Information Provision and Price Competition

Ettore Damiano
University of Toronto

Li, Hao
University of Toronto

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Abstract: Two sellers with ex-ante identical products, whose qualities can be either high or low, first choose a binary information structure, modeled as the probability that the signal reveals the state. After the buyer independently draws one private signal from each information structure, the sellers then each choose a price in the second stage. We identify two equilibria in information structures, a symmetric equilibrium with two perfectly informative structures, and an asymmetric equilibrium with one perfectly informative structure and one completely uninformative structure. The symmetric equilibrium is efficient while the asymmetric equilibrium is not, but the latter generates a greater revenue to the sellers because price competition is less fierce due to a greater quality difference when they compete.

1. Introduction

When a buyer is privately informed about his valuation for competing products, the outcome of price competition between the sellers naturally depends on the information structure of the buyer. For example, if the buyer's private signals are relatively uninformative, the quality difference between the products varies little across signal realizations. This means that there is little horizontal differentiation and therefore price competition is fierce. The informativeness of the buyer's private signals increases the degree of horizontal differentiation and softens price competition. The issue of price competition with informed buyers has received scant attention in the literature; a notable exception is Moscarini and Ottaviani (2001).

In this paper we present a model where sellers can use the information structure of the buyer as an additional instrument in price competition. This environment arises naturally in many buyer-seller relationships when the sellers can control the quality of the buyer's private information. This is especially true in independent private value situations, as in Bergemann and Pesendorfer (2001) and Eso and Szentes (2006).

We introduce a model in which two sellers with ex-ante identical products choose an information structure in the first stage and, after the buyer independently draws one private signal from each information structure, choose a price in the second stage. For each product there are two equally likely states, high and low, and two possible signals, high and low. An information structure is modeled as the probability that the signal reveals the state. We characterize subgame perfect equilibria of the two stage game between the two sellers.

Whether a seller has incentive to increase the informativeness of his signals depends on the rival seller's choice of information structure. If the rival is already giving less information, then a more informative information structure leads to a greater degree of product differentiation in each demand state. This softens price competition and tends to increase the revenue of both sellers. In contrast, when the rival is giving more information, then an increase in the informativeness of the signal structure generally has ambiguous effects on the degree of differentiation. In particular, there is less differentiation in the demand state where the buyer draws two high or two low signals, while there is more differentiation in the other two demand states. The effects on the revenues of the two sellers depend on the nature of price competition.

We find two subgame perfect equilibria. In both equilibria, price competition in the second stage involves randomization. In the symmetric equilibrium sellers provide full information. Each seller is a local monopolist when the buyer draws a high signal for his product and a low signal for his competitor's product, in the sense that equilibrium prices do not affect the purchase decision. Price competition is relevant only when the buyer draws a high signal from each seller and there is no sale when the buyer draws two low signals. Neither seller has an incentive to slightly downgrade the information structure because doing so would change the price competition subgame in such a way that the revenue as a local monopolist is reduced while the revenue from price competition is unaffected. The equilibrium outcome is efficient, with the total surplus evenly split among the two sellers and the buyer. In the asymmetric equilibrium, one seller gives no information, while the other seller provides full information. The first seller is a local monopolist when the buyer draws a low signal from the second seller; otherwise the two sellers compete in prices. The first seller has no incentive to provide just a little information because it would force him to charge a lower price in order to compete regardless of his own signal realization. The second seller has no incentive to reduce the informativeness of his signal because it would not affect his ability to sell when the signal realization is low, while it would reduce his competitive edge when the signal realization is high. The equilibrium outcome is inefficient because the buyer obtains the good of unknown quality with a positive probability even when the other good is known to be of high quality. Compared to the symmetric equilibrium, the sellers are jointly better off while the buyer is worse off.

This paper is organized as follows. Section 2 describes the model in which the two sellers simultaneously choose information structures in the first stage, and then after observing each other's choice, the sellers simultaneously choose prices in the second stage. A few comments on some of the modeling choices appear at the end of the section. In section 3, we first characterize the equilibria in price competition for three types of subgames of the second stage: i) symmetric subgames following an identical choice of information structure by the two sellers in the first stage; ii) asymmetric subgames in which one seller has chosen the perfectly informative information structure; and iii) asymmetric subgames in which one seller has chosen the completely uninformative structure. We then use these characterization results to establish the symmetric subgame perfect equilibrium with two perfectly informative structures and the asymmetric subgame perfect equilibrium with one perfectly informative structure and one completely uninformative structure. We discuss the welfare properties of the two equilibria and argue that both equilibria are robust. Section 4 contains some brief concluding remarks.

2. A Two-stage Model of Competition in Information Structures and Prices

There are two sellers, A and B, each with one good for sale to a potential buyer. The two goods are ex ante identical: the unobserved quality of each good is either H or L, with equal probabilities, and the two qualities are independent random variables. The buyer has a unit demand, and values a good of quality H at 1, and a good of quality L at 0. The sellers have 0 reservation value for their goods.

We consider a class of symmetric binary information structures for each good. Denote the two signals of each information structure as h and l. Each information structure is represented by a parameter α, which is both the probability of the signal being h conditional on the quality of the good is H, and the probability of the signal being l conditional on L. Without loss of generality, assume that α lies between 1/2 and 1. The parameter α measures the informativeness of the information structure, with $\alpha = 1$ corresponding to a perfectly informative structure, and $\alpha = 1/2$ corresponding to a completely uninformative structure. Note that the expected value of the good is α conditional on the signal h, and $1 - \alpha$ conditional on l. Further, given that the two qualities are equally likely, the ex ante probability of signal h equals the probability of l.

The game between the two sellers has two stages. In the first stage, the sellers simultaneously endow the buyer with information structures α_A and α_B. The quality of each good is realized, unobserved to both sellers and to the buyer, and the buyer receives a private signal about the quality of each good according to the two information structures. In the second stage, after observing each other's choice of information structure, the two sellers simultaneously choose prices p_A and p_B.

There are 4 equally likely demand states, corresponding to the 4 possible combinations of the signal realizations. In each demand state, if we denote as q_A and q_B the expected values of the two goods, then the payoff to A is p_A if $q_A - p_A > \max\{q_B - p_B, 0\}$ and 0 if $q_A - p_A < \max\{q_B - p_B, 0\}$. The payoff to A when $q_A - p_A = \max\{q_B - p_B, 0\}$ is determined by a tie-breaking rule, which is part of equilibrium. The payoff to seller B can be symmetrically defined. We adopt subgame perfect equilibrium as the solution concept of the two-stage game.

A few comments on the modeling choices are in order. First, our model assumes that the sellers observe each other's choice of information structure before price competition takes place. This allows us to examine how information affects product differentiation and price competition, which in turn determines the equilibrium choice of information structure. The two stage feature of our model differentiates our paper from Forand (2007), who also studies competition in information structure in a setup where buyers must choose among sellers based on the sellers' promise of information (before private signals are realized). Second, our model of binary states and binary signals for each product provides a simple way of modeling competition in information structures. Instead, the difficulty of our analysis lies in characterizing of the outcome of price competition, because the revenue functions are discontinuous as in Hotelling (1929) and Osborne and Pitchick (1987). Third, by focusing on price competition, we are restricting the sellers to direct mechanisms. In general this is not an innocuous assumption as shown in Epstein and Peters (1999), but it allows us to extend the analysis of price competition to environments where sellers have control over the buyers' access to private information. Finally, by modeling the tie-breaking rule as part of equilibrium, we remove the buyer as a strategic player in the game. This helps to simplify the notation. Alternatively, we can add a third stage in which the buyer makes purchase decisions conditional on the private signals and the prices. Any equilibrium outcome of our two-stage game between the two sellers with the tie-breaking rule determined in equilibrium corresponds to an equilibrium of the three-stage game with the buyer's equilibrium purchase strategy chosen to replicate the tie-breaking rule.

3. Equilibrium Analysis

To characterize subgame perfect equilibria of the two stage model of competition in information structures we must first characterize the equilibrium in the price competition stage of our model. Rather than characterizing the equilibrium for each possible choice of information structures α_A and α_B in the first stage, we focus on three scenarios: i) both sellers have chosen the same information structure; ii) one seller has chosen a perfectly informative structure; iii) one seller has chosen a perfectly uninformative structure. In each of these scenarios we provide a characterization of the outcome of price competition in terms of both the support of the equilibrium strategies and the equilibrium revenues of both sellers. For any relevant values of α_A and α_B, we denote with $P_A(\alpha_A, \alpha_B)$ and $P_B(\alpha_A, \alpha_B)$ the support of the equilibrium strategies of seller A and B respectively, in a Nash equilibrium of the subgame in which A has chosen α_A and B has chosen α_B. Similarly, $R_A(\alpha_A, \alpha_B)$ and $R_B(\alpha_A, \alpha_B)$ denote the equilibrium revenue to A and B. For convenience, we will treat seller B as the seller who chooses the (weakly) less informative structure. Scenario i) corresponds to the case where $\alpha_A = \alpha_B = \alpha$, while scenarios ii) and iii) correspond to the cases where $\alpha_A = 1, \alpha_B = \alpha$ and $\alpha_A = \alpha, \alpha_B = 1/2$ respectively. For each scenario and each value of $\alpha \in [1/2, 1]$, we identify an equilibrium in the price competition stage.

The analysis of the three scenarios will allow us to identify two distinct subgame perfect equilibria of the two stage game. While we cannot rule out that other equilibria exist because at this point we do not have a characterization of the equilibrium for each pricing subgame, the two equilibria we find illustrate the main insights about the interaction between competing information provision and price competition. Furthermore, the pricing equilibria of the three scenarios are identified by construction and we are unable at this point to prove the conjecture that they are unique in all three scenarios. This task is left to future research.

3.1. Pricing equilibrium under symmetric information structures

We first consider price competition after histories in which both sellers have chosen two information structures with an identical level of informativeness α. As can be expected, for any feasible value of $\alpha > 1/2$, the equilibrium in price competition is in mixed strategies. The only exception is when $\alpha = 1/2$. In this extreme case, the buyer receives no private information and values the two goods identically with probability 1. That is, two good are effectively homogeneous and Bertrand competition then leads both seller to price at the marginal cost of zero. The following lemma provides a characterization of the support of the equilibrium strategies for the two sellers. A complete description of the equilibrium strategies, together with the proof of the lemma, can be found in the Appendix. For notational convenience, we define

$$z = 2\alpha - 1.$$

For any given level of informativeness α, the parameter z is the difference in the expected value of the good between a buyer that receives a signal h and a buyer that receives a signal l.

LEMMA 1. *For any value of* $\alpha \in [1/2, 1]$, *there exists a symmetric mixed strategy equilibrium of the price competition game with* $\alpha_A = \alpha_B = \alpha$ *where the support of the equilibrium strategy is given by*

$$P_A(\alpha,\alpha) = P_B(\alpha,\alpha) = \begin{cases} \left[\frac{1}{2}z, \frac{3}{2}z\right] & \text{if} \quad \alpha \leq \frac{5}{8}; \\ \left[\frac{1}{2}z, \frac{1}{2}(1-z)\right] \cup \left[\frac{3z(1-z)}{1+2z}, \frac{3}{2}z\right] & \text{if} \quad \frac{5}{8} < \alpha \leq \frac{3}{4}; \\ \left[\frac{1}{4}(1+z), \frac{1}{2}(1+z)\right] & \text{if} \quad \frac{3}{4} < \alpha. \end{cases}$$

Figure 1 illustrates the equilibrium price support. Lemma 1 distinguishes between three qualitatively different cases. For low values of α, the upper bound of the support of the equilibrium strategy is smaller than $1-\alpha$. Note that as α goes to $1/2$, the support converges to a single mass point at zero. Since the quality difference z after opposite signal realizations is small in this range of α values, it is not worthwhile to charge a price above $1-\alpha$. This is because the additional surplus that can be extracted from a high valuation buyer does not justify the loss from a reduced buyer base. Further, the difference between the upper bound and the lower bound of the equilibrium price support is exactly z. This implies that price competition is only present in the demand states in which the buyer receives identical signals for both goods. Regardless of the equilibrium price realization,

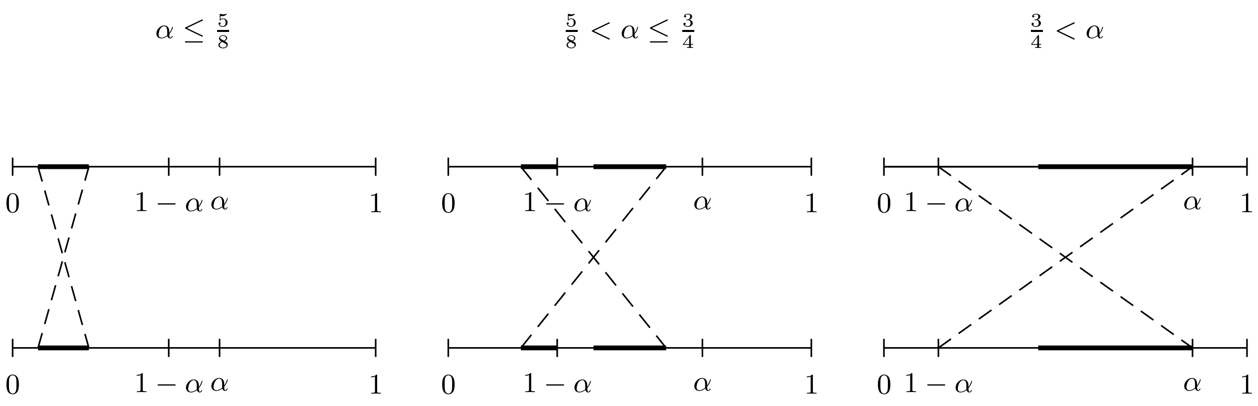

Figure 1: $P_A(\alpha,\alpha)$ and $P_B(\alpha,\alpha)$

the buyer will purchase the high signal product in the other demand states. Finally, a sale will always be made by one of the two sellers.

For high values of α, the lower bound of the support of the equilibrium strategy is higher than $1-\alpha$. In this region, the expected value of the product for a buyer who has received a signal l is very low and the sellers find it more profitable to target only the high valuation buyer. Price competition is only present in the demand state when the buyer has received a high signal for each product.

For intermediate values of α, the support of the equilibrium strategy is the union of two disjoint intervals. The two sellers randomize between prices below $1-\alpha$ and prices above $1-\alpha$ in equilibrium. As in the case of low α values, the difference between the upper and lower bound of the price support is z, and price competition is only present in the states in which the buyer receives identical signals for both goods. However, in equilibrium there is no sale with positive probability.

The equilibrium strategy has no atom and hence the equilibrium revenue of the two sellers can be easily identified from the lowest price in the support. For low and intermediate α's, at the lowest price, a seller makes a sale in all demand states but the demand state in which the buyer receives an l signal on his product and an h signal on his opponent's product. For high α's, at the lowest price, each seller is guaranteed to sell when the buyer draws an h signal about his own product and will not sell otherwise. The equilibrium revenue as a function of α is given by

$$R_A(\alpha,\alpha) = R_B(\alpha,\alpha) = \begin{cases} \frac{3}{4}\left(\alpha - \frac{1}{2}\right) & \text{if} \quad \alpha \leq \frac{3}{4}; \\ \frac{1}{4}\alpha & \text{if} \quad \frac{3}{4} < \alpha. \end{cases} \tag{1}$$

The above revenue formula has an intuitive interpretation. For high values of α, each seller is a monopolist in the demand state when the buyer receives a high signal for his product and a low signal for the rival's product, and extracts all surplus α in that state. In the demand states where the buyer receives two high signals, the two sellers compete to sell undifferentiated goods and give away all surplus to the buyer. In contrast, for low and intermediate values of α, in equilibrium the threat of price competition means that each seller cannot extract the all surplus of α in the most favorable demand state. The highest price that the seller can charge to ensure a sale in that state is $3z/2$, which is bounded away from α, with the rest of the surplus going to the buyer.

3.2. Pricing equilibrium with a perfectly Informative structure

We next consider price competition after seller A has chosen $\alpha_A = 1$. For every value of α chosen by B, the following lemma provides a characterization of the support of the equilibrium strategies for the two sellers. Again, the complete description of the equilibrium strategies is in the the proof of the lemma, which is in the Appendix. In general the support of equilibrium prices for both A and B is a union of two intervals. The proof constructs two bounds on the prices charged by seller B: the lowest price $\underline{p}_B$, and the lowest price $\underline{\underline{p}}_B$ above $1-\alpha$, both as continuous function of α.

Lemma 2. *For any value of $\alpha \in [1/2, 1]$, there exists a mixed strategy equilibrium of the price competition game with $\alpha_A = 1$ and $\alpha_B = \alpha$ in which the support of the equilibrium strategy for B is*

$$P_B(1,\alpha) = \begin{cases} [\underline{p}_B(\alpha), 1-\alpha] & \text{if} \quad \alpha \leq \frac{2}{3}; \\ [\underline{p}_B(\alpha), 1-\alpha] \cup [\underline{\underline{p}}_B(\alpha), \alpha] & \text{if} \quad \frac{2}{3} < \alpha \leq \frac{3}{4}; \\ [\underline{\underline{p}}_B(\alpha), \alpha] & \text{if} \quad \frac{3}{4} < \alpha, \end{cases}$$

and the support for A is $P_A(1,\alpha) = P_B(1,\alpha) + (1-\alpha)$.

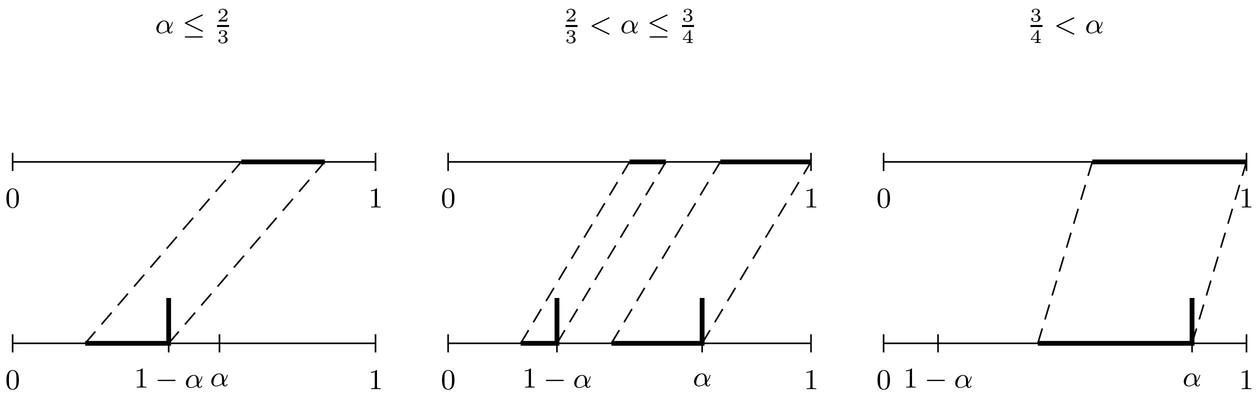

Figure 2: $P_A(1,\alpha)$ and $P_B(1,\alpha)$.

Figure 2 illustrates the equilibrium price supports for A and B. As in the symmetric case, there are three qualitatively different cases. For low values of α, in equilibrium seller B competes regardless of the signal on his own product; in equilibrium B never charges a price above $1-\alpha$. Unlike the symmetric case, B is a monopolist in the demand states where the buyer draws a low signal on A's product, which explains why the upper bound on B's price is exactly $1-\alpha$. For intermediate values of α, the quality difference between the two signals is too large for B to forego the opportunity of charging a high price in the demand state that favors his product. In equilibrium, B randomizes both below $1-\alpha$ and above. The upper bound is equal to α, because as before, B faces no competition in his most favorable demand state. For high values of α, B competes only in the demand states where the buyer receives a high signal on his product. The asymmetry between the two sellers is reflected in the fact that the price support of seller A is a pointwise rightward shift of that of B by $1-\alpha$, which is the difference in expected qualities when the buyer receives high signals for both goods.

The equilibrium price distribution for seller A is atomless. This allows us to compute the equilibrium revenue of B by using the upper bound on B's equilibrium price distribution, which is $1-\alpha$ for low α's and equals α otherwise. This gives

$$R_B(1,\alpha) = \begin{cases} \frac{1}{2}(1-\alpha) & \text{if} \quad \alpha \le \frac{2}{3}; \\ \frac{1}{4}\alpha & \text{if} \quad \frac{2}{3} < \alpha. \end{cases} \tag{2}$$

The equilibrium price distribution for seller B has an atom at the upper bound for all values of $\alpha < 1$. Thus, it is easier to compute the equilibrium revenue of A by using the lower bound on A's equilibrium strategy, which gives

$$R_A(1,\alpha) = \frac{1}{2}(\underline{p}_B(\alpha) + 1 - \alpha).$$

An explicit solution for $\underline{p}_B(\alpha)$ is obtained in the Appendix for values of $\alpha \geq 2/3$. For low values of α, the value of $\underline{p}_B(\alpha)$ is bounded from above by $1-\alpha$ and converges to $1/4$ as α goes to $1/2$.

The above revenue formulas can be interpreted as follows. As in the case of symmetric information structures, for high values of α, both sellers give up the surplus in the demand states where the buyer gets a low signal on his product, and extracts the full surplus in the most favorable demand state. Unlike the symmetric case, where they compete away the full surplus in the demand state with two high signals, the asymmetry in the information structures implies that seller A extracts an amount of surplus equal to the quality difference $1-\alpha$ in this state. For intermediate values of α, the support of the equilibrium strategies is the union of two disjoint intervals. While seller B is still a monopolist in his most favorable demand state and extracts the full surplus in that state, seller A is unable to do the same in his most favorable demand state because seller B randomizes among prices below $1-\alpha$ as well as prices above $1-\alpha$ in equilibrium. Finally, for low values of α, seller B while still a monopolist in demand states where the buyer receives a low signal on A's product, finds it more profitable to leave some rent to the high signal buyer and instead extract all surplus when the buyer receives a low signal. For seller A, as in the symmetric information structure case, the threat of price competition means that it is not possible to extract the all surplus in the most favorable demand state. However, the asymmetry in the information structures gives seller A a quality advantage when the buyer draws a high signal on his product, which prevents full surplus dissipation as the value of α becomes close to $1/2$.

3.3. Price equilibria with a completely uninformative structure

Finally, we consider price competition after seller B has chosen $\alpha_B = 1/2$. The following lemma characterizes the support of the equilibrium strategies for the two sellers, for every value of α chosen by A. Similarly to the case of Lemma 2, the proof of the following

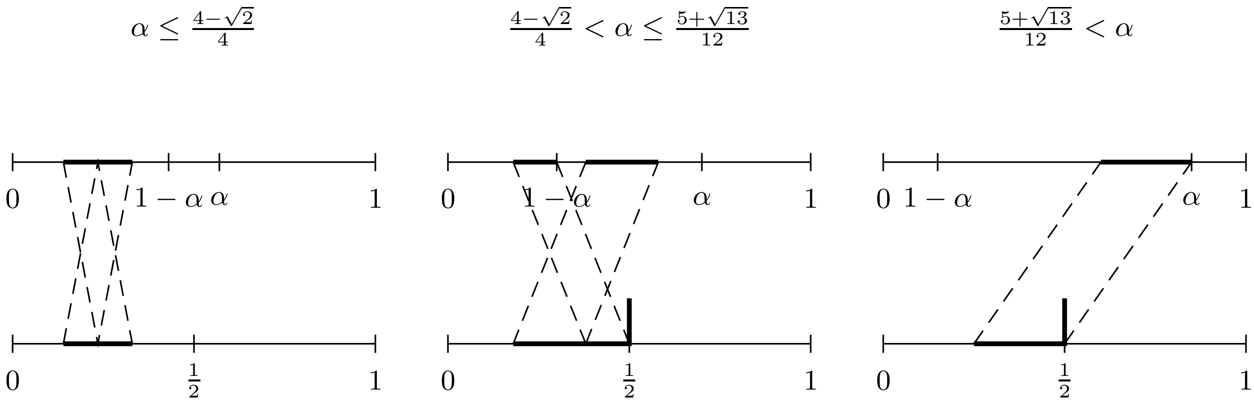

Figure 3: $P_A(\alpha, 1/2)$ and $P_B(\alpha, 1/2)$.

lemma constructs two bounds on the prices charged by seller A: the lowest price $\underline{p}_A$, and the lowest price $\underline{\underline{p}}_A$ above $1-\alpha$, both as continuous function of α.

Lemma 3. *For any value of* $\alpha \in [1/2, 1]$, *there exists a mixed strategy equilibrium of the price competition game with* $\alpha_A = \alpha$ *and* $\alpha_B = 1/2$ *in which the supports of the equilibrium strategies are:*

$$P_A(\alpha, 1/2) = \begin{cases} [\underline{p}_A(\alpha), \underline{p}_A(\alpha) + 2\alpha - 1] & \textit{if} \quad \alpha \leq \frac{4-\sqrt{2}}{4}; \\ [\underline{p}_A(\alpha), 1-\alpha] \cup [\underline{\underline{p}}_A(\alpha), \underline{p}_A(\alpha) + 2\alpha - 1] & \textit{if} \quad \frac{4-\sqrt{2}}{4} < \alpha \leq \frac{5+\sqrt{13}}{12}; \\ [\underline{\underline{p}}_A(\alpha), \alpha] & \textit{if} \quad \frac{5+\sqrt{13}}{12} < \alpha; \end{cases}$$

and

$$P_B(\alpha, 1/2) = \begin{cases} [\underline{p}_A(\alpha), \underline{p}_A(\alpha) + 2\alpha - 1] & \textit{if} \quad \alpha \leq \frac{4-\sqrt{2}}{4}; \\ [\underline{\underline{p}}_A(\alpha) - \frac{1}{2}(2\alpha - 1), \frac{1}{2}] & \textit{if} \quad \frac{4-\sqrt{2}}{4} < \alpha \leq \frac{5+\sqrt{13}}{12}; \\ [\frac{1}{4}, \frac{1}{2}] & \textit{if} \quad \frac{5+\sqrt{13}}{12} < \alpha. \end{cases}$$

Figure 3 illustrates the equilibrium price supports for A and B. Since the information structure of seller B is completely uninformative, the buyer's valuation for B's product is known and equal to 1/2. Therefore, unlike the previous two cases, there are only two demand states depending on whether the buyer receives a high or low signal on seller A's product.

As in Lemma 1 and 2, there are three qualitatively different cases. For high values of α, A competes only in the demand states where the buyer receives a high signal on his product. As in Lemma 2, the asymmetry between the two sellers is reflected in the fact that the price support of seller A is a pointwise rightward shift of that of B by $\alpha - 1/2$, which is the difference in expected qualities when the buyer receives high signal on A's product. For low values of α price competition is present in both demand states. High prices in A's support compete with low prices in B's support when the buyer receives a high signal, while low prices in A's support compete with high prices in B's support in the other demand state. Because the quality difference is the same whether the buyer receives a high or low signal, the equilibrium strategies are identical for the two sellers. For intermediate values of α, it remains true that A and B compete in both demand states. However, the quality difference is too large so that each seller would want to charge a higher price and gamble on the favorable demand state. As a result, the equilibrium strategies are asymmetric in this case with a gap in A's support and a mass point at $1/2$ for B.

The equilibrium revenues of seller A and seller B as functions of α can be obtained from the above characterization of the equilibrium strategies. They are given by the following revenue formulas

$$(3)\qquad \begin{cases} R_A(\alpha,1/2) = R_B(\alpha,1/2) = \frac{1}{2}\underline{p}_A(\alpha) + \frac{1}{4}z & \text{if} \quad \alpha \le \frac{4-\sqrt{2}}{4}; \\ R_A(\alpha,1/2) = \frac{1}{2}\underline{p}_A(\alpha), \quad R_B(\alpha,1/2) = \frac{1}{2}\underline{p}_A(\alpha) + \frac{1}{4}z & \text{if} \quad \frac{4-\sqrt{2}}{4} < \alpha \le \frac{5+\sqrt{13}}{12}; \\ R_A(\alpha,1/2) = \frac{1}{2}\underline{p}_A(\alpha), \quad R_B(\alpha,1/2) = \frac{1}{4} & \text{if} \quad \frac{5+\sqrt{13}}{12} < \alpha. \end{cases}$$

We can interpret the above revenue formulas as follows. For high values of α, seller B is a monopolist in the demand state where the buyer gets a low signal and extracts full surplus. In the other demand state in which the buyer draws a high signal or A's product, price competition with A having a quality advantage means that B expects no profit while A must leave part of the surplus to the buyer. For low values of α, price competition is fierce in both demand states, and neither seller can extract the full surplus in the favorable demand state. Finally, for intermediate values of α, the two sellers compete in both demand states, as in the low α case, but competition is less fierce. Seller B expects no profit when the buyer receives a high signal, but sells at the highest price of $1/2$ with a positive probability.

3.4. Equilibrium information structures

With the characterization results about price competition in Lemmas 1, 2 and 3, we are now ready to examine the first stage competition in terms of information provision. The first result is:

PROPOSITION 4. *There exists no symmetric subgame perfect equilibrium in which* $\alpha_A = \alpha_B < 1$.

The proof of the above result is in the appendix. The argument involves applying the revenue formulas (1) and (2) to show that for any $\alpha \in [1/2, 1)$,

$$R_A(1, \alpha) > R_A(\alpha, \alpha).$$

There is a simple intuition that explains the proposition. Suppose that initially both sellers choose some $\alpha \in [1/2, 1)$. As seller A increases the quality of the buyer's private signal on his product, the quality difference between the two products increases in each of the four demand states. In particular, when the buyer draws two high signals or two low signals, the quality difference increases from 0 to $\alpha_A - \alpha$, and when the buyer draws one high signal and one low signal, the quality difference is either $\alpha_A - (1 - \alpha)$ in the demand state that favors A, or $\alpha - (1 - \alpha_A)$ in the opposite demand state. Price competition becomes less fierce as a result of an increased quality difference in the every state. Seller A as well as seller B benefits from such an increase in α_A. To gather further intuition, consider the case where α is large. ¿From Lemma 1 we know that when $\alpha_A = \alpha_B = \alpha$, the two sellers are monopolist and extract all surplus in their most favorable demand state respectively, and price competition leads to the dissipation of the entire surplus in the state with two high signals. In contrast, when $\alpha_A = 1$ and $\alpha_B = \alpha$, both sellers remain monopolist and extract all surplus in the corresponding demand state. However, seller A benefits from increasing α_A from α to 1 in two ways: first, the the amount the buyer is willing to pay in A's favorable demand state is higher; second, A has a quality advantage over B when the buyer receives two high signals and gets a positive share of the surplus. Note also that the greater profit of A comes at no cost to B.

The unique symmetric equilibrium in information structure is given by $\alpha_A = \alpha_B = 1$. The proof of the next result follows immediately from the revenue formula (2), which implies that

$$R_B(1,1) \geq R_B(1,\alpha)$$

for any α, so that a perfectly informative structure is a best response against $\alpha_A = 1$ for seller B.

PROPOSITION 5. *There exists a symmetric subgame perfect equilibrium in which $\alpha_A = \alpha_B = 1$.*

¿From equation (2) we can see that against a perfectly informative structure, seller B's revenue is not monotone in his choice of information structure. In particular, it initially decreases in α_B and then becomes increasing when α_B reaches $2/3$. To understand this non-monotonicity, note that when α_B increases, the quality difference between the two products increases in the two demand states where the buyer draws one high signal and one low signal, and decreases when the buyer receives two high or two low signals. This is in contrast to the case when the more informative structure becomes even more informative. The effects on B's revenue of an increase in α_B given $\alpha_A = 1$ depend on the nature of price competition, and hence on the value of α_B. In particular, for high values of α_B, seller B's profit comes from the monopolist revenue in his favorable demand state and therefore he benefits from an increase in α_B. In contrast, for low values of α_B, seller B targets the demand states that favors A, and since an increase in α_B increases A's quality advantage in price competition, B's profit decreases.

Besides the symmetric equilibrium with perfect information structures, there is also an asymmetric equilibrium with one seller, say seller A, choosing a perfectly informative structure, and seller B choosing the completely uninformative structure.

PROPOSITION 6. *There exists an asymmetric subgame perfect equilibrium in which $\alpha_A = 1$ and $\alpha_B = 1/2$.*

Part of the intuition behind the above result is anticipated by our discussion after Proposition 4. There we argued that by making the more informative structure even more informative, the seller increases his profit by increasing quality differences in all demand states and softening price competition. Formally, using the revenue formula (3) and the characterization of the price support in the proof of Lemma 3 in the appendix, one can easily verify that $R_A(\alpha, 1/2)$ is strictly increasing in α. The other half of the argument for the proposition, that $\alpha_B = 1/2$ is a best response against $\alpha_A = 1$, is already in the revenue formula (2). More precisely, against $\alpha_A = 1$ seller B has exactly two best responses:

$$R_B(1, 1/2) = R_B(1, 1) > R_B(1, \alpha_B)$$

for any $\alpha_B \in (1/2, 1)$. Under either the perfectly information structure or the completely uninformative structure, seller B is a monopolist when the buyer gets a low signal for A's good, and extract full surplus, while expecting no surplus when the buyer gets a high signal for A's product. Due to the special feature of our simple model that a monopolist seller has a U-shaped revenue in the information structure and is indifferent between the two extremes, both the perfectly informative structure and the completely uninformative structure are best responses against $\alpha_A = 1$.

3.5. Discussion

Because both the perfectly informative and the completely uninformative structures are best responses to a perfectly informative one, the two equilibria identifies in Propositions 5 and 6 are not strict. However, we argue that they are robust in the following sense. First, there can be a lower bound on the informativeness of the signal structure that the sellers can choose, perhaps because the buyer is endowed with some private information about the two goods. In this case, the symmetric equilibrium with perfectly informative structures becomes a strict equilibrium. Second, there could be an upper bound on the informativeness of the signal structure that the sellers can choose, perhaps because of some technological constraint. For example, suppose that sellers can only choose between a completely uninformative structure and some other structure $1/2 < \alpha < 1$. Then the symmetric profile is no longer an equilibrium, because $1/2$ does strictly better against α than α, regardless of how close α is to 1. This can be verified using the revenue formulas

(1) and (3) together with the characterization of the equilibrium strategies in the proof of Lemma 3 in the Appendix.

In the case of a monopolist seller facing one buyer, both the perfectly informative information structure and the completely uninformative structure are optimal. The outcomes are both efficient and furthermore the buyer gets no surplus in either case. In contrast, the symmetric equilibrium identified in Proposition 5 and the asymmetric equilibrium in Proposition 6 are different. In the symmetric equilibrium, each seller's expected revenue is 1/4, which is the total surplus available when his good is of high quality and the competitor's good is of low quality, and the consumer's expected surplus is also 1/4, which is the total surplus available when both goods are of high quality. Since there is no surplus when both goods are of low quality, the outcome under the symmetric equilibrium is efficient. In contrast, the asymmetric equilibrium is inefficient. The reason is that in equilibrium, after receiving a high signal from the seller with the perfectly informative structure, the buyer has a positive probability of purchasing the other good, which is of an inferior quality because the seller has a completely uninformative structure. However, the asymmetric equilibrium yields a Pareto improvement to the sellers. In particular, the expected revenue to the seller with the completely uninformative structure remains 1/4, which is the monopolist revenue when the buyer gets a low signal for the competitor's product. The expected revenue to the seller with the perfectly informative structure grows to 3/8, because the seller appropriates part of the surplus from the high signal buyer due to softened price competition that arises from the quality difference. Of course, the loss in the buyer's surplus is larger than what is appropriated by the seller because of the allocative inefficiency.

4. Concluding Remarks

The paper closest to ours is Moscarini and Ottaviani (2001). They also use a model of binary states and binary signals to study the structure of competition for an informed buyer. However, in their model the qualities of the sellers' products are perfectly negatively correlated and the buyer receives a single signal about the relative quality. The solve for pricing equilibrium and use it to address the issue of value of information in comparative statics analysis. Because of the negative perfect correlation assumption, their model cannot be used to address the issue of competition in information structures. Further, in our model the presence of four demand states, makes the analysis of price competition more complicated. For example, while they get pure strategy equilibria in price competition for sufficiently informative signals, in our model this is never the case.

Bergemann and Pesendorfer (2001) offer a general model of optimal information provision for a monopolist sellers facing multiple potential buyers. They show that the monopolist has incentive to restrict the buyers' access to private information if the gain from reduction in information rent outweighs the loss from allocative efficiency. In the extreme of a single buyer, there is no incentive for the monopolist to provide any information. Our paper is motivated by the simple observation that when sellers compete for a buyer, no information from either sellers results in Bertrand competition for undifferentiated goods and hence zero profits. The main message of the paper is that competition between sellers provides a powerful incentive for information provision through the effect of information on product differentiation in the pricing game.

Although our model of competing in information structures and prices is rather simplistic, our insights about the interaction between information structures and price competition appears to be robust. It is possible than another class of information structures with continuous signals and/or continuous states could provide a more elegant characterization of this interaction. Whether a more general, yet tractable model can be constructed is left to future investigation.

Appendix

A.1. Proof of Lemma 1

The proof is constructive. For each value of $\alpha \geq 3/4$, consider the distribution function F^α defined by

$$F^\alpha(p) = \begin{cases} 0 & \text{if} \quad p \leq \frac{1}{2}\alpha; \\ 2 - \alpha/p & \text{if} \quad \frac{1}{2}\alpha < p \leq \alpha; \\ 1 & \text{if} \quad \alpha < p. \end{cases}$$

We claim that the mixed strategy profile with the common price distribution function F^α constitutes a symmetric Nash equilibrium in the subgame in which $\alpha_A = \alpha_B = \alpha$. Given the strategy of his opponent, when charging a price $p \in [\alpha/2, \alpha]$, a seller: i) never sells when the buyer receives a low signal for his product; ii) sells with probability 1 when the buyer receives a high signal for his product and a low signal for his rival's product; iii) and sells with probability $(1 - F^\alpha(p))$ when the buyer gets two high signals. The seller's expected revenue from charging p when the opponent uses the proposed equilibrium strategy is then equal to

$$R(p; F^\alpha) = p\left(\frac{1}{4} + \frac{1}{4}(1 - F^\alpha(p))\right) = \frac{1}{4}\alpha.$$

No seller would ever want to charge a price larger than α, which is the most the buyer is willing to pay. When deviating to a price lower than $\alpha/2$ the seller's expected revenue is maximized at $p = 1 - \alpha$ and it is equal to $R(1 - \alpha; F^\alpha) = 3(1 - \alpha)/4$, which is smaller than $\alpha/4$ for the range of α's considered. This establishes the claim for $\alpha \geq 3/4$.

For $5/8 < \alpha < 3/4$, let

$$F^\alpha(p) = \begin{cases} 0 & \text{if} \quad p \leq \frac{1}{2}z; \\ \frac{3}{2} - \frac{3}{4}z/p & \text{if} \quad \frac{1}{2}z < p \leq \frac{1}{2}(1-z); \\ \frac{3}{2} - \frac{3}{2}z/(1-z) & \text{if} \quad \frac{1}{2}(1-z) < p \leq \frac{3z(1-z)}{1+2z}; \\ 2 - \frac{3}{2}z/p & \text{if} \quad \frac{3z(1-z)}{1+2z} < p \leq \frac{3}{2}z; \\ 1 & \text{if} \quad \frac{3}{2}z < p. \end{cases}$$

The distribution function is well defined for the range of α values under consideration. It is easy to verify that for any price in the support of F^α, the expected revenue to a seller who charges that price is $3z/8$. For any price $p < z/2$, the expected revenue is given by

$$R(p; F^\alpha) = p\left(\frac{3}{4} + \frac{1}{4}(1 - F^\alpha(p + z))\right)$$

and is non-decreasing in p. For any price $p \in (3z/2, (1+z)/2]$, the expected revenue is given by

$$R(p; F^\alpha) = \frac{p}{4}(1 - F^\alpha(p - z))$$

and is non-increasing in p. Finally, no deviation to a price $p \in ((1-z)/2, 3z(1-z)/(1+2z))$ can be profitable, because the likelihood of selling is the same as when charging the price $3z(1-z)/(1+2z)$. This establishes the claim for α between 5/8 and 3/4.

For $\alpha \leq 5/8$, let

$$F^{\alpha}(p) = \begin{cases} 0 & \text{if} \quad p \leq \frac{1}{2}z; \\ \frac{3}{2} - \frac{3}{4}z/p & \text{if} \quad \frac{1}{2}z < p \leq \frac{3}{2}z; \\ 1 & \text{if} \quad \frac{3}{2}z < p. \end{cases}$$

For all $\alpha \leq 5/8$ the upper bound $3z/2$ of the support of F^{α} is smaller than $1-\alpha$. A seller who charges a price $p \in [z/2, 3z/2]$ will sell with probability $1-F^{\alpha}(p)$ in the demand states in which the buyer receives the same signal on both products; will sell with certainty when his own product receives a high signal while his opponent's product receives a low signal; and will not sell otherwise. The expected revenue from charging p is then

$$R(p; F^{\alpha}) = p\left(\frac{1}{2}(1 - F^{\alpha}(p)) + \frac{1}{4}\right) = \frac{3}{8}z.$$

For any price $p < z/2$, the expected revenue is $p\,(3/4 + (1 - F^{\alpha}(p+z))/4)$, which is increasing in p and equal to $3z/8$ at $p = z/2$. Finally, for any $p > 3z/2$ the expected revenue is no larger than $(1-F^{\alpha}(p-z))p/4$, which is decreasing in p and equal to $3z/8$ for $p = 3z/2$. This establishes the claim for $\alpha \leq 3/8$ and concludes the proof of the lemma. *Q.E.D.*

A.2. Proof of Lemma 2

The proof is constructive and consists of three cases.

Case 1: High values of α. For each value of $\alpha \geq 3/4$, consider the distribution functions F_A^{α} and F_B^{α} defined by

$$F_A^{\alpha}(p) = \begin{cases} 0 & \text{if} \quad p \leq 1 - \frac{1}{2}\alpha; \\ 2 - \alpha/(p - (1-\alpha)) & \text{if} \quad \frac{1}{2}\alpha < p \leq 1; \\ 1 & \text{if} \quad 1 < p; \end{cases}$$

and

$$F_B^{\alpha}(p) = \begin{cases} 0 & \text{if} \quad p \leq \frac{1}{2}\alpha; \\ 2 - (2-\alpha)/(p + (1-\alpha)) & \text{if} \quad \frac{1}{2}\alpha < p \leq \alpha; \\ 1 & \text{if} \quad \alpha < p. \end{cases}$$

If the buyer purchases A's product whenever indifferent between the two goods, the mixed strategy profile represented by the two distribution functions F_A^α and F_B^α is a Nash equilibrium in the subgame in which $\alpha_A = 1$, and $\alpha_B = \alpha$. Straightforward calculations reveal that the expected revenues of A and B are constant and equal to $(2-\alpha)/4$ and $\alpha/4$ in their respective price supports. Neither seller would deviate to a price higher then their respective upper bound because they would never sell. Seller A has no incentive to reduce his price below $1-\alpha/2$ since this would have no effect on his probability of making a sale. Seller B's most profitable deviation is to charge a price equal to $1-\alpha$. Such deviation would yield an expected profit of $3(1-\alpha)/4$ to seller A, which is no larger than $\alpha/4$ when $\alpha \geq 3/4$.

Case 2: Intermediate values of α. For $2/3 < \alpha < 3/4$, we construct an equilibrium in which the equilibrium strategy of A is atomless. Given the support of the equilibrium strategies, the expected revenue to B must be $\alpha/4$. Since $1-\alpha$ is in the support of B's strategy, we must have

$$R_B(1-\alpha, F_A^\alpha) = (1-\alpha)\left(\frac{1}{2} + \frac{1}{4}(1 - F_A^\alpha(2(1-\alpha)))\right) = \frac{1}{4}\alpha.$$

It follows that

$$1 - F_A^\alpha(2(1-\alpha)) = \alpha/(1-\alpha) - 2,$$

which is between 1 and 0 if and only if α is between 2/3 and 3/4. Next from

$$R_B(\underline{\underline{p}}(\alpha), F_A^\alpha) = \underline{\underline{p}}(\alpha)\left(\frac{1}{4} + \frac{1}{4}(1 - F_A^\alpha(2(1-\alpha)))\right) = \frac{1}{4}\alpha,$$

we have that $\underline{\underline{p}}(\alpha) = \alpha(1-\alpha)/(2\alpha-1)$ which is between $1-\alpha$ and α in the range of α's considered. To solve for $\underline{p}(\alpha)$ we must consider two cases. If $\underline{p}(\alpha) + z \leq \underline{\underline{p}}(\alpha)$, then

$$R(\underline{p}(\alpha), F_A^\alpha) = \underline{p}(\alpha)\left(\frac{3}{4} + \frac{1}{4}(1 - F_A^\alpha(2(1-\alpha)))\right),$$

and hence $\underline{p}(\alpha) = \alpha(1-\alpha)$. This solution is valid if

$$\frac{\alpha(1-\alpha)}{2\alpha-1} - \alpha(1-\alpha) \geq 2\alpha - 1,$$

which is equivalent to

$$2\alpha(1-\alpha)^2 \geq (2\alpha-1)^2.$$

The left-hand-side of the above inequality is decreasing in α while the right-hand-side is increasing in α. Moreover at $\alpha = 2/3$ the left-hand-side is strictly larger than the right-hand-side and the opposite holds at $\alpha = 3/4$, hence there exists a unique $\alpha^* \in (2/3, 3/4)$ such that for all $(\alpha \in 2/3, \alpha^*]$, $\underline{p}(\alpha) = \alpha(1-\alpha)$. If otherwise $\underline{p}(\alpha) + z < \underline{\underline{p}}(\alpha)$, note that $\underline{p}(\alpha) + 2\alpha - 1$ is in the support of F_B^α, from

$$R(\underline{p}(\alpha), F_A^\alpha) = \underline{p}(\alpha)\left(\frac{3}{4} + \frac{1}{4}(1 - F_A^\alpha(\underline{p}(\alpha) + \alpha))\right) = \frac{1}{4}\alpha,$$

and

$$R(\underline{p}(\alpha) + z, F_A^\alpha) = (\underline{p}(\alpha) + z)\left(\frac{1}{4} + \frac{1}{4}(1 - F_A^\alpha(\underline{p}(\alpha) + \alpha))\right) = \frac{1}{4}\alpha.$$

Then, we have that $\underline{p}(\alpha)$ must solve

$$\frac{\alpha(2\alpha-1)}{\underline{p}(\alpha)(\underline{p}(\alpha) + 2\alpha - 1)} = 2.$$

To verify that this is a valid solution for values of α in $(\alpha^*, 3/4)$, it is sufficient to check that it is larger than $\alpha(1-\alpha)$ in that range. This is equivalent to

$$\frac{\alpha(2\alpha-1)}{\alpha(1-\alpha)(\alpha(1-\alpha) + 2\alpha - 1)} > 2.$$

By definition of α^*, the above is satisfied for $\alpha > \alpha^*$. Thus, we have

$$\underline{p}(\alpha) = \begin{cases} \alpha(1-\alpha) & \text{if} \quad \frac{2}{3} < \alpha \leq \alpha^*; \\ \frac{1}{2}((\sqrt{(2\alpha-1)(4\alpha-1)} - (2\alpha-1)) & \text{if} \quad \alpha^* < \alpha \leq \frac{3}{4}. \end{cases} \tag{A.1}$$

Given $\underline{p}(\alpha)$ and $\underline{\underline{p}}(\alpha)$ defined above, an atomless strategy for A with distribution function F_A^α can be constructed to satisfy $R_B(p, F_A^\alpha) = \alpha/4$ for all prices in the support of B's strategy. Since the lower bound for A's strategy is $\underline{p}(\alpha) + 1 - \alpha$, seller A's equilibrium revenue must be equal to $(\underline{p}(\alpha) + 1 - \alpha)/2$. Next we must construct a strategy for B such that for

$$p\left(\frac{1}{4}(1 - F_B^\alpha(p - (1-\alpha))) + \frac{1}{4}\right) = \frac{1}{2}(\underline{p}(\alpha) + 1 - \alpha)$$

all $p \in [\underline{p}(\alpha) + (1-\alpha), 2(1-\alpha))$,

$$p\left(\frac{1}{4}(1 - F_B^\alpha(\underline{\underline{p}}(\alpha))) + \frac{1}{4}\right) = \frac{1}{2}(\underline{p}(\alpha) + 1 - \alpha)$$

for $p = 2(1-\alpha)$, and

$$p\left(\frac{1}{4}(1 - F_B^\alpha(p - (1-\alpha))) + \frac{1}{4}(1 - F_B^\alpha(p - \alpha))\right) = \frac{1}{2}(\underline{p}(\alpha) + 1 - \alpha)$$

for $p \in [\underline{\underline{p}}(\alpha) + (1-\alpha), 1)$. The equilibrium B's strategy could have a mass point at $p = \alpha$ and will have an atom at $p = 1-\alpha$ when $\underline{\underline{p}}(\alpha) > 1-\alpha$. The mass point at $1-\alpha$ can be obtained as the difference between $F_B^\alpha(\underline{\underline{p}}(\alpha))$ from the first equation and the limit for p going to $2(1-\alpha)$ of $F_B^\alpha(p-(1-\alpha))$ from the first equation. It can be shown to have a mass be smaller than 1 within the range of α considered. The mass point at α is obtained similarly. Straightforward calculations show that neither seller has an incentive to deviate by charging a price outside of their respective equilibrium's support. This establishes the claim for $\alpha \in (2/3, 3/4)$.

Case 3: Low values of α. The case of $\alpha \leq 2/3$ is the most involved, and we do not obtain an explicit solution for $\underline{p}(\alpha)$. However, we can show that an equilibrium exists with the following properties: i) the support of B's equilibrium strategy is an interval; ii) the upper bound of B's strategy is $1-\alpha$; iii) A's equilibrium strategy is atomless; iv) the support of A's strategy is a rightward shift of that of B by $1-\alpha$.

In this proposed equilibrium, seller B's revenue can be easily obtained by noting that when charging $1-\alpha$, seller B only sells in the demand states where A's good receives a low signal. Thus, $R_B(1,\alpha) = (1-\alpha)/2$, which is also equal to $(1-z)/4$. A necessary equilibrium condition is that the lower bound price $\underline{p}(\alpha)$ satisfies

$$R_B(p, F_A^\alpha) = \underline{p}(\alpha)\left(\frac{3}{4} + \frac{1}{4}(1 - F_A^\alpha(\underline{p}(\alpha) + \alpha))\right) = \frac{1}{4}(1-z). \tag{A.2}$$

The next definition characterizes equilibria in terms of the distance between bound and lower bound of the price distribution relative to z.

Definition 1. *A price equilibrium of the subgame where* $\alpha_A = 1$ *and* $\alpha_B = \alpha > 1/2$ *has the T-step property, if* $\inf P_B(1,\alpha) + Tz \leq \sup P_B(1,\alpha)$ *and* $\inf P_B(1,\alpha) + (T+1)z > \sup P_B(1,\alpha)$.

A T-step equilibrium that also respects properties i)-iv) must satisfy the following $T+1$ equations:

$$\frac{3}{4}+\frac{1}{4}(1-F_A^\alpha(\underline{p}(\alpha)+\alpha)) = \frac{1}{4}\frac{(1-z)}{\underline{p}(\alpha)}, \tag{A.3}$$

for each $0<t<T$

$$\frac{1}{2}+\frac{1}{4}(1-F_A^\alpha(\underline{p}(\alpha)+tz+(1-\alpha)))) + \frac{1}{4}(1-F_A^\alpha(\underline{p}(\alpha)+tz+\alpha))) = \frac{1}{4}\frac{(1-z)}{\underline{p}(\alpha)+tz}, \tag{A.4}$$

and

$$\frac{1}{2}+\frac{1}{4}(1-F_A^\alpha(\underline{p}(\alpha)+TZ+(1-\alpha)))) = \frac{1}{4}\frac{(1-z)}{\underline{p}(\alpha)+Tz}. \tag{A.5}$$

Since $\underline{p}(\alpha)+tz+(1-\alpha)=\underline{p}(\alpha)+(t-1)z+\alpha$, the above is a system of $T+1$ equations in $T+1$ unknowns. Moreover, by adding up separately the equations with t odd and those with t even, we have that when T is odd, the value of $\underline{p}(\alpha)$ that solves the above system satisfies

$$\sum_{i=0}^{(T-1)/2} \frac{z(1-z)}{(\underline{p}(\alpha)+2iz)(\underline{p}(\alpha)+(2i+1)z)} = 1, \tag{A.6}$$

and when T is even it satisfies

$$\sum_{i=1}^{T/2} \frac{z(1-z)}{(\underline{p}(\alpha)+2(i-1)z)(\underline{p}(\alpha)+(2i-1)z)} + \frac{1-z}{\underline{p}(\alpha)+Tz} = 3. \tag{A.7}$$

For every z, both (A.6) and (A.7) have a unique positive solution. Let $p(z;T)$ denote the solution to the corresponding equation. Note that $p(z;T)$ is a valid lower bound to B's equilibrium strategy if it satisfies the T-step property, that is, if

$$(1-\alpha)-z < p(z;T)+Tz \leq 1-\alpha.$$

Moreover, since $p(z;T)$ also solves (A.3), we have

$$(1-z)/3 \geq p(z;T) \geq (1-z)/4. \tag{A.8}$$

The next series of lemmas characterizes additional properties of $p(z;T)$.

LEMMA A.2. *Suppose T is an odd number and $z<z'$. If $p(z;T)$ and $p(z';T)$ are both valid, then $p(z;T)<p(z';T)$.*

LEMMA A.3. *Suppose T is an even number and $z < z'$. If $p(z;T)$ and $p(z';T)$ are both valid, then $p(z;T) > p(z';T)$.*

LEMMA A.4. *For all z, if T is an odd number, then $p(z;T) \leq p(z;T+2)$; and if T is an even number, then $p(z;T) \geq p(z;T+2)$.*

LEMMA A.5. *If $p(z;T) + (T+1)z = 1-\alpha$, then $p(z;T+1) = p(z;T)$.*

Lemma (A.2)-(A.5) imply that there exists a decreasing sequence of positive numbers converging to zero, $\{z_t\}_{t=0}^{\infty}$, such that $(1-\alpha) - z < p(z;t) + tz \leq (1-\alpha)$ if and only if $z \in (z_{t+1}, z_t]$. Moreover, when $t = 0$, from (A.3) we get $p(z;0) = 2(1-\alpha)/3$. from the inequality $1 - \alpha - 2(1-\alpha)/3 \leq 2\alpha - 1$ we get that $z_0 = 2/3$. This establishes that the conjectured equilibrium strategy can be constructed for A. Once $\underline{p}(\alpha)$ is determined, the equilibrium strategy of A and B can be obtained. The strategy of A will be atomless by construction, while that of B will have an atom at $1-\alpha$. It can be verified that neither seller has an incentive to charge a price outside the equilibrium support. This concludes the proof of the claim. *Q.E.D.*

A.3. Proof of Lemma 3

The proof is constructive. We distinguish three cases.

First, for each $\alpha \geq (\sqrt{13}+5)/12$ let the strategies of A and B be given by

$$F_A^{\alpha}(p) = \begin{cases} 0 & \text{if} \quad p \leq \alpha - \frac{1}{4}; \\ 2 - 1/(2p - (2\alpha - 1)) & \text{if} \quad \alpha - \frac{1}{4} < p \leq \alpha; \\ 1 & \text{if} \quad \alpha < p; \end{cases}$$

and

$$F_B^{\alpha}(p) = \begin{cases} 0 & \text{if} \quad p \leq \frac{1}{4}; \\ 1 - (\alpha - 1/4)/(p + \alpha - 1/2) & \text{if} \quad \frac{1}{4} < p \leq \frac{1}{2}; \\ 1 & \text{if} \quad \frac{1}{2} < p. \end{cases}$$

Note that while A's strategy is atomless, B's strategy has a mass point at $p = 1/2$. We claim that if the buyer purchase A's good when indifferent, the mixed strategy profile represented by the distribution functions F_A^α and F_B^α is a Nash equilibrium in the subgame in which $\alpha_A = \alpha$ and $\alpha_B = 1/2$. It is easy to verify that both A's and B's revenues are constant and equal to $\alpha/2 - 1/8$ and $1/4$ respectively, for all prices in the support. Seller B's revenue would be zero for any price above $1/2$, and at $p = 1/4$ seller B sells with certainty. No deviation would be profitable. Seller A would never find it profitable to deviate to a price in $(1-\alpha, \alpha - 1/4)$, nor to a price larger than α. Moreover, it can be verified that A's expected revenue increases for prices below $1-\alpha$. Finally, when charging a price of $1-\alpha$, seller A's expected revenue is no greater than $\alpha/2 - 1/8$ for all $\alpha \geq (\sqrt{13}+5)/12$.

For $(4-\sqrt{2})/4 < \alpha \leq (5+\sqrt{13})/12$ we construct an equilibrium with

$$P_A(\alpha, 1/2) = [\underline{p}_A(\alpha), 1-\alpha] \cup [\underline{\underline{p}}_A(\alpha), \underline{p}_A(\alpha) + 2\alpha - 1]$$

and

$$P_B(\alpha, 1/2) = [\underline{\underline{p}}_A(\alpha) - z/2, 1/2],$$

such that A's strategy is atomless and B's strategy has an atom at $1/2$. The tie breaking rule favors A. Denote as m_B the mass at $1/2$ for B. The necessary equilibrium conditions are

$$\begin{aligned}
R_A &= \frac{1}{2}\underline{p}_A(\alpha)(1 + (1 - F_B^\alpha(\underline{p}_A(\alpha) + z/2)));\\
R_A &= \frac{1}{2}(1-\alpha)\,(1 + m_B)\,;\\
R_A &= \frac{1}{2}\underline{\underline{p}}_A(\alpha);\\
R_A &= \frac{1}{2}(\underline{p}_A(\alpha) + z)(1 - F_B^\alpha(\underline{p}_A(\alpha) + z/2));
\end{aligned}$$

and

$$\begin{aligned}
R_B &= \frac{1}{2}(\underline{\underline{p}}_A(\alpha) - z/2)(1 + (1 - F_A^\alpha(1-\alpha)));\\
R_B &= \frac{1}{4}(1 - F_A^\alpha(1-\alpha));\\
R_B &= \frac{1}{2}(\underline{p}_A(\alpha) + z/2).
\end{aligned}$$

The above system of equation can be reduced to two equations in $\underline{p}_A(\alpha)$ and $\underline{\underline{p}}_A(\alpha)$:

$$\begin{aligned}
&\underline{p}_A(\alpha) + z/2 = (\underline{\underline{p}}_A(\alpha) - z/2)(1 + 2(\underline{p}_A(\alpha) + z/2));\\
&(\underline{p}_A(\alpha) + z)\underline{p}_A(\alpha) = z\underline{\underline{p}}_A(\alpha).
\end{aligned}$$

The solution is valid if $\underline{p}_A(\alpha) \leq 1-\alpha \leq \underline{\underline{p}}_A(\alpha)$. It can be verified that a unique valid solution exists if $(4-\sqrt{2})/4 < \alpha \leq (5+\sqrt{13})/12$, with $\underline{p}_A(\alpha)$ greater than $z/\sqrt{2}$ and increasing. The distribution functions F_A^α and F_B^α can be constructed and the no deviation conditions can be verified.

For $\alpha \leq (4-\sqrt{2})/4$, let the strategies of A and B be given by

$$F_A^\alpha(p) = F_B^\alpha(p) = \begin{cases} 0 & \text{if} \quad p \leq \frac{1}{\sqrt{2}}z; \\ 1 - \frac{(\sqrt{2}+1)z}{2p+z} & \text{if} \quad \frac{1}{\sqrt{2}}z < p \leq \frac{\sqrt{2}+1}{2}z; \\ 2 - \frac{(\sqrt{2}+1)z}{2p-z} & \text{if} \quad \frac{\sqrt{2}+1}{2}z < p \leq \frac{\sqrt{2}+2}{2}z; \\ 1 & \text{if} \quad \frac{\sqrt{2}+2}{2} < p. \end{cases}$$

Given the above equilibrium strategies, and under the assumption that

$$\frac{\sqrt{2}+1}{2}z \leq 1-\alpha, \tag{A.9}$$

the expected revenue is constant for all price in the support of the distribution, and given by

$$R_A(\alpha, 1/2) = R_B(\alpha, 1/2) = \frac{1}{4}(\sqrt{2}+1)z.$$

Straightforward calculations reveal that the expected revenue after a unilateral deviation to a price $p > (\sqrt{2}+2)z/2$, is decreasing in p. Similarly, the expected revenue after a unilateral deviation to a price $p < z/\sqrt{2}$, is increasing in p. The inequality (A.9) ensures that by charging a price equal to $(\sqrt{2}+1)z/2$ each seller sells with probability 1 in the demand state that favors him. The inequality holds if and only if $\alpha \leq (4-\sqrt{2})/4$.

Q.E.D.

A.4. Proof of Proposition 4

We prove the claim by establishing that $R_A(1,\alpha) > R_A(\alpha,\alpha)$ for all $\alpha < 1$. For $\alpha \leq 2/3$, we have

$$R_A(1,\alpha) = \frac{1}{2}(\underline{p}_B(\alpha) + 1 - \alpha) \geq \frac{3}{4}(1-\alpha) > \frac{3}{4}(\alpha - 1/2) = R_A(\alpha,\alpha),$$

where the second inequality follows from the bound $\underline{p}_B(\alpha) \geq (1-\alpha)/2$ established in the proof of Lemma 2. For $2/3 < \alpha \leq 3/4$, an explicit solution for $\underline{p}_B(\alpha)$ is given by (A.1). Direct comparison of the revenue formulas for $R_A(1,\alpha)$ and $R_A(\alpha,\alpha)$ reveals that the former is larger for all α in this range. For $3/4 < \alpha$, we have that

$$R_A(1,\alpha) = \frac{1}{4} \geq \frac{1}{4}\alpha = R_A(\alpha,\alpha).$$

This concludes the proof of the proposition. *Q.E.D.*

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