

Misinformation*

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Abstract

A political candidate has private information about his own qualifications and his rival's qualifications. He can choose to target himself in a positive campaign or his rival in a negative campaign, and he can choose how informative his campaign is. A more informative positive (negative) campaign generates a more accurate public signal about his (the rival's) qualifications, but is more costly. In the basic two-type model, a high type candidate has a comparative advantage in negative campaigns if he can lower the voter's opinion about his rival more effectively than raise her opinion about himself than the low type; and a comparative advantage in positive campaigns otherwise. In equilibrium, this comparative advantage, not the qualifications of the candidate or his rival, determines whether the high type candidate goes positive or negative. Additional ex post public information about the candidate (his rival) strengthens (weakens) the comparative advantage in positive (negative) campaigns. Allowing both positive and negative campaigns does not help the high type to separate, while allowing information campaigns by both candidates does.

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1 Introduction

Much attention has been paid to the use of positive versus negative campaigns in electoral campaigns, partly due to the explosive growth of negative campaigns in recent years.¹ Broadly speaking, a candidate uses positive campaigns to praise one's own qualifications and negative campaigns to discredit a rival's qualifications. The focus of earlier research in this area is on how negative campaigns may alienate voters because they lower the voters' opinions of the candidates involved and consequently depress voter turnout (Ansolabehere, Iyengar, Simon, and Valentino 1994, Skaperdas and Grofman 1995). More recent work, however, suggests that negative campaigns provide valuable information to the voters and they may not alienate the voters (Kahn and Geer 1994, Lau and Rovner 2007, Sides, Lipsitz, and Grossman 2010). This view is also supported by practitioners and political consultants.²

This paper focuses exclusively on the information provision role of political campaigns. Viewing through the lens of what information, and how much information a candidate allows the voters to observe through his campaigns to signal his qualifications, we can address an important question currently under debate. Namely, what leads a candidate to run positive or negative campaigns, and how the voter evaluates the candidate from his campaigns. We show that, in addition to the individual characteristics of a candidate vis a vis those of his rivals, the voter's prior knowledge of the possible characteristics a candidate may possess also matters in campaign choices. In particular, our results suggest that voters do not necessarily think well of a candidate who runs positive campaigns, nor do they think poorly of one who runs negative campaigns. Instead, the more informative a given campaign is, positive or negative, the stronger a candidate is perceived to be.

In the basic model, a candidate can be one of two types, with each type having imperfect

¹ For instance, in the 2006 midterm Congressional election, 90% of ads run in the final 60 days of all the House and Senate campaigns nationwide were negative. Susan Page, "Nasty Ads Close Out a Mud-Caked Campaign," USA Today online article (<http://tinyurl.com/29k9xs>), November 2, 2006. Also, using Wisconsin Advertising Project analysis of CMAG (Campaign Media Analysis Group) data from 2000-2008 as points of comparison, Wesleyan Media Project found that negative ads have steadily increased from about 27% per party in the 2004 election to about 50% per party in the 2010 House and Senate midterm election.

² In US News & World Report, October 6, 2008, Dick Morris pointed out: "Negative ads work and have their place....Negative ads are often the only way voters can penetrate the claims of the various campaigns and get the facts. Voters always tell pollsters that they hate negative ads, but politicians continue to run them. That's because the same polls show that they work."

information about his own qualifications and his rival's qualifications. The candidate's type is modeled as a pair of beliefs of how likely he and his rival are qualified, which we refer to as his own strength and the rival's strength respectively. The median voter knows the values of these strengths associated with each type, but the candidate alone knows which type he is. The candidate signals his type to the median voter by running a campaign, which generates a publicly observed campaign signal. His choices include the target of his campaign, which can be either his own qualifications ("positive campaign") or his rival's ("negative campaign"); and the informativeness ("level") of his campaign, which is the precision of the campaign signal the voter observes. We assume that the campaign signals are noisy but unbiased. The noise may, for instance, result from random shocks to the voter's preference. Running a more informative campaign reduces such noise, but does not make the campaign signals systematically more favorable to the candidate or less favorable to his rival. A more informative campaign is assumed to cost more, because it takes more research and time for a candidate to establish (or to refute) detailed, specific claims than to provide feel-good sound bites. The voter is able to observe the candidate's campaign choices—whether the campaign is positive or negative and whether it is informative—and the realized campaign signal. Then the voter rationally forms her own opinions about the candidates' qualifications. Each candidate seeks to maximize the expected difference between the voter's opinion about his qualifications over that about his rival, net of the cost.

In this signaling model with two-dimensional types, who is overall the stronger candidate, or the high type, is determined by who should come out ahead if the candidate's type was known. The high type has a greater difference between his strength and that of the rival, which we refer to as his overall strength. Clearly, the high type candidate has incentives to inform the voter of his overall strength; but the low type prefers to misinform the voter. Consider the scenario in which the high type is stronger than the low type, but he also faces a stronger rival than the low type does. Suppose that he runs an informative positive campaign in equilibrium so that the voter can learn more about himself. Should the low type imitate? The answer is "no." The low type is more likely to get an unfavorable campaign signal than the high type, because he is less likely to be qualified. In expectation, the more informative the positive campaign is, the less successful the low

type is in pretending to be the high type. This smaller and decreasing benefit for the low type to imitate the high type gives rise to a least cost separating equilibrium in positive campaigns.

It may seem that the high type can signal his overall strength by simply running a sufficiently informative campaign, whether it is positive or negative. This, however, is not true. Suppose our high type candidate in the above scenario goes negative, then he ends up showcasing his rival's qualifications. Because he actually faces a stronger rival than the low type does, under any negative campaign the voter will have a higher opinion of his rival in expectation. Because the high type candidate is more successful in persuading the voter that he is strong than his rival is weak, he should run positive campaigns. In this case, we say that the high type candidate has a comparative advantage in running positive campaigns. Similarly, if the high type himself is weaker than the low type but faces a weaker rival than the low type does, then he should go negative because he has a comparative advantage in negative campaigns.

The issue of where the comparative advantage lies is more subtle when the candidate can use either positive or negative campaigns to separate. This occurs when the high type candidate is in the best position: he is stronger than the low type, and he faces a weaker rival than the low type does. Unlike standard models, we cannot study how a high type's comparative advantage varies with only his own strength or his rival's because his type is two-dimensional. Rather, we must hold the overall strength of a high type candidate constant—the candidate's own strength and his rival's strength change at the same rate—to isolate the comparison between his advantage in positive versus negative campaigns. We find an intuitive, sufficient condition that ensures that, for a fixed overall strength, the high type's comparative advantage in positive campaigns increases in his own strength. Under this condition, the stronger the high type candidate is, the more difficult it is for the low type to run positive campaigns because he suffers more from a downgrade in the voter's opinion of him after an unfavorable campaign signal than an upgrade after a favorable one. As a result, the high type candidate needs to run a less informative positive campaign to separate. At the same time, because the overall strength is fixed, the corresponding increase in the high type's rival's strength implies that it is easier for the low type to run negative campaigns to imitate the high type because he can more successfully lower the voter's opinion of his rival. Thus, the

high type needs to run a higher level of negative campaigns to separate, leading to a monotonic characterization of the high type's comparative advantage.

The insight that comparative advantage determines the choice of positive versus negative campaigns is not driven by the restriction that only one kind of campaign may be used. The candidate may not use "contrast ads" in which he runs both a positive and a negative campaign even if he can. This is intuitive if the high type can only run a positive or a negative campaign to signal his overall strength. Allowing him to run a kind of campaign that he avoids in the first place does not help separation. It also holds when both positive and negative campaigns can be used for separation if the campaign cost is concave in campaign levels. Intuitively, as the high type "substitutes" one kind of campaign for another, say by increasing the positive campaign level and simultaneously decreasing the negative campaign level to deter the low type from imitation, the deterrence through the positive campaign becomes more effective relative to the negative campaign, while the impact on the cost of positive campaign declines relative to the negative campaign due to concavity.

The basic model is then extended to understand the candidate's campaign choices under more realistic settings. In one extension, the voter expects to receive independent evidence about the candidates' qualifications. We show that the presence of additional information hurts the high type candidate if it reduces his comparative advantage in a particular campaign, for instance, if the additional information is about his rival when the candidate would rather run a positive campaign about himself. We also show that our characterization of the comparative advantage is robust when the candidate and his rival can run competing campaigns, and further, competition lowers campaign levels due to strategic substitution. Finally, in a winner-take-all model we show that separation becomes more difficult, but is still driven by the high type candidate's comparative advantage.

Our model predicts that a negative campaign could be effective for a high type candidate, and in fact, the voter's perception of the candidate's qualifications may not deteriorate, and could even improve, following an informative negative campaign. This is because a candidate's own qualifications or those of his rival's are not sufficient to predict the use or the effectiveness of negative campaigns. Instead, how negative campaigns are used in equilibrium depends on the voter's prior knowledge of the candidate's alternative types. Unfortunately, the latter is difficult to measure

and observe, which may lead to seemingly conflicting empirical and experimental findings when the only available data is on the candidates' qualifications and their campaign choices. In general, our analysis suggests that any policy enhancing a high type candidate's comparative advantage in a kind of campaign makes it easier for him to signal his type and reduces his campaign cost, and vice versa. For example, banning negative campaigns cannot make a high type candidate better off, and it can make him strictly worse off if his comparative advantage lies in negative campaigns.

The majority of existing literature has focused on the effect of negative campaigns on voter behavior. Skaperdas and Grofman (1995) assume that negative campaigns reduce voter support for both the target and the sponsor of such campaigns without formally modeling why such a negative effect arises. In a complete information model, Harrington and Hess (1996) consider a Hotelling model in which a candidate's characteristics are known, but a candidate can, via campaign expenditures, move toward the swing voter's preferred ideology through a positive campaign; or he can move the rival's ideology away from the voter through a negative campaign.³ Our model differs from Harrington and Hess (1996) in that the candidate has private information about both his and his rival's characteristics, which are exogenously given. He can signal his characteristics through informative campaigns that produce unbiased evidence in expectation, but he cannot alter these characteristics to influence the voter.

Two recent papers are more closely related to the present model in that they focus on the role of information in electoral campaigns. Polborn and Yi (2006) consider a disclosure model in which a candidate is assumed to know the characteristics of both himself and his rival, but he can only verifiably disclose one dimension. Their main result is that the higher is the value of the disclosed characteristics, the lower is the expected value of the undisclosed dimension inferred by the voter in equilibrium. This implies that a candidate is more likely to choose a negative campaign when his own characteristics are bad, and a positive one when his rival's characteristics are good. While their result relies on the restriction that only one dimension can be disclosed, we have a signaling model in which the candidate is imperfectly informed. In our model, the level of campaign plays a critical

³ In a related model, Heidhues and Lagerlof (2003) show that when two candidates have correlated private information about a one-dimensional state variable, they do not reveal their private information truthfully in their platforms. Rather, they bias their platforms toward the voter's prior beliefs.

role in generating the comparative advantage characterization that does not rely on the restriction. An informative negative campaign is not an attempt to hide one’s own lack of qualifications, but rather an effective way to signal the candidate’s overall strength. More recently, Lovett and Shachar (2010) consider a model in which a candidate with both good and bad traits needs to allocate his budget optimally: if the voter has more knowledge of his good traits, he spends more money on negative ads about the rival’s bad traits and vice versa. Their model differs from ours in that the voter’s learning is non-Bayesian.

Our work is also related to the extant literature on advertising. Nelson (1974) shows that advertising may inform consumers directly through hard information that reduces search cost, and suggests that advertising may also provide soft information to signal product quality. In a study of electoral competition, Coate (2004) takes the former view and assumes that a candidate can provide truthful information about himself to win over the swing voters, rendering signaling useless by assumption. In contrast, many models assume that the actual content of advertisement is uninformative, but the advertising expenditure can be a costly signal of quality. For instance, Prat (2002) considers a model in which voters see the amount of campaign contributions as a costly signal of interest groups who have private information about the candidate’s qualifications. The present model incorporates both roles of advertising: the informativeness of a campaign is endogenously chosen as a signal of quality; but voters are also informed directly through the realized campaign signal over which the candidate only has imperfect control.

A distinguishing theoretical feature of our model is that the privately informed sender uses information structure—both the kind and the informativeness of a campaign—as a signal. Kamenica and Gentzkow (Forthcoming) share with this paper the feature that both the sender’s choice of signal and the realized signal are observable. The main difference is that in their model, the sender has no private information and chooses an optimal information structure to improve his expected payoff by changing the distribution of the receiver’s posterior beliefs. Therefore they have a model of “persuasion”, while ours is a signaling model. The optimal choice of information structure has also been studied in the auction design model of Bergemann and Pesendorfer (2007); and in duopoly games by Ottaviani and Moscarini (2001) and Damiano and Li (2007).

2 The Basic Model

There are two political candidates, a and b . Each candidate is either qualified or unqualified for a political office. In the basic model, only candidate a is a player (the sender) in the signaling game described below. Candidate a may be one of two types, denoted as type (α_L, β_L) and type (α_H, β_H) respectively.⁴ Each type is a pair of beliefs about the qualifications of a and b : the first component represents a 's private belief that he is qualified, while the second component represents his private belief that his rival is qualified. These beliefs are referred to as the *strength* of a and b respectively. Candidate a is type (α_L, β_L) with probability $\lambda \in (0, 1)$ and type (α_H, β_H) with probability $1 - \lambda$. The candidate's type is private information, but the values of (α_L, β_L) and (α_H, β_H) , as well as the type distribution are common knowledge between the voter and candidate a .

Define an *information campaign* as an observable choice of information structure—a distribution of a public signal about the qualifications of candidate a or b . An information campaign is positive if it generates a signal about candidate a 's qualifications (the *target* is a), and negative if it is about b (the target is b). Each information campaign generates either a favorable signal \bar{s} or an unfavorable signal \underline{s} about the target of the campaign. The precision of this campaign signal is $k \in [\frac{1}{2}, 1)$, which is the *level* of the campaign. More specifically, k is both the probability of the signal being \bar{s} conditional on that the targeted candidate is qualified and the probability of the signal being \underline{s} conditional on that the target is unqualified.

The median voter, the receiver in our signaling game, first observes candidate a 's campaign choices, which includes both the kind and the level of the campaign, and then observes the realized campaign signal. To focus on information provision, we assume that the voter is not a strategic player of this game: she simply uses Bayes' rule to form a pair of posterior beliefs about the qualifications of both candidates. These beliefs, denoted as π_a and π_b , together with a campaign cost function $C(k)$, determine the payoff to candidate a . In the basic model, candidate a maximizes the difference of the voter's posterior belief about himself over b , net of any campaign cost. The

⁴ No restriction or ordering is placed on parameter values α_L, α_H and β_L, β_H to allow for a full characterization. In the analysis, we show explicitly the condition that identifies a candidate as the high type or low type.

payoff to a is

$$\pi_a - \pi_b - C(k),$$

where C is continuous and strictly increasing, with $C(\frac{1}{2}) = 0$.⁵

For simplicity, candidate a 's private type is modeled directly as a pair of beliefs about whether he and his rival are qualified. Instead, we can explicitly model how candidate a forms his beliefs— (α_L, β_L) and (α_H, β_H) —after observing a private, imperfect signal. To do so, we need to specify a signal structure conditional on the four underlying states, which are the candidates' true qualifications. This is done in Section 4 to study electoral competition between the candidates, but in the basic model, such structure is unnecessary and merely complicates the notation.

In this model, candidate a cannot directly control the realization of the campaign signal, which is consistent with the idea of information provision. Although we assume that the voter can perfectly observe the informativeness of a campaign for simplicity, all our results hold qualitatively if the voter only observes a noisy measure of the true informativeness of a campaign. The fact that voters can judge the relative informativeness of a campaign is supported by empirical research in marketing and media studies. For instance, using survey and advertising data from the 2000 presidential campaign and two 1998 gubernatorial races, Sides, Lipsitz, and Grossman (2010) show that citizens separate judgments about the tone of a campaign (positive or negative) from judgments about the quality of information they have received.⁶ Further, we have implicitly assumed that candidate a cannot simultaneously run both a positive and a negative campaign in order to focus on his choice of campaign target. Section 3.3 extends the analysis to “contrast campaigns” in which a can run both a positive and a negative campaign, and shows that the candidate prefers to run only one campaign under reasonable assumptions.

Information campaigns are assumed to be costly; and a higher level of campaign, whether positive or negative, costs more than a lower level one. The idea is that it costs little for the candidate to gloat about himself; but much more is required to establish or to refute detailed claims based on the biographical, legal, educational, financial, or the voting records of a candidate.

⁵ The assumptions of strict monotonicity and zero fixed cost on the function C ensure the existence of a least cost separating equilibrium. They are made to simplify the analysis and are not crucial to our results.

⁶ In particular, the voter can judge whether a political campaign “gave voters a great deal of useful information, some, not too much, or no useful information at all?”

Such research cost, which depends on the informativeness of the campaign, is a non-negligible part of campaign expenditures.⁷ To focus on how the candidate's campaign choices depend on his characteristics, we assume that there is one continuous and strictly increasing cost function in both kinds of campaigns. Our analysis extends easily to allow different cost for positive and negative campaigns, capturing possible adverse social effect of negative campaigns such as turnout suppression (Ansolabehere, Iyengar, Simon, and Valentino 1994).

In this paper, the candidates' payoffs are modeled in a reduced form, which can be endogenized without affecting the results qualitatively. The chosen payoff specifications have natural interpretations in the context of political campaigns. The basic model is appropriate in a parliamentary system where the number of seats is proportional to the voter's support. In comparison, Section 5 considers a winner-take-all model to study how a plurality electoral system affects the candidate's information campaign in equilibrium and the impact on the voter.

Before turning to the analysis, we want to briefly mention two other possible applications of this model. First, the senders, a and b , are two companies competing for market share in a given market and the decision maker represents the consumers. Company a aims to increase its market shares at the expense of its rival. Each company has private information about the quality of both products, perhaps through past interactions and market analysis. A company can let the consumer observe a signal about the quality of his own product through advertising, free trials and other promotions. It can also adjust the informativeness of its signal by, for instance, varying the frequency of its advertisements or the numbers of features available in the free trials. Alternatively, he can send a signal about the rival's product such as bad safety records or low consumer protection agency ratings. The company chooses the signal that is most likely to sway the average consumer opinion in its favor. In the second application, the senders are two defendants charged for a certain crime and the decision maker is the judge. A defendant (or his legal representation) needs to decide whether to present the judge with evidence about himself such as possible alibis or testimony from expert witnesses. A defendant can also point toward motives or opportunities of the other defendant. The judge evaluates these evidence rationally.

⁷ For instance, using Federal Election Commission data, Center for Responsive Politics shows that in the 2008 presidential campaign, such research cost and consultant fee amounted to approximately \$7 million.

3 Equilibrium Information Campaigns

We look for Perfect Bayesian equilibria of this game in which the candidate makes campaign choices to maximize his expected payoff and the voter updates her belief according to Bayes' rule on the equilibrium path. The main modeling innovation is that the signal here is not an action as in a typical signaling game, but an information structure. This feature creates an important role for the voter's *interim* belief—the belief she forms about the candidate after observing the campaign choices, but before observing the realized campaign signal. In equilibrium, the voter forms the correct interim beliefs about the candidate from his campaign choices, and then adjusts her beliefs after she observes a favorable or an unfavorable signal by Bayes' rule.

Since candidate a only has imperfect information about both candidates' qualifications, the realized campaign signal can be favorable or unfavorable. Therefore we begin the analysis by investigating how candidate a 's campaign choices affect the voter's expected posterior beliefs about the qualifications of the candidates.

Suppose candidate a runs an informative positive campaign of level k . Further, suppose that the candidate's own strength is α and the voter's interim belief about him is $\tilde{\alpha}$.⁸ Then the voter's expected posterior belief about candidate a , $\Pi(\alpha, \tilde{\alpha}; k)$, is given by

$$\begin{aligned} \Pi(\alpha, \tilde{\alpha}; k) = & (\alpha k + (1 - \alpha)(1 - k)) \frac{\tilde{\alpha} k}{\tilde{\alpha} k + (1 - \tilde{\alpha})(1 - k)} \\ & + (\alpha(1 - k) + (1 - \alpha)k) \frac{\tilde{\alpha}(1 - k)}{\tilde{\alpha}(1 - k) + (1 - \tilde{\alpha})k}, \end{aligned} \quad (1)$$

where the first fraction gives how the voter upgrades her posterior belief about a 's qualifications after observing a favorable signal \bar{s} , and the second fraction is how she downgrades her opinion after an unfavorable signal \underline{s} . Clearly, the function Π increases in the candidate's strength: the higher is α , the more likely the voter will observe a favorable signal. The function Π also increases in the voter's interim belief $\tilde{\alpha}$: the stronger she thinks the candidate is, the more favorably she interprets each realized campaign signal. Moreover, because a more informative signal is more convincing,

⁸ The voter's interim belief and campaign level k are endogenously determined in equilibrium. The current exercise aims to illustrate the incentives of a candidate to manipulate the voter's expected posterior beliefs for fixed values of α , $\tilde{\alpha}$ and k .

the voter’s opinion of the candidate after a favorable signal increases in k while her opinion after an unfavorable signal decreases in it. Inspection of expression (1) leads to the following result.

Lemma 1 *(i) $\Pi(\alpha, \tilde{\alpha}; k) = \alpha$ if $\tilde{\alpha} = \alpha$; (ii) $\Pi(\alpha, \tilde{\alpha}; k)$ decreases in k if $\alpha < \tilde{\alpha}$; and (iii) $\Pi(\alpha, \tilde{\alpha}; k)$ increases in k if $\alpha > \tilde{\alpha}$.*

Albeit simple, Lemma 1 is important in understanding the direction of a candidate’s attempt to influence the voter.⁹ Part (i) shows that if the voter has the correct interim belief of the candidate’s strength, there is no value to an information campaign. No campaign can change the voter’s expected posterior belief by the law of iterated expectations, as the expected upgrade of the voter’s opinion is cancelled by the downgrade.¹⁰ If instead, the voter’s interim belief is different from the candidate’s private belief, candidate a can influence the voter’s perception by adjusting how informative his campaign is. If candidate a is privately less confident about his own qualifications than the voter, part (ii) of Lemma 1 shows that he would like to partly “hide” the bad news by reducing the informativeness of his campaign signal. Intuitively, if the voter overestimates the candidate given his campaign choices, the later observed informative campaign signal can only lower her opinion of the candidate on average. But a less informative campaign signal, favorable or unfavorable, is less effective in lowering the voter’s belief. Part (iii) shows that if candidate a is privately more confident about his qualifications than the voter, then he would like to choose a more informative campaign to highlight the good news about himself.

The voter’s expected posterior belief $\Pi(\beta, \tilde{\beta}; k)$ for candidate b after a negative campaign of level k , given private belief β and interim belief $\tilde{\beta}$, can be similarly derived. Naturally, in a negative campaign, candidate a lowers (raises) the voter’s perception about his rival by running a more informative campaign if he has worse (better) news about the rival than the voter believes.

⁹ Lemma 1 also holds if the voter can only observe a noisy, but unbiased, signal of campaign level k . In particular, the noisier is the voter’s observed signal, the less incentive the candidate has in running informative campaigns.

¹⁰ The marginal value of information to the voter, however, is always positive if we model the voter as choosing an action x to minimize expected loss $(x - q)^2$ where $q = 1$ if the candidate is qualified and $q = 0$ if he is unqualified.

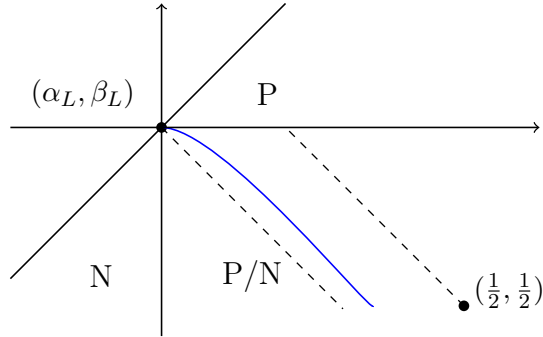


Figure 1: Equilibrium campaign choices of type (α_H, β_H)

3.1 Least cost separating equilibrium

Because campaign choices are a signal of the candidates' qualifications, we focus on separating equilibria in which the voter learns the candidate's private type. Unlike the assumption implicit in some of the political science literature, in the present model incentives to separate are not determined by whether a candidate type is stronger or weaker than his rival candidate b , for instance, whether α_H is larger or smaller than β_H . Rather, they depend on the comparison of the payoffs that the two types receive under complete information. If the candidate's type was known, the high type is the one that has a greater difference in strength between himself and the rival, or

$$\alpha_H - \beta_H > \alpha_L - \beta_L.$$

We refer to the above difference $\alpha_H - \beta_H$ as the *overall strength* of the high type.

There are three cases regarding the location of type (α_H, β_H) in the α_H - β_H parameter space, holding type (α_L, β_L) fixed. In the first case, referred to as the P-region, we have $\alpha_H > \alpha_L$ and $\beta_H > \beta_L$: the high type candidate himself is stronger than the low type but also faces a stronger rival than the low type does. In the second case, referred to as the N-region, we have the opposite scenario of $\alpha_H < \alpha_L$ and $\beta_H < \beta_L$. In the third case, referred to as P/N-region, both $\beta_H \leq \beta_L$ and $\alpha_H \geq \alpha_L$ hold, with at least one strictly. The high type candidate a is stronger than the low type and faces a weaker rival than the low type does. These three regions are illustrated in Figure 1.

Regardless of the location of the high type relative to the low type, there is always a separating

equilibrium in which the low type runs an uninformative, costless campaign, while the high type uses an informative campaign as a costly sign of his overall strength. As is standard in the signaling literature, we focus on the least cost separating equilibrium. The existence and uniqueness of the least cost separating equilibrium is then a direct consequence of Lemma 1.¹¹

Proposition 1 *In any separating equilibrium, the high type candidate runs a positive campaign in the P-region and a negative campaign in N-region, and he may run either kind of campaign in P/N-region. Further, a least cost separating equilibrium exists and is generically unique.*

Should the high type candidate, the type with greater overall strength, inform the voter about his own strength or about his rival's? Proposition 1 shows that the location of the high type candidate a , as represented by the three regions, is important in answering this question. To begin with, suppose that the high type is in the P-region and suppose that there exists a separating equilibrium of level k^p . Then in equilibrium, the high type candidate a receives $\Pi(\alpha_H, \alpha_H; k^p)$, which is simply α_H by part (i) of Lemma 1. If the low type candidate runs the same campaign to imitate him, then because $\alpha_H > \alpha_L$, the low type gets $\Pi(\alpha_L, \alpha_H; k^p)$ which is strictly smaller than α_H by part (ii) of Lemma 1. Intuitively, the low type candidate is less successful in raising the voter's expected posterior belief of his qualifications than the high type for any informative positive campaign. Moreover, the greater is k^p , the smaller is the low type's gain from imitating the high type. Therefore there exists a unique campaign level k_H^p , given by

$$\alpha_L - \beta_L = \Pi(\alpha_L, \alpha_H; k_H^p) - \beta_H - C(k_H^p), \quad (2)$$

such that the low type is indifferent between running an uninformative campaign and imitating the high type by running a positive campaign. In essence, the high type candidate can signal his overall strength in a positive campaign because he is informing the voter in the dimension in which he is stronger than the low type. Similarly, in the N-region, the high type should signal his overall strength in a negative campaign because he faces a worse rival than the low type: $\beta_H < \beta_L$. The

¹¹ Lemma 1 also implies that the interim belief specified in the proof of Proposition 1 is the only one satisfying the Intuitive Criterion of Cho and Kreps (1987). Further, under Lemma 1, the same refinement rules out other separating equilibria in which type (α_H, β_H) runs a higher level of campaign than the least cost separating level.

least cost separating level of negative campaign, k_H^n is given by

$$\alpha_L - \beta_L = \alpha_H - \Pi(\beta_L, \beta_H; k_H^n) - C(k_H^n). \quad (3)$$

At level k_H^n , the low type is indifferent between running an uninformative campaign and imitating the high type in running a negative campaign.

Separation of the two types is not just a matter of the high type candidate running a sufficiently informative campaign. In the P-region, for instance, there does not exist an equilibrium in which the high type candidate can separate from a low type candidate by running a negative campaign. Suppose separation is possible in a negative campaign of level $k^n > \frac{1}{2}$. Then in this putative equilibrium, type (α_H, β_H) gets $\alpha_H - \beta_H$. If the low type imitates the high type by running the same negative campaign, he gets $\alpha_H - \Pi(\beta_L, \beta_H; k_H^n)$, which is strictly larger than the high type's payoff because $\beta_L < \beta_H$. The reason is simple: the low type faces a worse rival and can thus lower the voter's posterior belief about his rival more successfully than the high type. Therefore whenever the high type prefers to run a negative campaign, the low type also prefers to run the same campaign, which is a contradiction. A symmetric argument establishes that in the N-region, the high type can not separate from the low type by running a positive campaign.

Finally, in the P/N-region, either positive or negative campaigns can be used for separation at sufficiently high levels. Therefore the high type candidate runs a positive campaign of level k_H^p if $k_H^p \leq k_H^n$ and a negative campaign of level k_H^n otherwise. An immediate implication of Proposition 1 is then that banning negative campaign never benefits the voter.

Corollary 1 *Banning negative campaign has no effect on the equilibrium campaign choices in the P-region, and in the P/N-region when the least cost separating equilibrium is a positive campaign. Otherwise, the high type candidate runs a positive campaign at a higher cost in the P/N-region, and pools with the low type with an uninformative campaign in the N-region.*

In the US, the marked increase in the amount and intensity of negative advertising in recent elections, especially since the 2004 presidential election, has lent support to the policy proposal of banning or at least limiting negative campaigns. Corollary 1 suggests that banning negative campaigns can only hurt the high type candidate, by either raising his cost of separation or making

separation altogether impossible. To evaluate the welfare impact on the voter, we need to provide an underlying structure for the reduced form payoff formulation used in the model. Suppose that the voter chooses two real-valued actions x_a and x_b to minimize the expected sum of losses $(x_a - q_a)^2 + (x_b - q_b)^2$ where, for each $i = a, b$, $q_i = 1$ if the candidate i is qualified and $q_i = 0$ if he is unqualified. Then, the voter's welfare is unaffected as long as the ban on negative campaigns still permits separation, but is reduced if the ban results in pooling through uninformative campaigns. Thus, when information provision is the main concern of an election, banning negative campaigns may hurt the voter by depriving her an opportunity to learn the qualifications of the candidates. We hasten to add that this model focuses exclusively on the information channel and is thus silent on any possible adverse effects of negative campaigns due to other factors in the political processes.

3.2 Comparative advantage in positive or negative campaigns

The high type candidate in the P/N-region is in the best position because he himself is stronger than the low type and he faces a weaker rival than the low type does: $\alpha_H > \alpha_L$ and $\beta_H < \beta_L$. In this region, Proposition 1 shows that the high type candidate can separate from the low type by running either a positive or negative campaign; and he chooses the less costly one in equilibrium. This result, however, is silent on what determines one kind of campaign is less costly than the other for a given high type candidate.

To answer this question, consider the following comparative statics exercise: fix the low type (α_L, β_L) and compare the equilibrium choice of campaign target by two different high type candidates in the P/N-region. For any type (α_H, β_H) in the P/N-region to deter the low type from imitating him, he could run a positive campaign such that, from rewriting (2):

$$\alpha_H - \beta_H - (\alpha_L - \beta_L) = \alpha_H - \Pi(\alpha_L, \alpha_H; k_H^p) + C(k_H^p), \quad (4)$$

or a negative one such that, from rewriting (3):

$$\alpha_H - \beta_H - (\alpha_L - \beta_L) = \Pi(\beta_L, \beta_H; k_H^n) - \beta_H + C(k_H^n). \quad (5)$$

The left-hand side of (4) and (5) is the same and represents the overall strength of the high type over the low type. Since $\alpha_H > \alpha_L$ and $\beta_H < \beta_L$ in the P/N-region, by Lemma 1 the right-hand side

of (4) and (5) are increasing in k_H^p and k_H^n respectively. Thus, for any high type candidate with the same overall strength, the term $\alpha_H - \Pi(\alpha_L, \alpha_H; k_H^p)$ represents his *advantage in positive campaigns*. The greater is this term, the less successful the low type is in imitating the high type, and thus the lower is the level k_H^p that the high type needs to deter the low type. Similarly, $\Pi(\beta_L, \beta_H; k_H^n) - \beta_H$ represents the high type's *advantage in negative campaigns*.

Because the candidate's type is two-dimensional, it is generally difficult to compare the campaign levels k_H^p and k_H^n for two arbitrary high types. To draw unambiguous conclusions from the present comparative statics exercise, we assume that $\alpha_H < \frac{1}{2} < \beta_H$. Under this assumption, we show that holding the overall strength of the high type $\alpha_H - \beta_H$ constant, his advantage in positive campaigns increases in α_H while his advantage in negative campaigns decreases in it. Intuitively, when $\alpha_H < \frac{1}{2}$, an increase in α_H raises the voter's upgrade of her opinion about the candidate qualifications after a favorable signal more than it reduces the downgrade after an unfavorable signal. Since the low type is a weaker candidate than the high type and is therefore less likely to generate a favorable signal in any positive campaign, an increase in α_H makes it harder for him to misinform the voter through positive campaigns.¹² Symmetrically, when $\beta_H > \frac{1}{2}$, a decrease in β_H makes it harder for the low type to misinform through negative campaigns, because it reduces the voter's downgrade of her opinion about the rival candidate qualifications after an unfavorable signal more than it reduces the upgrade after a favorable signal, but now the low type candidate faces a stronger candidate and is less likely to generate an unfavorable signal in any negative campaign. For this reason, when $\alpha_H < \frac{1}{2} < \beta_H$, we say that *misinformation incentives are monotone*.

Proposition 2 *Suppose that misinformation incentives are monotone. For the same overall strength of the high type, a simultaneous increase in candidates' strengths leads to a greater comparative advantage in positive campaigns for the high type, and results in a lower least cost separating level if the high type runs a positive campaign and a higher level if he runs a negative campaign.*

Formally Proposition 2 establishes the existence of a *boundary* that divides the P/N-region into

¹² If α_H is close to 1, a further increase in α_H can result in a decrease in $\alpha_H - \Pi(\alpha_L, \alpha_H; k^p)$ for any fixed k^p , so that the advantage of the high type in positive campaigns decreases. This happens because the voter downgrades her opinion after an unfavorable signal more than she upgrades it after a favorable signal, and because the voter's opinion about the candidate responds little to the realized campaign signal.

a positive campaign area adjacent to the P-region and a negative campaign area adjacent to the N-region (see Figure 1 for an illustration). For any overall strength of the high type candidate, there is a unique pair (α_H, β_H) on the boundary such that he is indifferent between a positive campaign and a negative campaign of the same level. At (α_H, β_H) , the high type candidate has the same advantage in positive and negative campaigns (the right-hand side of (4) and (5) are equal). As α_H and β_H increase at the same rate so that the overall strength remains constant, the assumption that the misinformation incentives are monotone guarantees that the least cost separating equilibrium involves a positive campaign of a decreasing level. Conversely, as α_H and β_H decrease at the same rate, the least cost separating equilibrium takes the form of a negative campaign of a decreasing level. This means that all the high type candidates above the boundary have a comparative advantage in positive campaigns and all those below the boundary have a comparative advantage in negative campaigns.

Along this boundary, as the overall strength of the high type candidate $\alpha_H - \beta_H$ increases, the least cost separating equilibrium level increases. This is clearly true if the boundary is monotonically decreasing in the P/N-region, since the right-hand side of condition (4) increases in α_H and the right-hand side of condition (5) decreases in β_H . But even if the boundary is not monotonically decreasing, the fact that for the same β_H , a higher α_H leads to a higher level of positive campaign, and that the high type is indifferent between a positive campaign and a negative campaign of the same level means that the equilibrium level has to increase along the boundary.¹³ Simple algebra can also show that the boundary falls between the lines defined by $\alpha_H + \beta_H = 1$ and $\alpha_H + \beta_H = \alpha_L + \beta_L$ in the α_H - β_H diagram. In the special case where $\alpha_L + \beta_L = 1$, the boundary is simply the line connecting (α_L, β_L) to $(\frac{1}{2}, \frac{1}{2})$.

Proposition 2 helps us think about a candidate's campaign choices when the voter has different

¹³ To see why the boundary may not be monotone, fix any (α_H, β_H) on the boundary, with the associated separating level k_H . Consider (α'_H, β_H) just to the right, with $\alpha'_H > \alpha_H$ and the associated positive separating level k_H^p given by (4) and (5). We have $k_H^p, k_H^n > k_H$. The boundary is non-monotone at (α_H, β_H) if (α'_H, β_H) is below the boundary, or equivalently, if $\partial k_H^p / \partial \alpha_H > \partial k_H^n / \partial \alpha_H$ at $k_H^p = k_H^n = k_H$. From equations (2) and (3), we have

$$\frac{\partial k_H^p}{\partial \alpha_H} = \frac{\partial \Pi(\alpha_L, \alpha_H; k_H) / \partial \alpha_H}{-\partial \Pi(\alpha_L, \alpha_H; k_H) / \partial k_H + C'(k_H)}; \quad \frac{\partial k_H^n}{\partial \alpha_H} = \frac{1}{\partial \Pi(\beta_L, \beta_H; k_H) / \partial k_H + C'(k_H)}.$$

Although $\partial \Pi(\alpha_L, \alpha_H; k_H) / \partial \alpha_H < 1$, we may have $-\partial \Pi(\alpha_L, \alpha_H; k_H) / \partial k_H < \partial \Pi(\beta_L, \beta_H; k_H) / \partial k_H$.

amount of prior knowledge about candidate a or b 's qualifications. Suppose that candidate b is well-known such that β_H is sufficiently close to β_L , then the high type candidate a is more likely to run a positive campaign due to his comparative advantage in positive campaigns (type (α_H, β_H) likely falls into the positive campaign area in the P/N-region if β_H is sufficiently close to β_L). Intuitively, in this case candidate a needs to convince the voter he is stronger than the average perception of the voter while he has little to reveal about candidate b . If candidate a himself is well-known such that α_H is sufficiently close to α_L , but the voter has a lot of uncertainty about candidate b , then candidate a 's comparative advantage is likely in negative campaigns because it is likely to generate an unfavorable signal and lower the voter's opinion about b . This conclusion is consistent with empirical findings: Kahn and Geer (1994) show that positive advertising increased the viewers' rating of an unknown candidate's capability in a study of how TV ads influence voters' impression of a candidate; and more recently, Lovett and Shachar (2010) find that if a candidate's traits are well-known by the voters, the candidate is more likely to go negative.

Propositions 1 and 2 show that, despite having the same overall strength over the low type, the high type candidate may nonetheless run different kinds of campaigns depending on his comparative advantage. In particular, even one in the best position of being a stronger candidate himself than the low type and facing a weaker candidate than the low type does may run a negative campaign, because it is the cost-effective way to boost the voter's opinion about him over his rival. Therefore in our model the high type candidate does not run a positive campaign because he wants to "hide" his rival's strong qualifications; nor a negative campaign to hide his own low qualifications, in contrast with the existing research such as Polborn and Yi (2006).¹⁴ An implication is that voters' opinion of a candidate depends on more than whether he runs a positive or a negative campaign: voters' prior knowledge of the strengths of different types of candidates also matters. In a given campaign, it is entirely plausible for voters to think well of a candidate running a negative campaign; or think poorly of the rival of a candidate running a positive campaign. Instead, the more informative a given campaign is, positive or negative, the stronger a candidate is perceived to be relative to the rival.

¹⁴ In our two-type model, it is impossible for a candidate to signal his strength but hide the strength of his rival.

3.3 Contrast campaigns

So far candidate a can run only a single campaign, we now turn to the case of “contrast campaigns” to see whether the high type candidate can do better by running both a positive campaign and a negative campaign. To avoid biasing our results, we assume that the costs of running two campaigns are additive with the same function C . That is, the total cost of running a positive campaign of level k^p and a negative campaign of level k^n is just $C(k^p) + C(k^n)$.

Proposition 3 *Suppose that candidate a can simultaneously run a positive and a negative campaign. In the least cost separating equilibrium, the high type candidate runs a single campaign in the P-region and the N-region, and if the campaign cost function is differentiable and concave, he also runs a single campaign in the P/N-region.*

The above result is straightforward in the P-region or the N-region, where the high type candidate can only signal his type successfully using one kind of campaign. Suppose, for instance, a high type candidate in the P-region runs both a positive campaign of level k^p and a negative campaign of level k^n . To prevent the low type from imitating, it must be that

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k^p) - \Pi(\beta_L, \beta_H; k^n) - C(k^p) - C(k^n). \quad (6)$$

From equation (2), the total campaign cost in running both k^p and k^n is smaller than the cost of running just k_H^p if $k^p < k_H^p$, and

$$\Pi(\alpha_L, \alpha_H; k^p) - \Pi(\beta_L, \beta_H; k^n) < \Pi(\alpha_L, \alpha_H; k_H^p) - \beta_H.$$

The above is impossible by Lemma 1 because $\alpha_H > \alpha_L$ and $\beta_H > \beta_L$ in the P-region. Intuitively, if the high type must also run an informative negative campaign, he has to run a higher level positive campaign than in the single-campaign case to deter the low type, who has a comparative advantage in proving that candidate b is less qualified. This increases his total cost of campaigning.

In the P/N-region, for the high type to separate from the low type with two campaigns, the total campaign cost must be high enough such that condition (6) is satisfied. From the previous analysis, we know that candidate a can signal his type using either campaign: condition (6) is satisfied with

equality by either $k^p = \frac{1}{2}$ and $k^n = k_H^n$ given by (3), or by $k^p = k_H^p$ given by (2) and $k^n = \frac{1}{2}$. Further, since $\alpha_H > \alpha_L$ and $\beta_H < \beta_L$ in P/N-region, Lemma 1 implies that positive and negative campaigns are substitutes in condition (6). Proposition 3 shows that when the cost function is concave, the total campaign cost is minimized by completely substituting one kind of campaign for the other. Intuitively, as the high type increases the positive campaign level and simultaneously decreases the negative campaign level to satisfy condition (6), the positive campaign becomes more effective in deterring the low type from imitation relative to the negative campaign. This follows because a higher level of positive campaign leads to a greater response by the voter to the realized campaign signal, and the opposite is true for a lower level of negative campaign. If the cost function is concave in campaign levels, then the marginal cost from a higher positive campaign level declines while the marginal saving from a lower negative campaign increases. As a result, more substitution of positive campaign for negative campaign reinforces the overall effectiveness of separation of the former over the latter, leading to complete substitution.

Proposition 3 demonstrates that our result that comparative advantage determines the high type's equilibrium choice of positive versus negative campaigns is not due to the restriction that only one kind of campaign may be used. This is in contrast with the existing research such as Polborn and Yi (2006). This provides further evidence that our characterization is obtained through the channel of misinformation rather than the choice between which dimension to reveal and which to conceal.

3.4 Independent information

Voters often have access to exogenous sources of information such as reports from the media that are outside the control of candidates. The effect of such additional information on a candidate's campaign choices, interesting in its own right, is also an important component in the analysis of the competing campaigns model in the next section. Specifically, suppose that the voter receives a public signal after the candidate has made his campaign choices, but before the voter forms her posterior belief about the candidates. Assume that this public signal s' is about a 's qualifications; the case when she receives a public signal about b is symmetric. To keep things simple, we assume

that the public signal s' is binary with a symmetric structure: s' is either \bar{s}' or \underline{s}' such that k' , the probability of $s' = \bar{s}'$ conditional on candidate a being qualified, equals the probability of $s' = \underline{s}'$ conditional on a being unqualified. In addition, s' is independent of the campaign signal s , conditional on whether the candidate is qualified or unqualified.

Since candidate a may run a positive campaign, the voter may observe two signals about candidate a . In this case, her expected posterior belief about a depends on both the candidate's own campaign level and the informativeness of the public signal. Denote this belief as $\Pi(\alpha, \tilde{\alpha}; k, k')$, which is a weighted average of her beliefs after observing both realized signals for a given private belief of the candidate α and interim belief of the voter $\tilde{\alpha}$, given by

$$\begin{aligned} & \Pi(\alpha, \tilde{\alpha}; k, k') \\ = & \frac{(\alpha k k' + (1 - \alpha)(1 - k)(1 - k'))\tilde{\alpha} k k'}{\tilde{\alpha} k k' + (1 - \tilde{\alpha})(1 - k)(1 - k')} + \frac{(\alpha k(1 - k') + (1 - \alpha)(1 - k)k')\tilde{\alpha} k(1 - k')}{\tilde{\alpha} k(1 - k') + (1 - \tilde{\alpha})(1 - k)k'} \\ + & \frac{(\alpha(1 - k)k' + (1 - \alpha)k(1 - k'))\tilde{\alpha}(1 - k)k'}{\tilde{\alpha}(1 - k)k' + (1 - \tilde{\alpha})k(1 - k')} + \frac{(\alpha(1 - k)(1 - k') + (1 - \alpha)k k')\tilde{\alpha}(1 - k)(1 - k')}{\tilde{\alpha}(1 - k)(1 - k') + (1 - \tilde{\alpha})k k'}. \end{aligned}$$

Straightforward algebra shows that the partial derivatives of $\Pi(\alpha, \tilde{\alpha}; k, k')$ with respect to k and k' have the same sign as $(\alpha - \tilde{\alpha})$. Thus we have the following counterpart of Lemma 1.

Lemma 2 (i) $\Pi(\alpha, \tilde{\alpha}; k, k') = \alpha$ if $\alpha = \tilde{\alpha}$. (ii) $\Pi(\alpha, \tilde{\alpha}; k, k')$ decreases in k and k' if $\alpha < \tilde{\alpha}$; and (iii) $\Pi(\alpha, \tilde{\alpha}; k, k')$ increases in k and k' if $\alpha > \tilde{\alpha}$.

Lemma 2 implies that when the additional public signal and the candidate's campaign signal have the same target, candidate a himself in this case, it becomes more difficult for a weak low type candidate to imitate a strong high type. By part (ii) of the above lemma, if $\alpha < \tilde{\alpha}$, then for any given level k of a positive campaign, the candidate's expected payoff is lower than when the voter has no additional information, that is, when $k' = \frac{1}{2}$. This is because the additional public signal reduces the low type's possible gain from imitating the high type at any campaign level.

When the additional public signal has a different target from the candidate's campaign signal, it might seem that the additional public signal has no impact because it does not affect the voter's evaluation of the campaign signal. This turns out to be false. When the high type runs a negative campaign, the additional public signal may help or hurt him depending on whether the high type he

himself is stronger than the low type. For the following result, we say that misinformation incentives are monotone in positive campaigns if $\alpha_L, \alpha_H < \frac{1}{2}$. This assumption is sufficient to ensure that if the low type is indifferent between running an informative campaign and an uninformative one, then the high type strictly prefers the former.¹⁵

Proposition 4 *Suppose the voter receives an additional signal of a sufficiently low level, and suppose that misinformation incentives are monotone in positive campaigns. In the least cost separating equilibrium, the high type runs a lower level positive campaign in the P-region and a higher level negative campaign in the N-region than when no additional signal is observed. Moreover, in the P/N-region he runs a lower level of campaign regardless of whether it is positive or negative.*

Proposition 4 shows that the additional public signal about candidate a enhances the high type's comparative advantage in positive campaigns. Therefore the high type candidate will continue to run positive campaigns if he runs a positive campaign in the absence of the additional public signal, but at a lower level. The condition on the level of the additional public signal in Proposition 4, given precisely in the proof, is imposed to simplify the exposition. If the additional public signal is very informative, then there is no need for the high type to use a costly informative campaign to separate so long as he is stronger than the low type: the equilibrium will be pooling at an uninformative level. If we rule out this rather uninteresting case, then Proposition 4 demonstrates the robustness of the results in Propositions 1 and 2. In particular, when the high type candidate finds it cheaper to run a negative campaign in the P/N-region without the additional signal, his comparative advantage may be now changed to positive campaigns. This happens because the additional signal increases the high type's comparative advantage in positive campaigns by giving the voter an opportunity to observe his own strength.

An interesting result of Proposition 4 is that the public's access to independent evidence outside the control of the candidate may hurt him by raising the cost of separation from the low type. This happens when the high type is in the N-region. The least cost separating equilibrium level in this

¹⁵ There is no need to impose any restriction on β_H . Also, the assumption of $\alpha_L, \alpha_H < \frac{1}{2}$ is not needed for this "single-crossing" condition when the voter has no additional information such as in the basic model.

case is $k_H^{n'}$, given by the low type's indifference condition:

$$\alpha_L - \beta_L = \Pi(\alpha_L, \alpha_H; k') - \Pi(\beta_L, \beta_H; k_H^{n'}) - C(k_H^{n'}).$$

Since $\alpha_H < \alpha_L$ and $\beta_H < \beta_L$, comparing the above with (3) immediately reveals that $k_H^{n'} > k_H^n$ by Lemma 1. In this case, the additional information gives the low type candidate a free opportunity to exploit the low type's comparative advantage in positive campaigns, increasing his gain from misinformation. This effect forces the high type to run a higher level of negative campaign to deter the low type. Of course, in the symmetric case of the voter having access to independent information about candidate b , the high type candidate a can be hurt if only positive campaigns can be used for separation, as the additional public information allows the low type to showcase his comparative advantage in negative campaigns.

4 Competing Campaigns

In this section we consider an extension of the basic in which candidate a and b simultaneously and independently choose a campaign to influence the voter. To identify any new effect arising solely from competition in information provision, we limit attention to an environment that is identical to the basic model except for the possible active campaign from b . In particular, there is no more exogenous private information available to the two candidates than in the basic model.

We introduce the following underlying information structure for the basic one-campaign model before introducing competing campaigns. For each candidate i , $i = a, b$, write $q_i = 1$ when i is qualified and $q_i = 0$ otherwise. We assume that candidate a and the voter share the same prior beliefs about the qualifications of the candidates, $\Pr(q_a, q_b)$. To maintain the two-type structure of the model, we assume that candidate a receives a private binary signal L or H , with probability λ and $1 - \lambda$ respectively, while the voter knows the value of each $\Pr(H|q_a, q_b)$ but does not observe the signal. Then, the private belief of candidate a given signal H that he is qualified and candidate b is qualified is respectively

$$\alpha_H = \frac{\sum_{q_b} \Pr(1, q_b) \Pr(H|1, q_b)}{\sum_{q_a, q_b} \Pr(q_a, q_b) \Pr(H|q_a, q_b)}; \quad \beta_H = \frac{\sum_{q_a} \Pr(q_a, 1) \Pr(H|q_a, 1)}{\sum_{q_a, q_b} \Pr(q_a, q_b) \Pr(H|q_a, q_b)}.$$

Candidate a 's private beliefs after signal L are similarly defined.

To ensure that the same amount of information is available as in the basic model, candidate b is assumed to share with candidate a (and hence with the voter) the same prior beliefs, and more importantly, receive the same private signal. In the analysis below, we need to specify the out-of-equilibrium interim beliefs of the voter about the qualifications of the candidates when their campaign choices suggest that their signals disagree according to their equilibrium strategies. To do so without making arbitrary assumptions, we consider the hypothetical scenario in which for each (q_a, q_b) , candidate b 's private signal is perfectly correlated with a 's signal with probability ρ , and is conditionally independent with probability $1 - \rho$, and then let ρ go to 1. Note that as long as $\rho < 1$, the belief about candidate a 's qualifications when the candidates' private signals disagree is well-defined, given by

$$\alpha_{HL} = \alpha_{LH} = \frac{\sum_{q_b} \Pr(1, q_b) \Pr(H|1, q_b) (1 - \Pr(H|1, q_b))}{\sum_{q_a, q_b} \Pr(q_a, q_b) \Pr(H|q_a, q_b) (1 - \Pr(H|q_a, q_b))}.$$

Due to this observation, let α_{HL} be the out-of-equilibrium interim belief of the voter about a . The out-of-equilibrium interim belief for b , $\beta_{HL} = \beta_{LH}$, is similarly defined.

Perfect correlation between the candidates' signals means that the strengths of the two candidates are either (α_H, β_H) or (α_L, β_L) . Observe that since $\alpha_H - \beta_H > \alpha_L - \beta_L$, type (α_H, β_H) candidate b has an incentive to pretend to be type (α_L, β_L) if candidate b ran the only campaign. For this reason, we refer to both type (α_H, β_H) candidate a and type (α_L, β_L) candidate b as the high type candidates, and correspondingly type (α_L, β_L) candidate a and type (α_H, β_H) candidate b as the low type. Note that under perfect correlation between the candidates' signals, regardless of the type realization there is one high type candidate and one low type. For ease of comparison with the basic one-campaign model, we continue to refer to the three regions in the α_H - β_H diagram from a 's perspective. An immediate implication of our setup is that Proposition 1 is directly applicable to candidate b 's campaign choices if he were the only one running a campaign. As an example, suppose that type (α_H, β_H) is in the P-region. If a is the only candidate making campaign choices, type (α_H, β_H) can only signal his overall strength over type (α_L, β_L) through a positive campaign. In contrast, if b is the only one making campaigning choices, type (α_L, β_L) must signal by running a negative campaign, because he is a weaker candidate than type (α_H, β_H) but he faces an even

weaker rival than the low type does.

Competing campaign choices by the rival candidate introduce two new elements in the basic one-campaign model. First, type (α_L, β_L) candidate a must now re-evaluate the relative effectiveness of positive versus negative campaigns, because ex post even a favorable campaign signal from a positive campaign might be undercut by an unfavorable signal from the negative campaign against him run by candidate b ; while his negative campaign targeting b might be neutralized by b 's own positive campaign in a similar fashion. Second, the way type (α_L, β_L) candidate a influences the voter's interim beliefs through his campaign choices is also affected, because unilateral deviations lead to out-of-equilibrium beliefs $(\alpha_{HL}, \beta_{HL})$ that we have put little restrictions on. To avoid biasing our results one way or the other, we focus on the case when α_{HL} is close to the unweighted average of α_H and α_L and symmetrically for β_{HL} . This is called the *neutrality* assumption in the sense that the out-of-equilibrium interim beliefs do not favor type (α_H, β_H) or type (α_L, β_L) .

Proposition 5 *In the competing-campaigns model, under the neutrality assumption, there is a separating equilibrium in which the low type candidate runs an uninformative campaign and the high type runs the same kind of informative campaign as in the one-campaign model. In the P/N-region, there is a separating equilibrium in which the high type candidate runs a lower level campaign.*

Proposition 5 suggests that our results on a candidate's choice of positive versus negative campaigns in the basic model are robust. In the P-region for example, just as in the basic one-campaign model, there is a separating equilibrium in which the low type runs an uninformative campaign while the high type candidate a runs a positive campaign and the high type b runs a negative campaign to signal their overall strength over the rival. Similarly, in the P/N-region, the result that high type candidate a or b can separate from the low type in either positive or negative campaigns continues to hold. This robustness result owes much to the neutrality assumption, which guarantees that if the low type candidate is just indifferent between running an uninformative campaign and imitating the high type in running an informative campaign, then the high type strictly prefers to separate. Moreover, the separating equilibria constructed in Proposition 5 are natural extensions of the least cost separating equilibria in the basic model: for the same pair of campaign targets

in informative campaigns run by the two high type candidates, there does not exist another separating equilibrium with lower campaign levels. Separating equilibria involving different kinds of campaigns, however, may exist.¹⁶

In the P/N-region, Proposition 5 offers a sharper characterization. Regardless of whether the equilibrium campaigns of the high type candidates have the same target or different targets, the level of their campaigns is lower than their separating levels k_H^p and k_H^n in the basic model, given by (2) and (3) respectively. One reason is that in the P/N-region, the campaign levels of the two high type candidates are *strategic substitutes* regardless of their targets. This is true even though the two informative campaigns are never run at the same time: in equilibrium one and only one informative campaign is run due to the perfect correlation between the candidates' signals. To see why the campaigns are strategic substitutes, suppose that the high type candidates a and b both run positive campaigns, at level k_a^p and k_b^p , respectively, targeting their own qualifications. Consider low type candidate a 's incentive to misinform the voter by imitating the high type candidate a . In a separating equilibrium we need

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_{HL}; k_a^p) - \Pi(\beta_L, \beta_{HL}; k_b^p) - C(k_a^p). \quad (7)$$

In the P/N-region, we have $\alpha_L < \alpha_{HL} < \alpha_H$ and $\beta_L > \beta_{HL} > \beta_H$. Thus, an increase in the high type candidate b 's positive campaign level k_b^p reduces the low type candidate a 's gain from imitating the high type a , and vice versa for high type candidate b . Intuitively, by making it more difficult for the low type to pretend to more qualified than he is when the campaigns have the same targets, or more difficult for him to make his rival look less qualified when the campaigns have different targets, the presence of the rival's campaign reduces the campaign levels required for separation.

Under competing campaigns, both candidates' campaign choices affect the voter's interim beliefs. More specifically, type (α_L, β_L) candidate a 's campaign choices have less impact on the voter's interim beliefs, because candidate b 's campaign choices also affect the voter's interim beliefs. This

¹⁶ For instance, in the P/N-region, there exist both a separating equilibrium in which the high type candidate a runs an informative positive campaign and the high type b runs an informative negative campaign, and another separating equilibrium in which both high type candidates run positive campaigns. It is not possible for us to compare these two equilibria in terms of the campaign levels because they are of two different kinds, and our model is generally asymmetric with respect to the two candidates. However, the proof of Proposition 5 establishes that each of the two equilibria is uniquely constructed by binding the equilibrium indifference conditions of the two low types.

is true even if candidate b runs no informative campaign, and thus the voter does not observe more realized campaign signals. The possibility of pretending to be an “intermediate” type $(\alpha_{HL}, \beta_{HL})$ is another difference from our basic model. To see this, compare condition (7) with condition (2) in the basic one-campaign model. Since $\alpha_L < \alpha_{HL} < \alpha_H$ and $\beta_L > \beta_{HL} > \beta_H$, the low type candidate a has less incentive to misinform the voter through a positive campaign compared to the one-campaign model even if $k_b^p = \frac{1}{2}$. In a more general model of competing campaigns, both the voter’s interim beliefs and the amount of information contained in the candidates’ signals will differ from those in the one-campaign model.¹⁷ The two cases we have studied, the case of additional public information and the case of competing campaigns with perfectly correlate signals, should be viewed as the two polar opposites.

5 Winner Takes All

In a winner-take-all political system, the candidate wins the election if he convinces the voter that he is more qualified than his rival. For simplicity, the payoff to candidate a is modeled as

$$\begin{cases} 1 - C(k), & \text{if } \pi_a \geq \pi_b \\ -C(k), & \text{otherwise.} \end{cases}$$

Assume that $\alpha_L, \alpha_H \leq \frac{1}{2}$ and $\beta_L, \beta_H \geq \frac{1}{2}$, or that candidate a is weaker than his rival regardless of his type to allow for a direct comparison with the basic model. The campaign cost $C(k)$ is assumed to be small for all relevant campaign levels, so that both types can afford any necessary campaigns.¹⁸

The first difference from the basic model is that a candidate in a winner-take-all system has an incentive to run an informative campaign under complete information: his campaign has value even if his type is known. No matter how far candidate a is lagging behind b , he always has a chance of winning if the voter observes a favorable campaign signal that is sufficiently informative

¹⁷ Although the strategic considerations in a more general model are similar to this model, a full equilibrium characterization of the general model depends on specific type distribution as well as the correlation between the candidates’ private signals, which is beyond the scope of the current paper.

¹⁸ Analysis in this section is valid even if campaigns are free. Unlike in the basic model where the campaign cost helps the high type separate from the low type, here it gives the low type more incentives to imitate the high type.

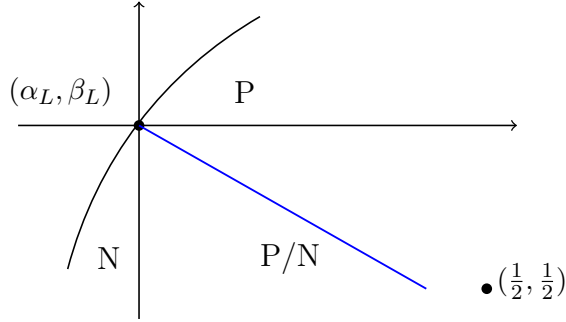


Figure 2: Winner-take-all Model

to overturn her low initial belief about him. For instance, for type (α_L, β_L) to win under complete information, the minimum level k_L^c of a positive campaign needs to satisfy

$$\frac{\alpha_L k_L^c}{\alpha_L k_L^c + (1 - \alpha_L)(1 - k_L^c)} = \beta_L.$$

That is, he wins if the realized campaign signal is \bar{s} and his campaign level is at least k_L^c .¹⁹ Similarly, k_H^c is the level type (α_H, β_H) runs under complete information.

The low type remains the one that receives a lower payoff under complete information, which implies in this case that he needs to run a higher level of campaign to catch up to candidate b . In other words, we assume that $k_L^c > k_H^c$ and identify (α_L, β_L) as the low type. Intuitively, the overall strength of the high type candidate is inversely related to the complete information level k_H^c : k_H^c is decreasing in α_H and increasing in β_H . Fix type (α_L, β_L) , then $k_L^c = k_H^c$ defines a curve such that type (α_H, β_H) is located to the right (and below) this curve in the α_H - β_H diagram. We classify the parameter space for the high type below this curve into three regions, P-region, N-region and P/N-region just as in the basic model. See Figure 2 for an illustration.

The second difference from the basic model is that, under complete information, each type prefers the kind of campaign that has a higher chance of winning. For instance, since $\alpha_L < \frac{1}{2} < \beta_L$, type (α_L, β_L) will run a negative campaign under complete information if and only if

$$\alpha_L k_L^c + (1 - \alpha_L)(1 - k_L^c) < (1 - \beta_L)k_L^c + \beta_L(1 - k_L^c),$$

¹⁹ It is easy to verify that k_L^c is the same level required of a negative campaign for the low type to win (when the realized campaign signal about candidate b 's qualifications is \underline{s}).

or $\alpha_L + \beta_L < 1$. In this case, we say that type (α_L, β_L) has a *preference* for negative campaigns; otherwise we say that he has a preference for positive campaigns. Intuitively, at the same campaign level, the candidate prefers the campaign in which the voter's prior belief is closer to $\frac{1}{2}$, and is thus more responsive to the relevant realized campaign signal. In either case, the low type candidate's winning chances decrease in his campaign level if it is above k_L^c , because a more informative campaign is more likely to generate an unfavorable signal about a or a favorable signal about b . Similar analysis applies to type (α_H, β_H) .

An immediate observation is that in the winner-take-all model, there does not exist a separating equilibrium in which both types of candidate run the same kind of campaign, positive or negative. This is because for the same kind of campaign, say positive campaigns, if the voter's interim belief is such that some level is sufficient for one type of candidate to win when the realized signal is favorable, then the same interim belief is also sufficient for the other type to win. But since a more informative campaign merely reduces a 's winning chances regardless of type, the type running a higher level of campaign in the putative equilibrium strictly prefers to deviate to the lower level run by the other type: it increases his winning chances at a lower cost. Therefore the two types of candidate a must run opposite kinds of campaigns to separate.

Throughout this section, we only discuss the case when the low type candidate has a preference for negative campaigns ($\alpha_L + \beta_L < 1$); the other case is similar. In any separating equilibrium (if it exists), the low type candidate must run his preferred campaign: a negative one at level k_L^c . Moreover, there exists a unique $k_L^p \in (\frac{1}{2}, k_L^c)$ such that type (α_L, β_L) is indifferent between his preferred negative campaign of level k_L^c and a positive campaign of a lower level k_L^p , determined by

$$(1 - \beta_L)k_L^c + \beta_L(1 - k_L^c) - C(k_L^c) = \alpha_L k_L^p + (1 - \alpha_L)(1 - k_L^p) - C(k_L^p). \quad (8)$$

Observe that the right-hand side is the low type's payoff if he pretends to be a high type by running a positive campaign, which is independent of the voter's interim beliefs so long as a favorable signal leads to a win.²⁰ Consequently, this (possible) separation level k_L^p does not depend on the high type's characteristics.

²⁰ In contrast with the basic model in which the low type's expected payoff varies continuously with the voter's interim belief, in the winner-take-all model, the voter's interim belief only matters in a discontinuous fashion, which makes it more difficult for a high type candidate to signal his type through choices of campaign levels.

The equilibrium condition for separation is that the high type prefers not to pool with the low type candidate in running a negative campaign at the level k_L^c :

$$\begin{aligned} & \alpha_H \max\{k_H^c, k_L^p\} + (1 - \alpha_H)(1 - \max\{k_H^c, k_L^p\}) - C(\max\{k_H^c, k_L^p\}) \\ \geq & (1 - \beta_H)k_L^c + \beta_H(1 - k_L^c) - C(k_L^c). \end{aligned} \quad (9)$$

The maximum operator on the left-hand side of condition (9) arises because if $k_L^p < k_H^c$, the campaign level k_L^p is not sufficiently high for the high type candidate a to convince the voter that he is more likely to be qualified than candidate b even after a favorable signal. If condition (9) is satisfied, we say that the high type has a *comparative advantage in positive campaigns*.

Proposition 6 *Suppose that the low type candidate prefers negative campaigns under complete information. There is a unique least cost separating equilibrium in which the low type candidate runs a negative campaign and the high type runs a positive campaign if the latter has a comparative advantage in positive campaigns; otherwise, there is a pooling equilibrium in which both types run a negative campaign of the same level.*

The high type candidate should run the kind of campaign in which he has a comparative advantage, as in the basic model. The least cost separating equilibrium, however, takes a different form due to the payoff discontinuity of the winner-take-all model. It is easiest to understand the least cost separating equilibrium when the overall strength of the high type candidate is so strong that he does not need to run a very informative campaign under complete information ($k_H^c \leq k_L^p$). In this case, condition (9) implies a linear *positive boundary* in the P/N-region, which is depicted in Figure 2. Above the positive boundary, the high type candidate separates with a positive campaign of level k_L^p from the low type, who runs a negative campaign of level k_L^c in the least cost separating equilibrium. Moreover, the weaker is the low type's preference for negative campaigns (as α_L increases and/or β_L decreases), the more tempted he is to imitate the high type, who then needs to run a higher level of positive campaign to separate (k_L^p increases). In the polar case where $\alpha_L + \beta_L = 1$ and thus the low type has no preference between the two kinds of campaigns, the positive boundary is simply $\alpha_H + \beta_H = 1$, a line connecting to (α_L, β_L) to $(\frac{1}{2}, \frac{1}{2})$ in the α_H - β_H

diagram. If the overall strength of the high type candidate is not sufficiently strong ($k_H^c > k_L^p$), however, he has to be willing to run a higher level of positive campaign than k_L^p to separate from the low type.²¹

Qualitatively similar to the basic model, above the positive boundary given by condition (9), type (α_H, β_H) has a comparative advantage in running positive campaigns; and conversely, below the boundary, he has a comparative advantage in negative campaigns. To see why, consider the example of the N-region, which lies below the boundary. Because $\alpha_H < \alpha_L$ and $\beta_H < \beta_L$ in the N-region, for any given level of campaign, the high type has a higher probability than the low type of getting an unfavorable signal for his rival and thus winning the election in a negative campaign; but a lower probability of getting a favorable signal for himself and winning the election in a positive campaign. Therefore, the high type strictly prefers a negative campaign of level k_L^c to a positive campaign of either level k_L^p or k_H^c . Intuitively, since $\alpha_H + \beta_H < 1$ in the N-region, the high type's preference for negative campaigns makes him unwilling to run a positive campaign of at least level k_L^p to separate from the low type. Similarly, above the boundary, the high type candidate prefers a positive campaign because he has a higher probability than the low type of getting a favorable signal for himself and thus winning the election in a positive campaign. The high type candidate's comparative advantage, however, is not only driven by his preference under complete information. Since $k_L^p < k_L^c$, the slope of the linear part of the positive boundary is greater than -1 , and thus there are (α_H, β_H) types that prefer negative campaigns under complete information but still have a comparative advantage in running positive campaigns.

When separation is impossible—if type (α_H, β_H) is located below the positive boundary—both types run the same negative campaign. In any pooling equilibrium, the high type candidate is more likely to win the election. Unlike the basic model, however, the “wrong” candidate may be elected ex post in a winner-take-all system. To see this, observe that at any such equilibrium, because the pooling campaign level is below the low type's complete information level k_L^c , the low type

²¹ There is a critical type (α_H, β_H) on the linear boundary such that condition (9) holds as an equality with the corresponding complete information level $k_H^c = k_L^p$. For all high types closer to (α_L, β_L) than this critical type, the boundary between a separating equilibrium and a pooling equilibrium is instead given by (9) with $\max\{k_H^c, k_L^p\} = k_H^c$. That is, when $k_H^c > k_L^p$, the high type may not run a positive campaign in a separating equilibrium above the boundary, unlike in the basic model. Since this does not affect our result qualitatively, we relegate the complete characterization to the proof of Proposition 6.

candidate wins with a positive probability. In contrast, under complete information, the low type will always lose if his campaign level is below k_L^c , regardless of the kind of campaign he runs.

Two consequences of Proposition 6 are immediate. First, banning one campaign can never increase the voter's welfare because doing so (weakly) increases pooling and hence the probability the wrong candidate is elected. Second, since separation is impossible within the same kind of campaign and since there are only two kinds of campaigns, only the lowest type can possibly be separated from the rest if there are more than two types. In that case, we should expect to see two groups of candidates each running one kind of campaign at the same level.

6 Concluding Remarks

In the basic model, with only two types of candidate, the equilibrium characterization in Proposition 1 needs no restriction on the candidate's type because the least cost separating equilibrium is determined by the incentives of the low type to misinform the voter. To further understand the nature of the least cost separating equilibrium, or to study the candidate's behavior when there are more than two types, it is necessary to rank a candidate's incentives to misinform the voter according to his type. Appendix B presents a single crossing condition, which is satisfied in the basic model if misinformation incentives are monotone (the sufficient condition for the result in Proposition 2). We can also use it to rule out pooling equilibria in the basic one-campaign model and to generalize the model to multiple types. In addition, we introduce a counterpart of this condition for the case of continuously distributed campaign signals.

Our separation result in the basic model relies on the assumption that campaign levels, possibly with some noise, are observable to the voter. If the campaign level is unobservable, that is, if the signal is jammed as in Holmström (1999) and Fudenberg and Tirole (1986), the realized campaign signal alone may fail to provide the voter with any information because the candidate may have no incentive to run an informative campaign. Consider the case of positive campaigns. When the campaign level is not observable, different types of candidate must receive the same posterior belief of the voter given the same realized campaign signal. Moreover, if the campaign is informative, the voter's posterior belief about the candidate must be higher conditional on a favorable realized

campaign signal. If candidate a 's probability of obtaining the favorable campaign signal is decreasing in his campaign level, all types of candidate run an uninformative campaign, contradicting the assumption that realized campaign signals are informative. If the candidate is sufficiently likely to be qualified such that his winning chance is increasing in his campaign level, then it is possible for the candidate to run an informative campaign in equilibrium. But because one is only judged on the observed campaign signal, the low type candidate may succeed in misinforming the voter, which cannot occur with observable campaign levels.

Finally, the present model is static while candidates often adjust their campaign choices throughout the election process. Unlike the case of contrast campaigns in Section 3.3, in a dynamic model the campaign choices are made sequentially in two stages. The candidate can potentially condition his second-stage choices on the realized campaign signal from his first-stage campaign. The same least cost separating equilibrium outcome remains, however, unless the voter's belief after the first-stage campaign has payoff implications to the candidate. In that case, the dynamics of the candidate's campaign choices should incorporate the value of information generated from learning about the candidate's qualifications.

Appendix A Proofs

Proof of Proposition 1. In any separating equilibrium, type (α_L, β_L) candidate a must run no informative campaign and receive a payoff of $\alpha_L - \beta_L$. Moreover, if in a separating equilibrium, type (α_H, β_H) candidate a runs a positive campaign of level $k^p > \frac{1}{2}$ or a negative campaign of $k^n > \frac{1}{2}$ to separate from type (α_L, β_L) , the following incentive constraints must be satisfied:

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k^p) - \beta_H - C(k^p) \quad (10)$$

$$\alpha_H - \beta_H - C(k^p) \geq \alpha_L - \beta_L \quad (11)$$

$$\alpha_L - \beta_L \geq \alpha_H - \Pi(\beta_L, \beta_H; k^n) - C(k^n) \quad (12)$$

$$\alpha_H - \beta_H - C(k^n) \geq \alpha_L - \beta_L \quad (13)$$

First, consider the case of $\alpha_H > \alpha_L, \beta_H > \beta_L$. Observe that at $k^p = \frac{1}{2}$, the left-hand side (10)

is smaller than the right-hand side; while at $k^p = 1$, the left-hand side is greater than the right-hand side. Also, the right-hand side of (10) decreases in k^p by Lemma 1, and thus the campaign level $k_H^p \in (\frac{1}{2}, 1)$ defined in (2) is the unique level such that (10) holds with equality. Moreover, substituting (10) at k_H^p into (11), we require

$$\alpha_H - \Pi(\alpha_L, \alpha_H; k_H^p) \geq 0,$$

which is always true when $\alpha_L < \alpha_H$. Now, we show that separation in negative campaigns is impossible in the P-region. Adding up (12) and (13), we require

$$\Pi(\beta_L, \beta_H; k^n) \geq \beta_H,$$

which contradicts the assumption that $\beta_L < \beta_H$ in the P-region. The interim belief supporting the equilibrium is: (α_L, β_L) if $k^p < k_H^p$ and (α_H, β_H) if $k^p \geq k_H^p$ for any positive campaign of some level k^p ; and (α_L, β_L) for any negative campaign.

By a symmetric argument, one can show that in the case of $\alpha_H < \alpha_L, \beta_H < \beta_L$, the unique least cost separating equilibrium level is k_H^n given by (3). Finally, if $\alpha_H \geq \alpha_L, \beta_H \leq \beta_L$, with at least one strict inequality, type (α_H, β_H) can separate from type (α_L, β_L) by either running a positive campaign of level k_H^p or by running a negative campaign of level k_H^n . The least cost separating level is the positive campaign of k_H^p if $k_H^p \leq k_H^n$ and a negative campaign of level k_H^n otherwise. The interim belief that supports this equilibrium is: (α_L, β_L) if $k^p < k_H^p$ and (α_H, β_H) if $k^p \geq k_H^p$ for any positive campaign of some level k^p ; and (α_L, β_L) for any negative campaign; and (α_L, β_L) if $k^n < k_H^n$ and (α_H, β_H) if $k^n \geq k_H^n$ for any negative campaign of some level k^n .

Proof of Proposition 2. Fix type (α_L, β_L) and suppose that $\alpha_L < \alpha_H < \frac{1}{2}$ and $\beta_L > \beta_H > \frac{1}{2}$. We claim that for each $\mu \in (\alpha_L - \beta_L, 0)$, there is a unique set of solutions (α_H, β_H) and k_H to

$$\alpha_H - \beta_H = \mu \tag{14}$$

$$\mu - (\alpha_L - \beta_L) = \alpha_H - \Pi(\alpha_L, \alpha_H; k_H) + C(k_H) \tag{15}$$

$$\mu - (\alpha_L - \beta_L) = \Pi(\beta_L, \beta_H; k_H) - \beta_H + C(k_H) \tag{16}$$

Define

$$\Delta(\alpha; k) = \frac{\alpha k}{\alpha k + (1 - \alpha)(1 - k)} - \frac{\alpha(1 - k)}{\alpha(1 - k) + (1 - \alpha)k}.$$

Then,

$$\begin{aligned}\alpha_H - \Pi(\alpha_L, \alpha_H; k_H) &= (2k_H - 1)(\alpha_H - \alpha_L)\Delta(\alpha_H; k_H) \\ \Pi(\beta_L, \beta_H; k_H) - \beta_H &= (2k_H - 1)(\beta_L - \beta_H)\Delta(\beta_H; k_H).\end{aligned}$$

It is straightforward to verify that

$$\Delta(\alpha; k) = \frac{\alpha(1 - \alpha)(2k - 1)}{(\alpha k + (1 - \alpha)(1 - k))(\alpha(1 - k) + (1 - \alpha)k)} > 0$$

for all $\alpha \in (0, 1)$ and $k > \frac{1}{2}$, and that

$$\frac{\partial \Delta(\alpha; k)}{\partial \alpha} = \frac{(1 - 2\alpha)(2k - 1)k(1 - k)}{(\alpha k + (1 - \alpha)(1 - k))^2(\alpha(1 - k) + (1 - \alpha)k)^2},$$

which has the same sign as $1 - 2\alpha$ for all $k \in (\frac{1}{2}, 1)$. Thus, at $\alpha_H = \beta_L + \mu$ and $\beta_H = \beta_L$, the right-hand side of (15) is strictly larger than the right-hand side of (16), and the opposite is true at $\alpha_H = \alpha_L$ and $\beta_H = \alpha_L - \mu$. Further, as α_H decreases from $\beta_L + \mu$ to α_L and simultaneously β_H decreases from β_L to $\alpha_L - \mu$ so that equation (14) remains satisfied, the right-hand side of (15) decreases for any fixed k_H while the right-hand side of (16) increases. The proposition then follows immediately from Lemma 1 as the right-hand side of both (15) and (16) increases in k_H .

Proof of Proposition 3. The argument in the text shows that running both a positive and a negative campaign cannot reduce the total cost of separation for type (α_H, β_H) candidate in the P-region or the N-region. Now, suppose that type (α_H, β_H) is in the P/N-region. In the least cost separating equilibrium, for type (α_H, β_H) to run two campaigns to separate from type (α_L, β_L) , the campaign levels k^p and k^n must satisfy condition (6) with equality. Suppose that the high type increases k^p while simultaneously decreasing k^n , starting from $k^p = \frac{1}{2}$ and $k^n = k_H^n$ given by (3), such that (6) continues to hold with equality. Then, the infinitesimal changes $dk^p > 0$ and $dk^n < 0$ must satisfy

$$dk^p \left(\frac{\partial \Pi(\alpha_L, \alpha_H; k^p)}{\partial k^p} - C'(k^p) \right) = dk^n \left(\frac{\partial \Pi(\beta_L, \beta_H; k^n)}{\partial k^n} + C'(k^n) \right).$$

Thus, the change in the total cost of campaigns $C(k^p) + C(k^n)$, given by $C'(k^p)dk^p + C'(k^n)dk^n$, has the same sign as

$$\frac{C'(k^p)}{C'(k^n)} + \frac{\partial \Pi(\alpha_L, \alpha_H; k^p)/\partial k^p}{\partial \Pi(\beta_L, \beta_H; k^n)/\partial k^n}. \quad (17)$$

The first ratio in expression (17) is always positive, and it is weakly decreasing as k^p increases and k^n decreases if C is concave. By Lemma 1, $\partial\Pi(\alpha_L, \alpha_H; k^p)/\partial k^p$ is negative and decreasing as k^p increases when $\alpha_L < \alpha_H$; and $\partial\Pi(\beta_L, \beta_H; k^n)/\partial k^n$ is positive and decreasing as k^n decreases when $\beta_L > \beta_H$. Therefore the second ratio in expression (17) is negative and decreasing as k^p increases and k^n decreases. Moreover, at $k^p = \frac{1}{2}$ and $k^n = k_H^n$ given by (3), expression (17) is positive because $\partial\Pi(\alpha_L, \alpha_H; k^p)/\partial k^p = 0$. Together, we have that when C is concave, expression (17) can change sign at most once from positive to negative. Thus the total campaign cost is minimized at either $k^p = \frac{1}{2}$ and $k^n = k_H^n$ given by (3), or $k^p = k_H^p$ given by (2) and $k^n = \frac{1}{2}$.

Proof of Proposition 4. Fix type (α_L, β_L) . We start by considering the case when type (α_H, β_H) is in the P-region. If there exists a separating equilibrium in positive campaigns, the following two conditions must hold for some campaign level k^p :

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k^p, k') - \beta_H - C(k^p); \quad (18)$$

$$\alpha_H - \beta_H - C(k^p) \geq \Pi(\alpha_H, \alpha_L; k') - \beta_L. \quad (19)$$

By Lemma 2, $\Pi(\alpha_L, \alpha_H; k^p, k')$ decreases in both k^p and k' since $\alpha_L < \alpha_H$ in the P-region. Let $k'' \in (\frac{1}{2}, 1)$ be uniquely defined by

$$\alpha_L - \beta_L = \Pi(\alpha_L, \alpha_H; k'') - \beta_H.$$

This is the upper-bound on k' in the statement of the proposition. Then, for all $k' < k''$, there exists a unique value of $k^p \in (\frac{1}{2}, 1)$, say $k_H^{p'}$, such that (18) holds with equality. Condition (19) holds for $k^p = k_H^{p'}$ if

$$\alpha_H - \Pi(\alpha_L, \alpha_H; k_H^{p'}, k') \geq \Pi(\alpha_H, \alpha_L; k') - \alpha_L.$$

Note that the left-hand side is greater than $\alpha_H - \Pi(\alpha_L, \alpha_H; k')$ by Lemma 2. Further,

$$\begin{aligned} \alpha_H - \Pi(\alpha_L, \alpha_H; k') &= (2k' - 1)(\alpha_H - \alpha_L)\Delta(\alpha_H; k') \\ &> (2k' - 1)(\alpha_H - \alpha_L)\Delta(\alpha_L; k') = \Pi(\alpha_H, \alpha_L; k') - \alpha_L, \end{aligned}$$

where Δ is defined in the proof of Proposition 2, and the inequality follows because $\alpha_L < \alpha_H < \frac{1}{2}$.

In the P-region, if there exists a separating equilibrium in negative campaigns, then we must have:

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k') - \Pi(\beta_L, \beta_H; k^n) - C(k^n); \quad (20)$$

$$\alpha_H - \beta_H - C(k^n) \geq \Pi(\alpha_H, \alpha_L; k') - \beta_L. \quad (21)$$

However, because $\Pi(\alpha_L, \alpha_H; k') > \Pi(\alpha_L, \alpha_H; k_H^{p'}/k')$ and $\Pi(\beta_L, \beta_H; k^n) < \beta_H$, the right-hand side of condition (20) is strictly larger than that of condition (18) for any k^n . It follows that if there exists a level k^n that satisfies condition (20) with equality, it must be that $k^n > k_H^{p'}$. Thus in the least cost separating equilibrium the high type runs a positive campaign of level $k_H^{p'}$. Depending on whether there is a separating equilibrium in negative campaigns, we can construct the interim belief accordingly, similar to the proof of Proposition 1. Also, because $\Pi(\alpha_L, \alpha_H; k_H^{p'}/k') < \Pi(\alpha_L, \alpha_H; k')$, the equilibrium campaign level $k_H^{p'}$ is strictly lower than k_H^p given by (2) in the basic model.

Second, suppose that type (α_H, β_H) is in the N-region. If there exists a separating equilibrium in which type (α_H, β_H) runs a negative campaign of level k^n , then (20) and (21) hold. Because $\beta_L > \beta_H$, the right-hand side of (20) strictly decreases in k^n . Next, because $\alpha_L > \alpha_H$, $\Pi(\alpha_L, \alpha_H; k')$ increases in k' and is larger than α_H . Therefore for any k' there exists a unique level $k_H^{n'} \in (\frac{1}{2}, 1)$ such that (20) holds with equality. At this level, condition (21) holds strictly. This is because in the N-region, $\beta_L > \beta_H$ implies $\Pi(\beta_L, \beta_H; k^n) > \beta_H$, and $\alpha_H < \alpha_L < \frac{1}{2}$ implies

$$\begin{aligned} \alpha_L - \Pi(\alpha_H, \alpha_L; k') &= (2k' - 1)(\alpha_L - \alpha_H)\Delta(\alpha_L; k') \\ &> (2k' - 1)(\alpha_L - \alpha_H)\Delta(\alpha_H; k') = \Pi(\alpha_L, \alpha_H; k') - \alpha_H. \end{aligned}$$

Further, using similar arguments as in the case of the P-region, we can show that either there is no separating equilibrium in positive campaigns, or else it involves a separating level higher than $k_H^{n'}$. Thus there exists a unique least cost separating equilibrium with $k_H^{n'}$. Finally, because $\Pi(\alpha_L, \alpha_H; k') > \alpha_H$ in the N-region, $k_H^{n'}$ is strictly greater than k_H^n given by (3) in the basic model.

Finally, in the P/N-region, similar arguments as above establish that there always exists a separating equilibrium in positive campaigns of level $k_H^{p'}$, and further $k_H^{p'} < k_H^p$. Also, if there exists a separating equilibrium in negative campaigns, then the separating level is $k_H^{n'}$, which is

strictly lower than k_H^n because $\alpha_H > \alpha_L$ implies $\Pi(\alpha_L, \alpha_H; k') < \alpha_H$ in the P/N-region. The least cost separating equilibrium is the less costly of the two campaigns (if there exists a separating equilibrium in negative campaigns).

Proof of Proposition 5. We construct separating equilibria where, for each realized type, the low type candidate run an uninformative campaign. For the high types, there are two cases: type (α_H, β_H) candidate a and type (α_L, β_L) candidate b run campaigns with the same target; or they run campaigns with different targets.

In the first case, suppose that in equilibrium, type (α_H, β_H) candidate a runs a positive campaign of some level k_a^p and (α_L, β_L) candidate b runs a negative campaign of level k_b^n ; the case of the high type a running a negative campaign and the high type b running a positive campaign is symmetric. We will argue there is always such an equilibrium when (α_H, β_H) is either in the P-region or in the P/N-region. The necessary equilibrium conditions are:

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_{HL}; k_a^p, k_b^n) - \beta_{HL} - C(k_a^p) \quad (22)$$

$$\alpha_H - \beta_H - C(k_a^p) \geq \alpha_{HL} - \beta_{HL} \quad (23)$$

$$\beta_H - \alpha_H \geq \beta_{HL} - \Pi(\alpha_H, \alpha_{HL}; k_a^p, k_b^n) - C(k_b^n) \quad (24)$$

$$\beta_L - \alpha_L - C(k_b^n) \geq \beta_{HL} - \alpha_{HL} \quad (25)$$

Suppose that conditions (22) and (24), respectively the incentive constraints for type (α_L, β_L) candidate a and type (α_H, β_H) candidate b , hold with equality. Substituting condition (22) into (23) and condition (24) into (25), we need the following conditions for (23) and (25) to hold:

$$\alpha_{HL} - 2\beta_{HL} + \Pi(\alpha_L, \alpha_{HL}; k_a^p, k_b^n) \leq \alpha_H - \beta_H + \alpha_L - \beta_L \leq \alpha_{HL} - 2\beta_{HL} + \Pi(\alpha_H, \alpha_{HL}; k_a^p, k_b^n).$$

By Lemma 2, $\Pi(\alpha_L, \alpha_{HL}; k_a^p, k_b^n) < \alpha_{HL} < \Pi(\alpha_H, \alpha_{HL}; k_a^p, k_b^n)$ in either P-region or P/N-region. Therefore under the neutrality assumption, the above conditions are satisfied. Thus each high type candidate does not want to pretend to be type $(\alpha_{HL}, \beta_{HL})$ when the respective low type candidate is indifferent. It is then straightforward to specify a complete set of interim beliefs to support the separating equilibrium.

Next, we show that there exists a unique pair of campaign levels $k_a^p, k_b^n \in (\frac{1}{2}, 1)$ such that conditions (22) and (24) both hold with equality, which are then the equilibrium levels for the

high type candidates. Let \bar{k}_a^p be such that condition (22) holds with equality at $k_b^n = \frac{1}{2}$. This is well-defined because $\alpha_L - \beta_L < \alpha_{HL} - \beta_{HL}$ in both the P-region and the P/N-region. Similarly, let \bar{k}_b^n be such that condition (24) holds with equality at $k_a^p = \frac{1}{2}$. Let $r_a(k_b^n)$ be the value of k_a^p such that condition (22) holds with equality for each $k_b^n > \frac{1}{2}$. Note that since $\alpha_L < \alpha_{HL}$ in the P-region and in the P/N-region, by Lemma 1 and 2, $r_a(k_b^n)$ decreases as k_b^n increases. We first claim that the function r_a is well-defined for all $k_b^n \leq \bar{k}_b^n$, that is, $r_a(\bar{k}_b^n) > \frac{1}{2}$. From condition (22), this claim is equivalent to

$$\alpha_{HL} - \Pi(\alpha_L, \alpha_{HL}; \bar{k}_b^n) \leq \alpha_{HL} - \alpha_L - \beta_{HL} + \beta_L.$$

By the definition of \bar{k}_b^n , we have

$$\Pi(\alpha_H, \alpha_{HL}; \bar{k}_b^n) - \alpha_{HL} = \alpha_H - \alpha_{HL} - \beta_H + \beta_{HL} - C(\bar{k}_b^n).$$

Using the function Δ defined in the proof of Proposition 2, we have

$$\begin{aligned} \alpha_{HL} - \Pi(\alpha_L, \alpha_{HL}; \bar{k}_b^n) &= (2\bar{k}_b^n - 1)(\alpha_{HL} - \alpha_L)\Delta(\alpha_{HL}; \bar{k}_b^n); \\ \Pi(\alpha_H, \alpha_{HL}; \bar{k}_b^n) - \alpha_{HL} &= (2\bar{k}_b^n - 1)(\alpha_H - \alpha_{HL})\Delta(\alpha_{HL}; \bar{k}_b^n). \end{aligned}$$

Thus, the claim is true if $\alpha_{HL} = \frac{1}{2}(\alpha_H + \alpha_L)$ and $\beta_{HL} = \frac{1}{2}(\beta_L + \beta_H)$, and hence by continuity also holds under the neutrality assumption. A symmetric argument establishes that the function $r_b(k_a^p)$ given by the value of k_b^n such that condition (24) holds with equality is well-defined for all $k_a^p \in \left[\frac{1}{2}, \bar{k}_a^p\right]$. Now, by taking derivatives we can verify that $r_a'(k_b^n) < r_b'(k_a^p)$ whenever they intersect under the assumption of $\alpha_{HL} = \frac{1}{2}(\alpha_H + \alpha_L)$. It follows immediately that r_a and r_b have a unique intersection at some k_a^p, k_b^n , with $k_a^p \in \left(\frac{1}{2}, \bar{k}_a^p\right)$ and $k_b^n \in \left(\frac{1}{2}, \bar{k}_b^n\right)$.

Finally, in the P/N-region, we have $\beta_H < \beta_{HL} < \beta_L$. Comparing (22) to (2), and (24) to (3), we immediately obtain that the equilibrium levels for the high type candidates are strictly lower than their respective levels k_H^p and k_H^n .

In the second case, suppose that in equilibrium, type (α_H, β_H) candidate a runs a positive campaign of some level k_a^p and (α_L, β_L) candidate b runs a positive campaign of level k_b^p ; the case of the two types running negative campaigns is symmetric. The equilibrium condition for the low type candidate a is (7); the condition for the low type b is

$$\beta_H - \alpha_H \geq \Pi(\beta_H, \beta_{HL}; k_a^p) - \Pi(\alpha_H, \alpha_{HL}; k_b^p) - C(k_b^p). \quad (26)$$

The rest of the arguments is analogous to the first case. Briefly, under the neutrality assumption, if the low type candidates are indifferent then the high type candidates strictly prefer their respective informative campaigns. Conditions (7) and (26) with equalities have a unique interior intersection in (k_a^p, k_b^p) . Since $\alpha_L < \alpha_H$ and $\beta_L > \beta_H$ in the P/N-region, the intersection involves levels that are strictly lower than k_H^p given by (2) for candidate a and for candidate b respectively.

Proof of Proposition 6. For each type (α, β) , denote the complete information level k^c as a function $\Theta(\alpha, \beta)$, given by

$$k^c = \Theta(\alpha, \beta) = \frac{(1 - \alpha)\beta}{\alpha(1 - \beta) + (1 - \alpha)\beta}.$$

Note that $\Theta(\alpha, \beta)$ is decreasing in α and increasing in β . Fix type (α_L, β_L) such that $\alpha_L + \beta_L < 1$. We have shown in the text that the unique campaign level $k_L^p \in (\frac{1}{2}, k_L^c)$ at which type (α_L, β_L) is indifferent between the negative campaign of level k_L^c and a positive campaign is given by (8).

We claim that there is a unique solution in (α_H, β_H) to the two equations $\Theta(\alpha_H, \beta_H) = k_L^p$, and

$$\alpha_H k_L^p + (1 - \alpha_H)(1 - k_L^p) - C(k_L^p) = (1 - \beta_H)k_L^c + \beta_H(1 - k_L^c) - C(k_L^c). \quad (27)$$

To see this, note that equation (27) is a downward sloping line in the α_H - β_H diagram going through (α_L, β_L) . At $\alpha_H = \alpha_L$ and $\beta_H = \beta_L$, by definition we have $\Theta(\alpha_H, \beta_H) = k_L^c > k_L^p$. At the intersection of (27) and $\alpha_H = \beta_H$, we have $\Theta(\alpha_H, \beta_H) = \frac{1}{2} < k_L^p$. Thus, there is a unique type $(\hat{\alpha}_H, \hat{\beta}_H)$ satisfying $\Theta(\hat{\alpha}_H, \hat{\beta}_H) = k_L^p$ and (27). Define the positive boundary by setting (9) to equality, with $k_H^c = \Theta(\alpha_H, \beta_H)$. Then, the boundary is given by (27) for all $\alpha_H \geq \hat{\alpha}_H$, and by

$$\alpha_H \Theta(\alpha_H, \beta_H) + (1 - \alpha_H)(1 - \Theta(\alpha_H, \beta_H)) - C(\Theta(\alpha_H, \beta_H)) = (1 - \beta_H)k_L^c + \beta_H(1 - k_L^c) - C(k_L^c)$$

for all $\alpha_H \in [\alpha_L, \hat{\alpha}_H]$. In both cases, for all (α_H, β_H) such that $\Theta(\alpha_H, \beta_H) = k$ with $k \in [\frac{1}{2}, k_L^c]$, there is a unique type, say (α'_H, β'_H) , that is on the positive boundary, and (9) is satisfied if and only if $\alpha_H > \alpha'_H$. Finally, on the part of the positive boundary with $\alpha_H \in [\alpha_L, \hat{\alpha}_H]$, for each (α_H, β_H) satisfying (27), we have $\Theta(\alpha_H, \beta_H) > k_L^p$, and thus

$$\alpha_H \Theta(\alpha_H, \beta_H) + (1 - \alpha_H)(1 - \Theta(\alpha_H, \beta_H)) - C(\Theta(\alpha_H, \beta_H)) < (1 - \beta_H)k_L^c + \beta_H(1 - k_L^c) - C(k_L^c),$$

implying (α_H, β_H) is below the positive boundary.

Consider any (α_H, β_H) such that $\Theta(\alpha_H, \beta_H) \leq k_L^c$ and $\alpha_H \leq \frac{1}{2} \leq \beta_H$. Suppose first that (α_H, β_H) satisfies (9). There are two cases. If $\Theta(\alpha_H, \beta_H) < k_L^p$, then the unique least cost separating equilibrium is for the low type to run a negative campaign of level k_L^c and for the high type to run a positive campaign of level k_L^p . If instead $\Theta(\alpha_H, \beta_H) \geq k_L^p$, then the unique least cost separating equilibrium is for the low type to run a negative campaign of level k_L^c and for the high type to run a positive campaign of level k_H^c . In either case, it is straightforward to construct the set of interim beliefs of the voter to support the equilibrium.

Suppose next that type (α_H, β_H) violates (9). There does not exist a separating equilibrium. Let $\alpha_m = \lambda\alpha_L + (1 - \lambda)\alpha_H$ and $\beta_m = \lambda\beta_L + (1 - \lambda)\beta_H$ be the pooling beliefs of the voter, and let $k_m = \Theta(\alpha_m, \beta_m)$. Then in any pooling equilibrium, the minimum campaign level that candidate a has to run is k_m . For any $\hat{k}^n \in [k_m, k_L^c]$, there is a pooling equilibrium in which both types run the negative campaign of level \hat{k}^n , supported by the out-of-equilibrium belief that the type is (α_L, β_L) after any deviation to a campaign level below k_m and (α_H, β_H) after any deviation to above k_L^c .

Appendix B Robustness of Least Cost Separation

In models with more than two types or with richer information structure, it is necessary to rank each type of candidate's incentives to imitate all types of candidate above him to study whether equilibrium separation is possible. In this section, we present a single crossing condition and show that the separation result in the basic model is robust if this condition is satisfied.

B.1 Single crossing condition

Suppose that the candidate chooses a positive campaign; the case of negative campaigns is similar. Let the candidate's type be (α, β) and the voter's interim beliefs be $(\tilde{\alpha}, \tilde{\beta})$. Then at a campaign level of k , the payoff to the candidate is $\Pi(\alpha, \tilde{\alpha}; k) - \tilde{\beta} - C(k)$. The single crossing condition requires that the marginal rate of substitution in the candidate's payoff function between k and the interim belief $\tilde{\alpha}$, given by $-(\partial\Pi(\alpha, \tilde{\alpha}; k)/\partial k - C'(k))/(\partial\Pi(\alpha, \tilde{\alpha}; k)/\partial\tilde{\alpha})$, is decreasing in α . Using

the function Δ defined in the proof of Proposition 2, we can write this single crossing condition as

$$\left(2\Delta(\tilde{\alpha}; k) + (2k - 1)\frac{\partial\Delta(\tilde{\alpha}; k)}{\partial k}\right) (1 - (2k - 1)\Delta(\tilde{\alpha}; k)) + (2k - 1)\frac{\partial\Delta(\tilde{\alpha}; k)}{\partial\hat{\alpha}}C'(k) > 0.$$

Because $\Delta(\tilde{\alpha}; k)$ increases in k , a sufficient but not necessary condition for the above to be true is if $\Delta(\tilde{\alpha}; k)$ increases in $\hat{\alpha}$, which is the case if $\alpha < \frac{1}{2}$. Similarly in a negative campaign, the single crossing condition is guaranteed if $\beta > \frac{1}{2}$.

In our basic model with two types of candidate, this single crossing condition is sufficient to rule out pooling equilibrium under the D1 refinement of Banks and Sobel (1987). Consider a pooling equilibrium in a positive campaign of level \hat{k}^p . Since in any equilibrium type (α_L, β_L) gets at least his complete information payoff, we have $\hat{k}^p < k_H^p$. The single crossing condition guarantees that for any deviation $k^p \in (\hat{k}^p, k_H^p)$ in a positive campaign, if the interim belief is such that the low type weakly benefits from the deviation, then under the same interim belief the high type strictly benefits. Applying the D1 refinement, the out-of-equilibrium belief should be (α_H, β_H) after the deviation to k^p . Under such beliefs, however, the pooling equilibrium fails because the high type benefits by deviating to a positive campaign of level just above \hat{k}^p .

B.2 Separation with more than two types

The single crossing condition also allows us to generalize our results to more than two types. We focus on separation by the level in positive campaigns. Let there be $T > 2$ types, denoted as (α_t, β_t) , $t = 1, \dots, T$. Assume that $\alpha_{t-1} \leq \alpha_t < \frac{1}{2}$ and $\beta_{t-1} \geq \beta_t$ with at least one strict inequality for all $t = 2, \dots, T$ so that the single crossing condition is satisfied. The least cost separating equilibrium levels of positive campaigns, k_t^p are defined iteratively by the indifference condition of type $(\alpha_{t-1}, \beta_{t-1})$ between its own equilibrium campaign of level k_{t-1}^p and type (α_t, β_t) 's level k_t^p , starting with $k_1^p = \frac{1}{2}$:

$$\alpha_{t-1} - \beta_{t-1} - C(k_{t-1}^p) = \Pi(\alpha_{t-1}, \alpha_t; k_t^p) - \beta_t - C(k_t^p). \quad (28)$$

Since $\alpha_{t-1} \leq \alpha_t$ and $\beta_{t-1} \geq \beta_t$ with at least one strict inequality, the above condition implies that $k_{t-1}^p < k_t^p$. Assume that $C(1)$ is sufficiently great, or $\beta_t - \beta_{t-1}$ is sufficiently small, so that all levels k_t^p are well defined. Consider first ‘‘upward’’ deviations; downward deviations can be

symmetrically analyzed. We claim that type (α_t, β_t) strictly prefers k_t^p to any $k_{t'}^p$ with $t' \geq t + 2$. To see this, note that type $(\alpha_{t'-1}, \beta_{t'-1})$ is indifferent between $k_{t'-1}^p$ and $k_{t'}^p$. From equation (28) for type $(\alpha_{t'-1}, \beta_{t'-1})$, we have that type (α_t, β_t) strictly prefers $k_{t'-1}^p$ to $k_{t'}^p$ if

$$\Pi(\alpha_{t'-1}, \alpha_{t'-1}; k_{t'-1}^p) - \Pi(\alpha_t, \alpha_{t'-1}; k_{t'-1}^p) < \Pi(\alpha_{t'-1}, \alpha_{t'}; k_{t'}^p) - \Pi(\alpha_t, \alpha_{t'}; k_{t'}^p),$$

which is true due to the single crossing conditions ($\alpha_{t'-1} \leq \alpha_{t'} < \frac{1}{2}$ and $k_{t'-1}^p < k_{t'}^p$). Since type (α_t, β_t) is indifferent between k_t^p and k_{t+1}^p , an iteration of the above argument establishes that type (α_t, β_t) strictly prefers k_t^p to $k_{t'}^p$.

B.3 Separation with a continuous campaign signal

Suppose that each information campaign generates a continuous campaign signal s about the target of the campaign, we can adapt the single crossing condition to ensure that different types of candidates separate by running different levels of campaigns. To fix ideas, consider the one-dimensional model in which a is the only candidate. Suppose that he runs a positive campaign. Recall that a is qualified ($q = 1$) or unqualified ($q = 0$). Let g_q^k be the density function of the campaign signal s conditional on state q , with the same support $[\underline{s}, \bar{s}]$, where k (to be described below) corresponds to the campaign level before. For any given interim belief $\tilde{\alpha}$, the voter's expected posterior belief is

$$\Pi(\alpha, \tilde{\alpha}; k) = \int_{\underline{s}}^{\bar{s}} \frac{\tilde{\alpha} g_1^k(s)}{\tilde{\alpha} g_1^k(s) + (1 - \tilde{\alpha}) g_0^k(s)} (\alpha g_1^k(s) + (1 - \alpha) g_0^k(s)) ds,$$

which remains a linear function of the candidate's private belief α . Assume further that the density function has the monotone likelihood ratio property (MLRP): $g_1^k(s)/g_0^k(s)$ increases with s so that a higher s leads to a greater posterior belief that the candidate is qualified (Milgrom 1981).

Next, we extend the symmetric binary information structure to model the idea that a higher campaign level k means a more informative signal structure. Denote the midpoint of the support $[\underline{s}, \bar{s}]$ as s_* . Suppose for each k , the two density functions $g_1^k(s)$ and $g_0^k(s)$ satisfy $g_1^k(s) = g_0^k(2s_* - s)$ and $g_0^k(s) = g_1^k(2s_* - s)$. Also, suppose that as k increases, $g_1^k(s)$ increases and $g_0^k(s)$ decreases for each $s > s_*$. The symmetry and MLRP together imply that $g_1^k(s) \geq g_0^k(s)$ for all $s \geq s_*$, and the opposite holds for $s \leq s_*$. Then, if $k' > k$, denoting G_q^k as the distribution function of s conditional

on q , we have

$$(G_1^{k'})^{-1}(G_1^k(s)) > (G_0^{k'})^{-1}(G_0^k(s))$$

for all $s \in (\underline{s}, \bar{s})$. The above inequality shows that the information structure parameterized by k' is more informative than the one by k in the sense of Lehmann (1988).

Since $\Pi(\alpha, \tilde{\alpha}; k)$ remains linear in α , the single crossing condition needed for separation is obtained if the partial derivative of $\Pi(\alpha, \tilde{\alpha}; k)$ with respect to α is increasing both in k and in $\tilde{\alpha}$. To see that this partial derivative is increasing in k , note that by the symmetric construction of g_q^k , we can write it as an integral of the product of two differences:

$$\int_{s_*}^{\bar{s}} \left(\frac{\tilde{\alpha} g_1^k(s)}{\tilde{\alpha} g_1^k(s) + (1 - \tilde{\alpha}) g_0^k(s)} - \frac{\tilde{\alpha} g_0^k(s)}{\tilde{\alpha} g_0^k(s) + (1 - \tilde{\alpha}) g_1^k(s)} \right) (g_1^k(s) - g_0^k(s)) ds.$$

As k increases, $g_1^k(s)$ increases and $g_0^k(s)$ decreases for each $s > s_*$, and thus both differences in the above integrand increase. To find a sufficient condition for the partial derivative to increase in $\tilde{\alpha}$, note that since $G_1^k(s)$ first-order stochastically dominates $G_0^k(s)$ by MLRP, it suffices if the derivative of the posterior belief $\tilde{\alpha} g_1^k(s) / (\tilde{\alpha} g_1^k(s) + (1 - \tilde{\alpha}) g_0^k(s))$ with respect to $\tilde{\alpha}$ is increasing in s . This is true if, at the highest campaign signal \bar{s} and the most informative campaign \bar{k} , we have

$$\tilde{\alpha} g_1^{\bar{k}}(\bar{s}) < (1 - \tilde{\alpha}) g_0^{\bar{k}}(\bar{s}).$$

Or the state is more likely to be 0 than 1 even after the most favorable campaign signal from the most informative campaign. This condition, the counterpart to $\alpha < \frac{1}{2}$ in the original binary model, imposes a joint upper bound on the private belief and the level of campaign.

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