and T_+ in $\left[\frac{1}{2}, 1\right]$ when $\alpha Y_B \leq \frac{1}{16}$, and no real root otherwise. Claims (ii) and (iii) follow.

Proof of Lemma 7. First, suppose that $S_A(\hat{r}) > 0$ at some $\hat{r}$, and is locally strictly increasing, and convex. Define a linear function $\underline{S}_A$ such that $\underline{S}_A(\hat{r}) = S_A(\hat{r})$ and $\underline{S}_A(r) < S_A(r)$ for all $r \neq \hat{r}$ in a neighborhood of $\hat{r}$. By Lemma 6, we can define a rank $\tilde{r}$ in B such that $S^*_B(\tilde{r}) = S_A(\hat{r}) + P(T^*)$. For each ϵ, we construct the following resource distribution schedule S^ϵ_B for B that coincides with S^*_B except when $r \in (\tilde{r}-\epsilon, \tilde{r}+\epsilon]$, in which case $S^\epsilon_B(r)$ is equal to $S^*_B(\tilde{r})$. By construction, for all ϵ, the schedule S^ϵ_B respects the resource constraint. For any T, the quantile-quantile plot $\bar{t}^T$ under S_A and S^ϵ_B is identical to that under S_A and S^*_B for all r outside the interval $(\tilde{r}-\epsilon, \tilde{r}+\epsilon]$, and is otherwise a constant equal to the value of the quantile-quantile plot under S_A and S^*_B at $\tilde{r}$, which is $1 - S_A^{-1}(S^*_B(\tilde{r}) - P(T))$. It follows that the change in the value of $\overline{D}(T)$, when B switches from S^*_B to S^ϵ_B, is given by

$$\int_{\tilde{r}-\epsilon}^{\tilde{r}+\epsilon} S_A^{-1}(S^*_B(r) - P(T))\ \mathrm{d}r - \int_{\tilde{r}-\epsilon}^{\tilde{r}+\epsilon} S_A^{-1}(S^*_B(\tilde{r}) - P(T))\ \mathrm{d}r.$$

At $T = T^*$, the second term of the above expression equals $2\epsilon\hat{r}$. For the first term, by definition of $\underline{S}_A$ we have

$$S_A^{-1}(S^*_B(r) - P(T^*)) \leq \underline{S}_A^{-1}(S^*_B(r) - P(T^*)),$$

for all $r \in (\tilde{r}-\epsilon, \tilde{r}+\epsilon]$, with strict inequality for all r different from $\tilde{r}$. Since both $\underline{S}_A$ and S^*_B are linear in the corresponding intervals,

$$\underline{S}_A^{-1}(S^*_B(r) - P(T^*)) = \hat{r} + (r - \tilde{r})\ \frac{S^{*\prime}_B}{\underline{S}'_A}$$

for all $r \in (\tilde{r}-\epsilon, \tilde{r}+\epsilon)$. Thus, the first term in the expression for the change in the value of $\overline{D}(T)$ is strictly smaller than $2\epsilon\hat{r}$. This implies that at $T = T^*$, for all small and positive ϵ, the value of $\overline{D}(T)$ decreases when B switches from S^*_B to S^ϵ_B. It follows that T^* is no longer a fixed point of $\overline{D}(T)$.

Denoting with $\overline{D}^{\epsilon}(T)$ the function defined by (4) when A offers the schedule S_A and B the schedule S_B^{ϵ}, and with $\overline{D}^{*}(T)$ the same function when B offers S_B^*, we have that when ϵ goes to 0: (i) $\overline{D}^{\epsilon}(T)$ converges uniformly to $\overline{D}^{*}(T)$; and (ii) the derivative of $\overline{D}^{\epsilon}(T)$ with respect to T converges uniformly to the derivative of $\overline{D}^{*}(T)$. Using property (ii), and the fact that the derivative of $\overline{D}^{*}(T)$ is strictly smaller than 1 at T^* and continuous, we can establish that there exists a small and positive γ, such that the derivative of $\overline{D}^{\epsilon}(T)$ is less than 1 for any $T \in (T^*, T^* + \gamma)$ and for all sufficiently small ϵ. It follows that $\overline{D}^{\epsilon}(T)$ is strictly less than T for $T \in (T^*, T^* + \gamma)$ for sufficiently small ϵ. Since by property (i) for all sufficiently small ϵ, the mapping $\overline{D}^{\epsilon}(T)$ has no fixed point greater than $T^* + \gamma$, the largest fixed point is strictly below T^*. Thus, S_B^* is not a best response to S_A.

Next, suppose that for some $\hat{r}$ such that $S_A(\hat{r}) > 0$, the schedule $S_A(r)$ is locally convex but $S_A(r)$ is constant just below $\hat{r}$. Then, for any small and positive η, construct the modification of S_B^* as above, but set $S_B^{\epsilon}(\tilde{r}) = S_A(\hat{r}) + P(T^*) + \eta$. Following the same argument as above, we can establish that this modification does strictly better for B than S_B^* for all η. As η goes to 0, the modified S_B^{ϵ} also satisfies B's resource constraint. Thus, S_B^* is not a best response to S_A.

Finally, suppose that for some $\hat{r}$ such that $S_A(\hat{r}) > 0$, the schedule $S_A(r)$ is locally concave. Define $\tilde{r}$ as in the first case of local convexity. Consider a modified schedule S_B^{ϵ} which is identical to S_B^* outside the interval $(\tilde{r} - \epsilon, \tilde{r} + \epsilon]$, and is equal to $S_B^*(\tilde{r} - \epsilon)$ for all $r \in (\tilde{r} - \epsilon, \tilde{r}]$ and to $S_B^*(\tilde{r} + \epsilon)$ for all $r \in (\tilde{r}, \tilde{r} + \epsilon]$. Following a similar argument as in the case of local convexity, we can show that for sufficiently small ϵ, the largest fixed point of the mapping $\overline{D}(T)$ under S_A and S_B^{ϵ} is strictly smaller than T^*. Thus, S_B^* is not a best response to S_A in this case.

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