The Role of Quality in Service Markets Organized as Multi-Attribute Auctions*

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Abstract

We develop a methodology for the empirical study of the markets for services. These markets are typically organized as multi-attribute auctions in which buyers take into account seller's price as well as his various characteristics, including quality. Our identification strategy exploits observed buyer and seller decisions to recover the distribution of seller quality conditional on observable characteristics, the distribution of seller's costs conditional on the full vector of characteristics including quality, and the distribution of buyers' tastes. These objects are central to understanding the functioning of these markets, their efficiency, and their optimal design. We propose an implementable econometric procedure based on our identification strategy and apply it to an on-line market for programming services. Our empirical results confirm that quality plays a central role in this market: for example, the variation in quality among the providers and the willingness of buyers to pay for quality account for over 50% of the variation in buyer choices, while the observable characteristics account for less than 20%.

Keywords: quality, markets for services, multi-attribute auctions, identification, unobserved bidder heterogeneity, unobserved buyers' tastes, participation in auctions

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1 Introduction

Recall the last time you hired someone to paint your house. Several painters evaluated the job and submitted their price quotes (bids). Since candidates potentially differed in quality, you interviewed each of them to assess their professionalism, the likelihood that the job would be completed on time and according to your expectations. When submitting their bids, painters might have been uncertain about your idiosyncratic willingness to pay for quality, and most likely uninformed about the identity and costs of their competitors. You selected the painter taking both the price and his quality into consideration. You probably went through a similar process when hiring a nanny for your kids, a piano tuner, a hairdresser, or when choosing a brokerage. In each of these examples, the price was an important, but quality considerations also played an important role in your ultimate choice. Formally, you conducted a multi-attribute auction, where the price and the quality dimensions of the bidder were relevant to your selection.

Transactions described above are not rare. Services account for around 80% of the U.S. gross domestic product, and many service markets, including private business sector procurement, are organized in the form of multi-attribute auctions. This is suggestive of the fact that, in addition to price, other bidder attributes, such as quality, play a role in choosing the provider. How important is that role? What is the distribution of buyer preferences for quality? How large is the dispersion of the quality of suppliers? How well do the observable (to an econometrician) supplier characteristics proxy for their quality? What is the distribution of supplier costs conditional on quality? More generally, how efficient are these markets? What is their optimal design? Unfortunately, despite the manifest importance of these questions the literature provides limited answers.

The primary constraint on the analysis of these markets has been the limited nature and availability of the data. These markets are decentralized, with many small players providing similar but not identical services. Fortunately, recently such markets have become increasingly organized on the Internet, opening an avenue for their study. Yet, even with the wider availability of centralized data, the characteristics of the service providers observed in these data likely provide a relatively poor measure of quality. For example, while the observed qualifications and experience of the painter mattered for your selection, the photographs of completed jobs probably provided considerably more information about quality. Similarly, you probably got more information about the quality of the nanny by observing her interaction with your kids than from knowing her age, and education. Thus, a buyer is likely to have richer information on the quality of a service provider than what is recorded in data set. The objective of this paper is to develop methodology that supplements quality-related characteristics recorded in data with information inferred from the economic choices observed made by buyers.

Our modeling choices are motivated by the data from an online market for programming services[1] that is explicitly structured as a multi-attribute auction and resembles an off-line service markets in many ways. While this market, like many other service markets, lacks a uniform and objective record of quality, it is observed that the buyers often do not award the contract to the lowest bidder and that the premium the buyers agree to pay is not well accounted for by the difference in other observed seller characteristics. In other words, buyers are willing to pay a

[1]Professional services such as computer services account for half of the 20% growth in GDP share of services over the last fifty years (Herrendorf, Rogerson, Valentinyi(2009)).

premium for desirable seller characteristics that are not recorded in the data.

In our model, each project advertised by a buyer is associated with a set of potential bidders. A subset of potential bidders chooses to participate the auction (to become actual bidders).[2] Bidders' idiosyncratic costs for completing the project are private information. The buyer's utility from a specific seller's services is a function of the seller's characteristics with a buyer-specific weights. It also includes seller-specific component that reflects the value of the match as perceived by buyer. One characteristic considered by the buyer captures seller's permanent quality which remains the same across projects. Buyers assess bidders' qualifications through private communications during the bidding process.

We begin by developing a strategy to identify the key objects of interest: the distribution of bidder quality, the distribution of the buyers' tastes for quality, and the distribution of the sellers' costs for completing the projects. While our environment is unique, it shares both the features of discrete choice and auction settings. Our identification strategy builds on the insights from these two literatures.

The discrete choice framework is typically used to model purchases in markets for differentiated physical products.[3] Similar to the environment we consider, in those markets multiple buyers choose from a finite set of products, buyers may be heterogeneous in their tastes for product attributes, and some product attributes may not be observed by an econometrician but are known to market participants. In the market for physical products, researchers observe decisions of a large number of consumers operating in the same market and, therefore, choosing from the same set of products. Thus, product market shares are observed and product qualities can be solved for from their relationship with the market shares.

These techniques cannot be applied in our setting. The main difficulty is that in service markets sellers' market shares conditional on the choice set are not observed since very few buyers are choosing from exactly the same set of alternatives. Moreover, while some generalized notion of the market shares can potentially be defined for the "permanent" sellers, in most auctions they compete with "transitory" competitors who enter the market only for a short time. This feature is typical of many service markets that are often characterized by high turnover of the providers. Our evidence suggests that, due to private communication between buyers and sellers, buyers are well-informed about the quality of even transitory sellers and take it into account when selecting a winner. Since a reliable estimate of market shares for transitory sellers is not available, and the market shares for the permanent sellers involve unobserved quality differences among both permanent and transitory bidders, it is infeasible to infer product qualities merely from their relationship with the market shares.

Our solution to this problem, which is the first step of our three-step identification strategy, is to use the long-run performance of permanent bidders to rank them in the order of increasing qualities conditional on observable characteristics. In particular, we construct a pairwise index

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[2]We utilize auction participation models, such as in Li and Zheng (2009), Krasnokutskaya and Seim (2011), Roberts and Sweeting (2010), Marmer, Shneyerov, and Xu (2010), in linking the set of potential competitors to the set of alternatives from which the buyer eventually makes his choices as well as in rationalizing pricing competition for a specific project.

[3]The identification of such models is studied in, e.g., Berry and Haile (2010b), Berry and Haile (2010a), Fox and Amit (2012), Bajari, Fox, Kim, and Ryan (2012).

that reflects one seller's probability of winning in the auctions where both sellers are among potential bidders. Since the sellers often do not fully know their actual competitors when they decide on their bids, the set of potential bidders captures the relevant competitive environment. We show that between two sellers, the seller with the higher quality has a higher value of this index. Since we apply this procedure to permanent bidders who complete a large number of projects, the probability of winning on which the index is based can be considered observed in the data.[4] This step effectively identifies (up to a monotone transformation) the distribution of quality for permanent bidders as a function of other covariates while bypassing the issues of unobserved buyers' tastes, the unavailability of information on market shares, and the presence of transitory bidders.

Next, we identify the distribution of buyers' tastes and sellers' quality levels. To do so, we exploit a feature of our environment often encountered in auction settings – variation in private sellers' costs. We consider auctions that are characterized by different configurations of the set of permanent actual bidders. For example, we consider auctions where only bidders with the same characteristics participate or a set of auctions with bidders that share same characteristics except for quality. After we fix the set of permanent actual bidders, bidders' prices vary exogenously across auctions due to the presence of private cost information. Then, the relationships between permanent seller's price and his probability of winning in the presence of different sets of permanent competitors identify distributions of various buyer's taste components and quality levels. To summarize, in our setting the distribution of buyers' tastes is identified non-parametrically out of the variation in the sets of permanent actual bidders combined with the exogenous variation in prices. We provide conditions under which our model generates sufficient price variation.

In the third and final step of our identification strategy, we recover the distribution of sellers' costs conditional on observable characteristics and quality. Note that in our environment it is natural to assume that buyers are heterogeneous in their requirements for the quality of the job and therefore their willingness to pay for quality. Moreover, the buyers' decision rule is not revealed to bidders at any time, so that there is uncertainty about the weight the buyer would assign to different characteristics. It is this unstructured nature of the auction format that distinguishes the market we study from those studied in the previous auction literature, including the recent literature on "non-standard auction formats" which assumes that the decision rule is known to the bidders.[5] When the award rule is not known, mark-up over costs charged by bidders is not linked directly to the distribution of competitors' prices. Instead, it also depends on the distribution of buyers' tastes. Therefore, we can proceed to identify the distribution of bidders' costs only after we identify the distributions of buyers' tastes and all relevant seller

[4]Note that, as discussed above, the market shares for permanent bidders defined on the basis of the set of potential bidders cannot be used to solve for quality of permanent bidders due to the presence of transitory bidders.

[5]These include standard auctions with discrimination or preferential treatment for particular types of participants in Marion (2007), Krasnokutskaya and Seim (2011), Swinkels (2009) and scoring auctions where the award is based on a scoring rule that aggregates several bid components, studied in, e.g., Asker and Cantillon (2010), Asker and Cantillon (2008), andBajari and Lewis (2011). Scoring auctions additionally differ from our environment by treating bidder's characteristics as auction-level choice variables. While a multi-attribute auction format is prevalent in industry procurement, we found only two papers that study it. Greenstein (1993) and Greenstein (1995) analyze IBM's procurement and documents different weights that IBM was revealed through its choices to assign to different attributes of bidders.

characteristics (including quality). We accomplish this by relying on the inversion method first proposed by Guerre, Perrigne, and Vuong (2000) and later applied in various environments by Li, Perrigne, and Vuong (2000), Jofre-Bonet and Pesendorfer (2003), Li, Perrigne, and Vuong (2002), Krasnokutskaya (2011), Athey and Haile (2002).

Having established the identification of the model, we develop econometric methodology. First we propose a nonparametric classification procedure that implements the first step of the identification strategy. It uses the index developed in the identification section to rank bidders with respect to quality, i.e., to effectively sub-divide them into groups of equal quality. Two practical issues may potentially arise in small samples. First, the transitivity of the quality ranking could be violated. Second, one may face indeterminacy with multiple compatible classifications. We propose a sequential algorithm that is designed to circumvent the transitivity issue, and to choose the classification which is most strongly supported by the data. Through our classification method, we consistently estimate a quality group structure. However, it provides no evidence on quality levels and therefore does not permit comparison across covariate values.

Next, we estimate quality levels and the distribution of buyers' tastes by method of moments. After parametrically specifying the distribution of buyers tastes, we use insights from the nonparametric identification strategies to form moment restrictions. The main challenge in the implementation of this step arises from the presence of transitory sellers. The relationship between the prices and qualities of these sellers has to be integrated out in theoretical moment conditions. The traditional approach that requires solving for sellers' strategies at every iteration is extremely computationally costly in our setting. Instead, we show that transitory sellers' strategies are non-parametrically identified from the data through the variation in the sets of permanent competitors. We then parametrically approximate the equilibrium distribution of the transitory sellers' bids and participation frequencies and estimate these objects jointly with buyers' tastes and quality levels. We also show that our methodology is amenable to semi-parametric extension that allows estimating equilibrium objects non-parametrically.

Our methodology has a number of attractive features. For example, it recovers the qualities of permanent sellers exactly. The recovered distribution of qualities is not restricted to be orthogonal to other relevant seller characteristics such as seller's country affiliation or measures of performance recorded in the data. This allows us to explore the economic importance of these relationships. In addition, our approach obviates the need for instrumental variables often required to identify buyers' price sensitivity conditional on quality. The instruments typically used are the characteristics of competing products. These instruments are not available in environments such as ours where all observable seller characteristics are potentially endogenous (e.g., reputation score or the number of projects completed). Note that as the data sets with transaction prices become available, our identification approach can also be used in the classic differentiated product markets. That is, in environments where transaction price deviations from wholesale price are based on private sellers' information, the variation in transaction prices can be exploited to identify buyers' tastes in the absence of other instruments. While our methodology is tailored to a specific service market, we believe it can be modified in a number of ways to apply to wide range of settings with similar data structures.

Applying our identification and inference strategies to the data from the online market for programming services, we find that quality plays an economically significant role. The model with quality explains 70% of buyers' choices, whereas the model without quality explains only

25%. There is also clear evidence for nontrivial quality heterogeneity among the sellers. The buyers exhibit a strong preference for quality: an average buyer is willing to pay a 50% premium to move from the lowest to the highest quality level. Thus, the variations in quality levels across the sellers and in buyers' willingness to pay for quality account for more of the variation in the data than all other covariates combined. We also estimate a statistically significant relationship between transitory sellers' bids and their qualities. This result demonstrates the strength of our identification strategy and fit of our model, since the relationship between transitory seller's quality and price is not directly observed in the data. While unobserved quality is commonly believed to be important in the service markets, our findings contribute to the literature by offering the first formal assessment of its role. Moreover, the presence of large quality heterogeneity provides a plausible explanation for why the market we study, and service markets more generally, tend to be organized as multi-attribute auctions.

Our empirical results also contribute to a better understanding of the information and enforcement issues studied in the literature on Internet auctions. Main questions in this literature are whether consumers are able to obtain credible information about the product sold and what mechanism incentivizes sellers to exert higher effort. One possibility emphasized in the literature is that reputation score proxies for the quality of the seller, or, in other words, it serves as an informative device that counteracts adverse selection problems. On the other hand, it may represent an enforcement mechanism that eliminates moral hazard issues. The emerging consensus in the literature seems to indicate that the latter role is probably more relevant (e.g., Cabral and Hortacsu (2010)). Our empirical results appear to reinforce this consensus, as we explain below.

First, we estimate a non-trivial quality heterogeneity within a narrow band of reputation scores, which suggests that the reputation score predicts seller's quality very imperfectly. Second, we estimate that the bid distribution of transitory bidders significantly depends on the bidder quality in addition to all the covariates. Recall, that this object is recovered from the variation in buyers choices since no direct link between transitory seller's bids and his quality is observed in the data. This suggests that the buyer must be informed about quality either directly or indirectly through signaling. It appears thus that information problems play only a limited role, at least in the segment of the market we study.[6] The reputation scores are nonetheless valued by the buyer, which must indicate that they fulfill their literal role, namely, that of creating reputation. In summary, evidence from the data appear to point more toward the role of reputation scores as addressing moral hazards rather than adverse selections.

The estimated bidding strategies show expected regularities: they are increasing in costs, and the prices of the low-quality bidders are noticeably lower than those of the higher-quality ones. A particularly interesting insight is that when we combine bid distribution with bid strategies to recover the distribution of costs conditional on seller's characteristics including quality, we find that a tight (low variance) cost distribution can rationalize the large variation in prices observed in this market. Intuitively, in a standard auction, the sensitivity of the winning probability to the increase in prices provides a high-cost bidder with an incentive to lower his mark-up. In contrast, our model allows for seller-specific match component of buyers' tastes that is unobserved (and therefore purely stochastic) from seller's point of view. The high-cost seller's winning probability depends to a large degree on favorable realization of this stochastic component. This gives seller incentive to drive the price up and gamble on a lucky outcome.

[6]Relatedly, Lewis (2011) finds little evidence of adverse selection in the context of the e-Bay auto market.

The paper is organized as follows. Section 2 describes our market and main features of the data that motivate our modeling choices. The model is described in Section 3. In Section 4 we develop the identification strategy. Sections 5 and 6 describe the econometric implementation of the proposed identification strategy. Section 7 presents the results of the empirical analysis based on the methodology developed in this paper. Section 8 summarizes the findings and outlines directions for further research.

2 Market Description and Some Features of Data

2.1 Market Description

We study a market mediated by an online platform that serves as a match-maker between the demand and supply for services of computer programming. This company provides an environment that allows buyers (the demand side) to post job announcements. At the same time it maintains the registry of potential sellers (the supply side). The registry provides limited information on verifiable "outside" credentials as well as information about the on-site performance of the seller. The later includes reputation scores or ratings, buyer's numerical feedback about working with a given seller, as well as instances of delays and disputes. In the case of a dispute, the company provides professional arbitration services which ensure that a seller is paid if only if the completed job satisfies industry standards.

The intermediary company allocates jobs through multi-attribute auctions. Under the rules of such auction, a buyer is allowed to take into account multiple seller characteristics in addition to the price quote. As a result, the selected seller is not necessarily the one who submits the lowest quote. While the buyer considers some (possibly all) available information about participating sellers when making his decision, the weights he assigns to different pieces of information are not known to other market participants.

Suppliers can communicate with the buyer before posting a price quotes. Such an exchange of messages is very common. On average each seller submits three messages per auction in our data. In some cases, a seller can attach an example of his work or a sketch of the proposed code. The number and the content of these communications are not observed by the other sellers. Hence while the buyer has an opportunity to form an informed opinion about each sellers' quality, competing sellers have much more limited knowledge of their competitors' quality. In principle, competitors can infer seller's quality from his long run rate of winning in the way similar to the one we propose in this paper.

When a seller contacts a buyer for any reason, his code name appears on the project webpage. Therefore, at any point in time, a visitor to the page can see the list of the sellers who contacted the buyer before this point. This list generally does not coincide with the set of the sellers who eventually submit price quotes since a few sellers may ask the buyer a question without submitting a quote. Therefore, a prospective seller does not observe the set of his competitors.[7] It is

[7]Another indication that the set of actual competitors is not observed is that bids are generally submitted throughout the time period allocated for the auction and once submitted are rarely revised.

possible, however, to form an opinion about the potential competition by observing participation activity across similar auctions.

Finally, the buyer has an opportunity to set a secret reserve price in these auctions. This means that some of the bids are rejected outright without being compared to the other bids.

2.2 Some Data Regularities

We have access to the data from the starting date of our online programming market and for the subsequent 6 years of this company's operation. The majority of buyers in these six years are one-time participants. Less than 2% of buyers return with multiple projects. In addition, repeated buyers do not return with the same type of project. As a result, they very rarely work with the same seller repeatedly.

The multi-attribute feature of the auction is strongly supported in the data. Indeed, in our sample, 58% of the projects are awarded to a seller who quotes a price above the lowest price submitted in the auction. Table 1 documents the share of such projects as well as an average mark-up over the smallest bid for some project types.

Table 1: Projects Awarded at a Price that Exceeds Lowest Price in Auction

Type of work	Projects Share	Price Mark-up
Database	64%	41.2%
Platforms	52%	37.9%
Graphics	71%	38.4%
Web-related	52%	41.2%

Note: The results in this table are based on full sample which includes 600,000 projects.

These results indicate that buyers consider seller characteristics other than price when choosing a winner. We explore the buyers' choices using a logit model with random coefficients (without choice-specific fixed effects). In this analysis, we set the dependent variable, Y_{li}, to be a project award dummy which is equal to one if the seller i won the project l and zero otherwise. The award depends on the buyer's utility from a specific alternative (seller), which is modeled as a linear function of seller characteristics, X_{ki}, (the number of ratings (experience), average score, delays, arbitration), seller location dummies, $\mu_{c(i)}$, and a seller-specific price quote, B_{li}:

$$Y_{li} = X_i\alpha_l + \gamma_l B_{li} + \mu_{c(i)} + \epsilon_{li} \tag{1}$$

Table 2 reports the results of this analysis. As can be seen from this table, the mean of the price coefficient is estimated to be positive and statistically significant. This result suggests an omitted variable bias since in most markets, buyers dislike paying higher prices other things equal. This means that some additional characteristic, not recorded in the data, affects buyers' choice in conjunction with the price, location and performance measures. Such an omitted variable should be positively aligned with the price and is, therefore, some vertical characteristic such as quality.

Table 2: Logit Model with Random Coefficients

Variable	Coefficient	Std.Error
Normalized Price (mean)	6.461	0.743

Note: The results in this table are based on full sample which includes 600,000 projects.

Another important feature of our data is summarized in Table 3. It shows that a large share of sellers in our data are transitory. In particular, for 65% of the seller population, the tenure with the intermediary is under one month and for 75%, under three months. On the other hand, 10% of the seller population stayed for more than two years. The share of transitory sellers is larger in the beginning years, but settles down so that the distribution of the tenure is almost constant over the last three years. In these years, 30% of the sellers stayed in the market for more than a year, whereas 65% of the sellers left the market in less than three months.

In contrast to other online markets, the sellers' performance does not appear to be related to their propensity to stay in the market. To see this closely, we define *permanent sellers* to be the sellers with a tenure longer than one year, and *transitory sellers* to be the sellers who left after less than one year. Table 3 documents no significant differences between permanent and transitory sellers in the number of bids submitted before the first success, as well as in the distribution of reputation scores received by these bidders for their first or last projects. We obtain similar results when transitory sellers are redefined to be those who left after six months.

An interesting regularity emerges concerning the number of the bids before the first success. When we compute the unconditional distribution of this variable, the time to the first success for the transitory sellers appears to be substantially shorter than that for the permanent seller. This discrepancy disappears when the distribution of this variable *conditional on* achieving at least one success is considered. This suggests that many transitory bidders do not wait for success, and that sorting into permanent or transitory groups is driven by an outside option rather than quality or performance differences among the sellers.

In general, transitory sellers appear to be quite successful: their rate of winning is comparable to that of permanent sellers, and they often beat permanent sellers at comparable prices. Given that the extensive communication between buyers and sellers is present, this suggests that buyers may be able to assess the quality of transitory sellers as accurately as they assess the quality of permanent sellers.

On the other hand, very little information about transitory sellers is publicly available. Indeed, public information is released when a seller completes a project, and transitory sellers usually complete one or two projects and leave the market. It is plausible, therefore, that competing sellers are not informed about transitory sellers' qualities. The situation is different for the permanent sellers. The market may infer their quality from the long-run rate of their successes through reasoning similar to that we use later in this paper.

Table 3: Analysis of Permanent vs. Transitory Sellers

	Tenure Distribution			
	$\leq$ 1m	$\leq$ 3m	$\leq$ 12m	$\leq$ 24m
overall	65%	75%	80%	90%
annual (last 3 years)	45%	65%	70%	75%
	Number of Bids before First Success			
	$\leq$ 10%	$\leq$ 25%	$\leq$ 50%	$\leq$ 75%
tenure$\geq$ 12m	5	9	17	42
tenure$\leq$ 12m	1	2	3	12
tenure$\leq$ 12m (success$\geq$ 1)	3	7	15	36
	First Reputation Score			
	8	9	10	
tenure$\geq$ 12m	5%	10%	85%	
tenure$\leq$ 12m	5%	10%	85%	
	Last Reputation Score			
	8	9	10	
tenure$\geq$ 12m	2%	30%	68%	
tenure$\leq$ 12m	2%	28%	70%	

Note: The results in this table are based on full sample which includes 600,000 projects.

3 The Model

A market is served by two types of sellers: the permanent sellers staying with the market for a long time completing many projects, and the transitory sellers present only for a very short time completing one or two projects during their tenure. Formally, a population of sellers $\mathcal{N}$ is partitioned so that $\mathcal{N} = \mathcal{N}^p \cup \mathcal{N}^t$ where $\mathcal{N}^p$, and $\mathcal{N}^t$ denote subpopulations of permanent and transitory sellers respectively.

Every seller is characterized by a vector of discrete observable characteristics, x_i, and a scalar quality level, q_i. The distribution of the qualities may potentially depend on seller's observable characteristics.[8] In particular, the quality of the seller with observable characteristics x is drawn from a discrete distribution with K_x distinct quality levels (denoted by $\{\bar{q}_{x,k}\}_{k \leq K_x}$ with $\bar{q}_{x,1} > \bar{q}_{x,2} > ... > \bar{q}_{x,K_x}$).[9]

The demand side of the market is represented by one-time buyers. Each of them brings a single project to the market and seeks to hire a service provider who would complete the job. We use l to index a buyer as well as the job he posted. A project, in turn, is summarized by a set of characteristics, z_l, such as the date when the project is posted, and the contract terms

[8]Note that discreteness assumption could be relaxed in many parts of the paper. In addition, this assumption suits well to our environment since all seller characteristics in our data are discrete.

[9]Discreteness of the quality distribution is not restrictive. We cannot hope to recover more quality levels than the number of observations we have at our disposal. On the other hand, our approach allows for every seller to have a distinct quality level.

(e.g., specification of coding tasks, deadlines, etc). The set of potential bidders for project l, $N_l = N_l(z_l) \subseteq \mathcal{N}$, depends on the project characteristics. This dependence is suppressed in most places in the paper to simplify notation. The set of potential sellers is further partitioned into potential permanent sellers, $N^p \subseteq \mathcal{N}^p$, and potential transitory sellers, $N^t \subseteq \mathcal{N}^t$.

Each potential bidder $i \in N$ observes his own private entry cost $S_{i,l} \in \mathbb{R}^1_+$ and the set of potential competitors for project l. He then decides whether to enter the auction, i.e. to submit a price quote to the buyer. The set of the entrants (or actual bidders) is denoted by $A = A^p \cup A^t \subseteq N$. Upon entry, an actual bidder observes the private cost $C_{i,l} \in \mathbb{R}^1_+$ for the project, and quotes a price/bid $B_{i,l}$. An entrant's private signals $S_{i,l}$ and $C_{i,l}$ may be dependent. Actual bidders do not observe the participation decisions of other potential bidders.

We assume that the buyer observes all relevant characteristics of the actual bidders including qualities regardless of whether they are permanent transitory. This assumption reflects extensive communications permitted to occur between the buyer and the sellers in these auctions. On the other hand, the sellers (who can observe public information only) know the quality of the permanent competing sellers, not that of the transitory ones.[10]

The projects are allocated through multi-attribute auctions. Under multi-attribute auction rules, a buyer chooses a seller whose service provision would maximize the buyer's utility. The utility of buyer l from services of seller i is given by

$$u_{i,l} = \alpha_l q_i + x_i' \beta_l - B_{i,l} + \epsilon_{i,l}. \tag{2}$$

Here α_l and β_l represent buyers' tastes for sellers' quality and observed characteristics, and $\epsilon_l \equiv (\epsilon_{i,l})_{i \in N}$ denotes the vector of seller-specific match components as perceived by the buyer l. The match components capture all relevant considerations other than those reflected by sellers' observable characteristics and qualities. For example, they may reflect buyers' impressions about the sellers' fit which are gleaned from communications with them.

Buyer l may set a secret reserve price R_l which from the seller's point of view is drawn from a distribution with a bounded support $[\underline{r}, \overline{r}]$. The reserve price may depend on buyer's tastes for quality and observable characteristics, α, β, but not on the vector of seller-specific components ,ϵ_l, which realizes after the reserve price is set. Only the sellers with price quotes below R_l belong to buyer l's choice set. We refer to the set of actual bidders with $B_{i,l} \leq R_l$ as the set of qualified bidders and denote it by $A_{Q,l}$ to distinguish it from the set of all actual bidders A_l.

To summarize, an actual bidder i wins the project l if and only if:

$$\text{for all } j \neq i \text{ in } A_l, \text{ either "}B_{j,l} > R_l\text{" or " } B_{i,l} \leq R_l \textit{ and } \alpha_l \Delta q_{ij} + \Delta x_{ij}' \beta_l + \Delta\epsilon_{ij,l} \geq \Delta B_{ij,l}\text{ ",}$$

where $\Delta q_{ij} \equiv q_i - q_j$, $\Delta x_{ij} \equiv x_i - x_j$, $\Delta\epsilon_{ij,l} \equiv \epsilon_{i,l} - \epsilon_{j,l}$, and $\Delta B_{ij,l} \equiv B_{i,l} - B_{j,l}$.

[10]This assumption is not essential for our methodology. Here we assume that permanent sellers quality could be inferred by competitors from their winning record. The methodology could easily be modified to suit an environment where competing bidders do not observe each others quality.

3.1 Assumptions

Next, we specify the model assumptions that are maintained throughout the paper. Let $S \equiv (S_i)_{i\in N}$, $C \equiv (C_i)_{i\in N}$, and let $F_{\alpha,\beta,\epsilon,C,S|N}$ denote the joint distribution of buyer tastes and sellers' private information. For a given set of potential bidders N, we make the following assumptions.

($A1$) Both N and $F_{\alpha,\beta,\epsilon,S,C|N}$ are common knowledge among the sellers.

($A2$) (i) $(S_i, C_i)_{i\in N}$ are independent across i. (ii) For all $i \in N$, C_i is continuously distributed with positive densities over $[\underline{c_i}, \overline{c_i}]$. (iii) (S_i, C_i) are identically distributed for all i with identical qualities.

($A3$) $(\alpha, \beta, \epsilon)$ are independent from (S, C). Suppliers do not observe $(\alpha, \beta, \epsilon)$ in entry and bidding stages.

($A4$) (i) ϵ_i's are i.i.d. across i with continuous and positive densities over $[\underline{\varepsilon}, \overline{\varepsilon}]$. (ii) α is continuously distributed with positive densities over $[0, \overline{\alpha}]$.

($A4'$) Condition (A4) holds with α, β, ϵ being mutually independent.

($A5$) The distribution of sellers' qualities, Q, conditional on observable characteristics is independent of sellers' permanent or transitory status:

$$P(Q_i = \bar{q}|i \in \mathcal{N}^t, x_i = \bar{x}) = P(Q_i = \bar{q}|i \in \mathcal{N}^p, x_i = \bar{x}).$$

(A1)-(A2) are restrictions on the information available to sellers. (A3) requires buyers' tastes to be independent of sellers' private information. (A4) requires that there be certain symmetry in $(\epsilon_i)_{i\in N}$, and that the distributions of buyers' tastes components have no gaps or points with probability mass. (A5) suggests that the quality of a seller is not related to her status as a permanent or transitory seller.

3.2 Equilibrium Characterization

We now characterize the pure-strategy, symmetric Bayesian Nash equilibria (PSBNE) of the multi-attribute auction game. In this section all objects are defined for a specific set of the potential bidders. We suppress the dependence on $N = N^p \cup N^t$ for brevity. We focus on type-symmetric equilibria. In such equilibrium, participants who are *ex ante* identical, i.e characterized by the same (x_i, q_i) and the permanent/transitory status adopt the same strategies.

A seller's strategy, σ_i, consists of two components: participation strategy, $\sigma_i^E : \Omega_{S_i} \to \{0, 1\}$, and bidding strategy, $\sigma_i^B : [\underline{c_i}, \overline{c_i}] \to \mathbb{R}^1_+$. Conditional on participation, a seller i's expected profit from bidding b is given by

$$\pi_i(b, c_i; \sigma_{-i}^E, \sigma_{-i}^B) \equiv (b - c_i)P(i \,\text{wins} \mid b; \sigma_{-i}^E, \sigma_{-i}^B),$$

where $(\sigma^E_{-i}, \sigma^B_{-i})$ is the profile of strategies of the other participants. Here,

$$P(i \,\text{wins} \mid B_i = b; \sigma^E_{-i}, \sigma^B_{-i}) = \tag{3}$$
$$\sum_{a \subseteq N \setminus \{i\}} \left(\begin{array}{l} P(\text{“}\alpha \Delta q_{ik} + \Delta x_{ik}\beta + B_k + \Delta\epsilon_{ik} \geq b \textit{ or } B_k > R\text{”} \; \forall k \in A_{-i}, \text{ and } b < R \mid A_{-i} = a) \\ \times \Pi_{k \in a} P(D_k = 1) \times \Pi_{k \in N \setminus \{i,a\}} P(D_k = 0) \end{array} \right).$$

where $A_{-i} \equiv A \setminus \{i\}$; $B_k = \sigma^B_i(C_k)$ and $D_k = \sigma^E_i(S_k)$. Further, in this expression $\Delta q_{ik} = q_i - q_k$ if $k \in A^p_{-i}$ and $\Delta q_{ik} = q_i - Q_k$ if $k \in A^t_{-i}$. This reflects the fact that from i's perspective, the quality levels of permanent competitors are known constants, while the quality levels of transitory competitors are random variables (denoted by $(Q_j : j \in N^t \setminus \{i\})$).

A PSBNE is a profile of strategies $\{(\sigma^E_i, \sigma^B_i)\}_{i \in N}$ such that: (i) $\sigma^B_i(c_i) = \arg\max_b \pi_i(b, c_i; \sigma^E_{-i}, \sigma^B_{-i})$; (ii) $E[\pi_i(\sigma^B_i(C_i), C_i) | S_i = s_i] - s_i \geq 0$ for all s_i such that $\sigma^E_i(s_i) = 1$ (and < 0 for all s_i such that $\sigma^E_i(s_i) = 0$).[11]

We assume that participation and bids in data are rationalized by a single PSBNE. While we do not formally prove equilibrium existence we believe it could be justified using an argument similar to the one presented in Athey and Levin (2001). We conclude this section by establishing some properties of the distribution of prices and entry decisions in PSBNE. These properties are direct consequences of assumptions maintained in (A1)-(A4) and the proof is omitted for brevity.

Lemma 1 *Suppose (A1)-(A4) hold. Then in a PSBNE:*

(i) $\{\alpha, \beta, \epsilon\}$ *is independent from entry decisions* $\{D_i\}_{i \in N}$ *(and therefore* A*);*

(ii) $\{D_i\}_{i \in N}$ *are mutually independent across all* $i \in N$*;*

(iii) conditional on N^t, N^p*, the bids* $\{B_i\}_{i \in a}$ *are mutually independent across* i *for a realization of entrants' set* $A = a$*;*

(iv) $\bar{b}_i \leq \bar{r}$ *for all* $i \in N$*; and*

(v) if $\bar{c}_i = \bar{r}$ *for all* i *and the support of* R *is* $[\underline{r}, \bar{r}]$*, then* $\bar{b}_i = \bar{r}$ *for all* i*.*

Lemma 1 essentially implies that properties (A1)-(A3) stated for the set of potential bidders are also inherited by the set of actual bidders. Point (v) refers to the properties of the support of bids in a special setting when the upper end of the support of costs is the same across all seller types and coincides with the upper end of the support of reserve price. We consider such environment later in the paper when we discuss requirements of our identification strategy. Finally, note that if $i \in N^p$ and $j \in N^t$, then entry probabilities and bidding distributions of i and j could be different even with $x_i = x_j$ and the realization of $q_j = q_i$. This is because the information available to i and j is different.

[11] Note (ii) implies that entry strategies are monotone (i.e., $\exists \bar{s}_i$ for all i such that $\sigma^E_i(s_i) = 1$ iff $s_i \leq \bar{s}_i$) if C_i is independent from S_i.

4 Non-parametric Identification

We outline our identification strategy in this section. We proceed under the assumption that we observe a large number of projects associated with the same set of characteristics, z_l, and the same set of potential bidders. In the subsequent exposition, reference to z_l and N_l is suppressed for brevity. For each project, the data contain information on the set of actual bidders, their price quotes, the set of qualified participants, and buyer's choices,[12] and the objective of the analysis is to recover the distribution of buyers' tastes, quality levels of permanent sellers, and the distributions of bidders' private costs. In our setting, the qualities of permanent sellers may naturally be viewed as sellers' fixed effects. In contrast, the qualities of transitory bidders can only be summarized at the distribution level, and are treated as random effects.

As discussed in the introduction, despite some similarities to the discrete choice, we cannot directly use identification strategies developed in that literature, primarily due to (1) the unavailability of market shares (probability of winning) of individual sellers conditional on buyers' choice set (the set of qualified bidders), and (2) a large number of transitory sellers present in the market and influencing permanent sellers' winning probability in any given auction.

Our solution is to discretize the unobserved qualities. This allows us to first recover the the quality group structure which informs us about the permanent sellers' quality distribution up to a finite set of parameters.[13] Once the quality structure is recovered we proceed to identify the distribution of buyers' tastes for quality. Our identification strategy relies on the variation in permanent sellers' prices as well as the variation in permanent actual bidders sets conditional on the set of potential bidders. The first source of variation is unusual in the choice literature. It is available to us due to private costs that typically characterize auction environment. Later in this section, we discuss conditions that ensure price variation required by our identification strategy.

4.1 Quality Group Structure

This sub-section explains how the quality group structure of the set of permanent sellers, $\mathcal{N}^p$, can be recovered from the data. This part of our identification strategy relies on the property of our model summarized in the proposition below.

Let Ω_{B_i} denote the support of the distribution of bids submitted by seller i. Further, for $b \in \Omega_{B_i}$, define:

$$r_{i,j}(b) \equiv P(i \,\text{wins}\, | B_i = b, i \in A, j \notin A). \tag{4}$$

Proposition 1 *Suppose (A1)-(A4) hold and* $P\{D_k = 1\} > 0$ *for all* $k \in N$. *Then:*

$$sign(r_{i,j}(b) - r_{j,i}(b)) = sign(q_i - q_j)$$

[12]While we assume that realizations of the reserve price in a specific auction are observed in the data, it is sufficient for most results to observe the set of qualified participants instead.

[13]In our setting this distribution coincides with the quality distribution of transitory bidders by assumption.

for all b in the interior of $\Omega_{B_i} \cap \Omega_{B_j}$ and any $\{i,j\} \in N^p$ with $x_i = x_j$.[14]

This proposition formalizes an observation that given the set of potential bidders, the uncertainty in competition that i faces when "$i \in A$ while $j \notin A$" is identical to that j faces when "$j \in A$ while $i \notin A$". If we further condition on i and j quoting the same prices, then the difference in winning probabilities must be solely due to the difference in their quality levels.

In our model the quality comparison is transitive, i.e. if $q_i > q_j$ and $q_j > q_k$ then $q_i > q_k$. Therefore, the proposition below allows us to rank permanent sellers in the order of weakly increasing qualities. In particular, it allows us to identify groups of equal quality or quality group structure of permanent sellers.[15]

Further, while our index is pairwise in nature[16] it's pairwise feature is very weak and is unlikely to cause problems in large datasets such as ours.

Our identification strategy requires that the supports of price distributions for bidders with different qualities (and the same x's) should overlap. We discuss this requirement at the end of this section.

4.2 Distribution of the Buyer-Seller Components

Using recovered quality group structure from Section 4.1, we identify quality levels and the buyers' taste distribution. We begin with the seller-specific component of buyers tastes and then proceed to identification of the taste for quality and the seller's observable characteristics.

Having identified the quality group affiliation of permanent sellers, we ignore sellers' identities from now on, and treat different sellers in the same (x, q)-group as independent "draws" from the (x, q)-group. In particular, we do not distinguish between bids submitted by the sellers i and j, as long as both sellers i and j belong to the same (x, q)-group.

We identify the distribution of the buyer-seller component, $\epsilon_{l,i}$, from the relationship between the permanent seller's price and his probability of winning against competitors from the same group. First, we outline identification strategy in a simple case where all qualified participants are permanent bidders. We then show how the method can be generalized to allow for presence of transitory bidders among qualified participants.

Consider auctions where the set of qualified participants consists of three permanent entrants $i, j, k \in N^p$ who belong to the same $(x, q)-$ group. Let $b_{i,j,k}$ denote the event "$B_i = b_i$, $B_j = b_j$,

[14]The *sign* function is defined in a usual way: $sign(a) > 0$ if $a > 0$, $sign(a) < 0$ if $a < 0$, and $sign(a) = 0$ if $a = 0$.

[15]Transitivity property maybe violated in estimation due to small sample issues. Our estimation strategy takes this possibility into account.

[16]It is potentially possible that the full ranking of permanent sellers may not be constructed if not all sellers could be linked through our index. In this case, partial ranking of sellers could be constructed by sub-dividing sellers into sets for which full ranking could be obtained. In parametric estimation these sets can then be linked in the second stage where the quality levels of various quality groups are recovered.

$B_k = b_k$. "The probability that i wins conditional on $b_{i,j,k}$ and $A_Q = \{i, j, k\}$ is

$$\begin{aligned} P(\epsilon_j - \epsilon_i &\leq b_j - b_i,\ \epsilon_k - \epsilon_i \leq b_k - b_i \mid i, j, k \in A, \min_{s \in A \backslash \{i,j,k\}} B_s > R \geq \max_{s \in \{i,j,k\}} B_s,\ b_{i,j,k}) \quad (5) \\ &= P(\epsilon_j - \epsilon_i \leq b_j - b_i,\ \epsilon_k - \epsilon_i \leq b_k - b_i) \end{aligned}$$

where we have used the independence between ϵ and R. The prices of permanent bidders (B_i, B_j, B_k) vary independently from $\{\epsilon\}_{i,j,k}$ under (A3). If the support of $(B_j - B_i, B_k - B_i)$ given "$R \geq \max\{B_i, B_j, B_k\}$ and $A_Q = \{i, j, k\}$" covers that of $(\epsilon_j - \epsilon_i, \epsilon_k - \epsilon_i)$ then the joint distribution of $(\epsilon_j - \epsilon_i, \epsilon_k - \epsilon_i)$ can be fully recovered. After that, F_{ϵ_i} can be identified up to a location normalization using, for example, Kotlarski Theorem (Kotlarski (1967)) under mutual independence of $(\epsilon_i, \epsilon_j, \epsilon_k)$.

Next, we generalize this argument to allow for presence of qualified, transitory bidders.[17] The main issue here is that prices of transitory bidders are correlated with their random qualities. Consider an auction where the set of qualified participants consist of four or more sellers, some of these sellers may be transitory. We require that three of these participants should be permanent sellers, $i, j, k \in N^p$,that belong to the same (x, q) group.[18] We rely on a modified probability of winning. More specifically, for any vector of bids (b_i, b_j, b_k) and the set of qualified entrants $A_Q = a$ such that $\{i, j, k\} \subset a$, we consider the following probability

$$P\left(i \,\text{wins}|\, A_Q = a,\ b_{i,j,k},\ \text{“}u_i \geq u_m\ \forall m \in a \backslash \{i, j, k\}\text{”}\right), \tag{6}$$

That is, we additionally condition on the event that the buyer chooses between i, j and k. Notice that, given $A_Q = a$ and $b_{i,j,k}$, the event that i wins is the intersection of two events "$\Delta\epsilon_{ij} \geq \Delta b_{ji}$, $\Delta\epsilon_{ik} \geq \Delta b_{ki}$" and "$u_i \geq u_m$ for all $m \in a_Q \backslash \{i, j, k\}$."Thus, expression in (6) is written as

$$P\left(\Delta\epsilon_{ji} \leq \Delta b_{ji},\ \Delta\epsilon_{ki} \leq \Delta b_{ki}|\, \omega^*,\ b_{i,j,k}\right),$$

where ω^* denotes "$A_Q = a$ and $u_i \geq u_m\ \forall m \in a \backslash \{i, j, k\}$".

The main insight is that under (A1)-(A4'), ϵ_j and ϵ_k are mutually independent, and are jointly independent from ϵ_i and the event "ω^* and $b_{i,j,k}$."In addition, ϵ_i is independent from (B_j, B_k) conditional on $B_i = b_i$ and ω^*. That is, $F_{\epsilon_i,\epsilon_j,\epsilon_k|\omega^*,b_{i,j,k}} = F_{\epsilon_i|\omega^*,b_i} F_{\epsilon_j} F_{\epsilon_k}$. Hence if for some $b_i \in \Omega_{B_i}$ the support of $(B_j - b_i, B_k - b_i)$ covers the support of $(\epsilon_j - \epsilon_i, \epsilon_k - \epsilon_i)$, then the joint distribution of $(\epsilon_j - \epsilon_i, \epsilon_k - \epsilon_i)$ conditional on $B_i = b_i$ and ω^* can be recovered over its full support. After this, we proceed as before to recover F_{ϵ_i} up to a location. Finally, notice that (6) can be obtained from data as

$$\frac{P(i \,\text{wins}|\, A_Q = a,\ b_{i,j,k})}{P(i \,\text{wins}|\, A_Q = a \backslash \{j, k\},\ b_i)} \tag{7}$$

by Bayes rule. We formalize these arguments by specifying two additional support conditions

[17]Note, that we can use the same approach to allow for presence of additional permanent bidders whose prices are not used in identification.

[18]The identification can also be achieved with qualified participants' sets consisting of three bidders with just two permanent bidders who belong to the same (x, q)- group.

and Proposition 2 below. The proof is included in the Appendix.

($A6$) There exists an (x, q)-group of permanent sellers such that at least three sellers in N belong to this group, and $b_i \in \Omega_{B(x,q)}$, such that for any two independent random variables, B_j and B_k, distributed according to $F_{B(x,q)}(.)$, the support of $(B_j - b_i, B_k - b_i)$ covers the support of $(\epsilon_j - \epsilon_i, \epsilon_k - \epsilon_i)$.

($R1$) The support of $B_{(x,q)}$ with (x, q) specified in ($A6$) intersects with that of B_m for some $m \in N$ other than three $(x, q)-$bidders from ($A6$).

(A6) ensures the existence of a triplet $\{i, j, k\}$ that delivers identification, and the identification of the joint distribution of $(\epsilon_j - \epsilon_i, \epsilon_k - \epsilon_i)$ given $B_i = b_i$ and ω^*.[19] Condition (R1) is a technical restriction that avoids conditioning on zero probability events in our argument.

Proposition 2 *Suppose A1, A2, A3, A4', A6 and R1 hold, $P(D_s = 1) > 0$ for all $s \in N$. Then F_{ϵ_i} is identified up to a location normalization, $E[\epsilon_i] = 0$.*

Identifying the distribution of ϵ_i requires sufficient variation of prices in the data. We describe conditions that yield such variation in Section 4.4.

4.3 Quality Differences and Buyer Tastes

Next, we show how to recover the distribution of buyers' tastes for quality (F_α), and quality levels for permanent sellers. As before, we exploit the relationship between the probability of winning and prices of permanent bidders. To illustrate the main idea, we first consider a simple case where the set of qualified entrants consist of two permanent entrants from the same x-group but with different quality levels. For the sake of exposition, we also maintain that R is independent from α. (We discuss extension to the case with more than two entrants, involving transitory bidders, and the case with correlation between R and α later in this subsection.) Then,

$$\begin{aligned} P(\, i \,\text{wins} \quad | \quad & b_i, b_j, A_Q = \{i, j\}) \qquad (8) \\ = \quad & P(\alpha \Delta q_{ji} + \Delta \epsilon_{ji} \leq \Delta b_{ji} |\, b_i, b_j, \min_{k \in A \setminus \{i,j\}} B_k > R \geq \max\{b_i, b_j\}, i, j \in A) \\ = \quad & P(\alpha \Delta q_{ji} + \Delta \epsilon_{ji} \leq \Delta b_{ji}) \end{aligned}$$

where we have used the fact that bids in equilibrium are functions of private costs only, which are independent from buyers' tastes $(\alpha, \beta, \epsilon)$ as well as reserve price R. Under (A3), the distribution of $\alpha \Delta q_{ji} + \Delta \epsilon_{ji}$ is independent from B_i and B_j.Provided that the support of $B_j - B_i$ covers that of $\alpha \Delta q_{ji} + \Delta \epsilon_{ji}$, the distribution of $\alpha \Delta q_{ji} + \Delta \epsilon_{ji}$ can be recovered through the variation in b_i and b_j in (8). With $F_{\Delta \epsilon_{ji}}$ already identified from Section 4.2, this leads to the recovery of

[19] In principle, such a condition can be verified in data.

the distribution of $\alpha \Delta q_{ji}$. This means that F_α and Δq_{ji} are identified up to a normalization, $E[\alpha] = 1$.

We now extend this idea to auctions that involve three sellers i, j, k, where i, j are permanent sellers known to belong to different quality groups within the same $x-$ group, and the third entrant k could be permanent or transitory. To simplify exposition, assume that k belong to the same x-group. (Further generalizations to cases with more than four entrants or with k from a different $x-$group do not pose additional challenges and are omitted for brevity.) We first formulate the support condition.

($A7$) There exists some $b_k \in \Omega_{B_k}$ such that conditional on $B_k = b_k$ and $A_Q = \{i, j, k\}$ the support of $(B_i - B_j, B_i - b_k)$ covers that of $(\alpha \Delta q_{ij} + \Delta \epsilon_{ij}\ ,\ \alpha\,(q_i - Q_k) + \Delta \epsilon_{ik})$.

The interpretation of (A7) differs according to whether k is permanent or transitory seller. If $k \in N^p$ then q_k is a scalar constant. Otherwise, Q_k is a random variable whose distribution is correlated with that of B_k, since B_k depends on seller k's quality, Q_k.

Proposition 3 *Suppose F_{ϵ_i} is identified and A1, A2, A3 and A4' hold. If A7 holds for some permanent i, j and some other transitory seller k, then F_α and Δq_{ij} are identified up to a scale normalization, $E[\alpha] = 1$.*

Proof of Proposition 3. By definition,

$$
\begin{aligned}
& P(i\,\text{wins}|\, b_{i,j,k}, A_Q = \{i, j, k\}) \\
= \;& P\left(Y_1 \geq b_i - b_j,\, Y_2 \geq b_i |\; b_{i,j,k}, \{i, j, k\} \subseteq A, \min_{s \in A \setminus \{i,j,k\}} B_s > R \geq \max_{s \in \{i,j,k\}} b_s \right). \qquad (9)
\end{aligned}
$$

where $Y_1 \equiv \alpha \Delta q_{ij} + \Delta \epsilon_{ij}$ and $Y_2 \equiv \alpha (q_i - Q_k) + \Delta \epsilon_{ik} + B_k$. This conditional probability can be recovered from the data. Under the assumptions stated, the joint distribution of (Y_1, Y_2) is independent from B_i, B_j given "$A_Q = \{i, j, k\}$, and $B_k = b_k$."Thus, under (A7), the joint distribution of (Y_1, Y_2) given "$A_Q = \{i, j, k\}$, b_i, b_j, b_k" can be fully recovered over its support through variation in (b_i, b_j). Under stated assumptions, the marginal distribution of Y_1 is independent from the event "$A_Q = \{i, j, k\}, b_i, b_j, b_k$."Thus the unconditional distribution of $\alpha \Delta q_{ij} + \Delta \epsilon_{ij}$ is identified. With F_{ϵ_i} identified for all $i \in N$, F_α and Δq_{ij} are also identified up to a scale normalization. *Q.E.D.*

For the case where R is correlated with buyers' tastes α, the intuition from the first paragraph in this subsection does not apply directly. To see this, note variations in b_i, b_j would affect the event $R \geq \max\{b_i, b_j\}$, which in turn would affect the distribution of Y_1, Y_2 through its impact on the distribution of α. Nonetheless, the logic above can be extended to show identification. Specially, we can use

$$
\begin{aligned}
& P(i\,\text{wins}, A = A_Q = \{i, j\}|\, b_i, b_j, i, j \in A) \\
= \;& P(\alpha \Delta q_{ji} + \Delta \epsilon_{ji} \leq \Delta b_{ji}, R \geq \max\{b_i, b_j\} | b_i, b_j) = P(\alpha \Delta q_{ji} + \Delta \epsilon_{ji} \leq \Delta b_{ji}, R \geq \max\{b_i, b_j\})
\end{aligned}
$$

With sufficient variations in b_i, b_j we can identify the joint distribution of $\alpha\Delta q_{ji} + \Delta\epsilon_{ji}$ and R. Of course this argument requires the use of a set of more restrictive support conditions.

Note that, once the group structure and quality levels for the set of permanent sellers are identified, the distribution of quality levels for transitory sellers can be recovered as well. Indeed, under (A5) the probability that quality of transitory seller is at a specific level is equal to the proportion of permanent sellers who are of this quality level:

$$P(Q_i = \bar{q}|i \in N^t, x_i = \bar{x}) = \frac{\#\{i \in \mathcal{N}^p : q_i = \bar{q},\ x_i = \bar{x}\}}{\#\{i \in \mathcal{N}^p : x_i = \bar{x}\}}.$$

The identification of buyers' tastes for observable characteristics follows steps similar to those used in the identification of buyers' taste for quality. Finally, once buyer's tastes, permanent sellers' quality levels, and transitory bidders distribution of qualities are recovered, the distribution of bidders' private information can be identified as well via an argument similar to that in Guerre, Perrigne, and Vuong (2000). Details could be found in the Supplementary Appendix.

4.4 Support Conditions

The identification of the distribution of buyers' tastes hinges on conditions which require that supports of prices submitted by different quality groups should intersect and that sufficient price variation should exist. In this sub-section we investigate whether a model such as ours is capable of generating data that satisfy these conditions. We present our analysis for the case where buyers' tastes are a summarized by seller-specific component only, i.e. $u_{i,l} = -B_{i,l} + \epsilon_{i,l}$.

We begin by deriving restrictions on the relationship between the support of prices and that of buyers' tastes that are sufficient for (A6)and (R1). Then, we explain how sufficient price variation could be generated by our model.

Remark 1 *Suppose $\bar{c}_i = \bar{c}$ for all i and $\bar{r} = \bar{c}$. Then R1 holds. If, in addition, for some $i \in N^p$, $\bar{b}_i - \underline{b}_i > 2(\bar{\varepsilon} - \underline{\varepsilon})$ and there exists $j, k \in N^p$ with $q_i = q_j = q_k$ and $x_i = x_j = x_k$, then (A6) holds.*

By part (v) of Lemma 1, $\bar{b}_i = \bar{c}$ for all $i \in N$ when $\bar{c}_i = c_i$ for all $i \in N$. Hence, R1 holds. This also implies the support of $(B_i - B_j, B_i - B_k)$ given $B_i = b_i$ does not become smaller after conditioning on "$B_i = b_i$ and $R \geq \max\{b_i, B_j, B_k\}$."

It is easy to visualize the second observation in Remark 1. Figure 1(1) depicts the unconditional support of $(\epsilon_j - \epsilon_i, \epsilon_k - \epsilon_i)$. The support is a subset of the square $[\underline{\varepsilon} - \bar{\varepsilon}, \bar{\varepsilon} - \underline{\varepsilon}] \otimes [\underline{\varepsilon} - \bar{\varepsilon}, \bar{\varepsilon} - \underline{\varepsilon}]$ that is formed by truncating the square by further restricting the difference between horizontal and vertical coordinates being bounded between $\underline{\varepsilon} - \bar{\varepsilon}$ and $\bar{\varepsilon} - \underline{\varepsilon}$. Figure 1(2) depicts the joint support of $(B_j - b_i, B_k - b_i)$ where $x_k = x_j$ and $q_k = q_j$. Recall that in a type-symmetric BNE, the support of B_i, B_j, B_k are the same if i, j, k belong to the same (x, q) group. Hence for the square in Figure 1(2) to cover the support in Figure 1(1), it is sufficient to have $\bar{b}_j - b_i > \bar{\varepsilon} - \underline{\varepsilon}$ and $\underline{b}_j - b_i < \underline{\varepsilon} - \bar{\varepsilon}$. Therefore with $\bar{b}_i - \underline{b}_i > 2(\bar{\varepsilon} - \underline{\varepsilon})$, A5 holds at least for $b_i = \frac{\bar{b}_i + \underline{b}_i}{2}$ because $\bar{\varepsilon} - \underline{\varepsilon} < \bar{b}_i - \frac{\bar{b}_i + \underline{b}_i}{2}$ and $\underline{\varepsilon} - \bar{\varepsilon} > \underline{b}_i - \frac{\bar{b}_i + \underline{b}_i}{2}$.

Figure 1: Support Conditions

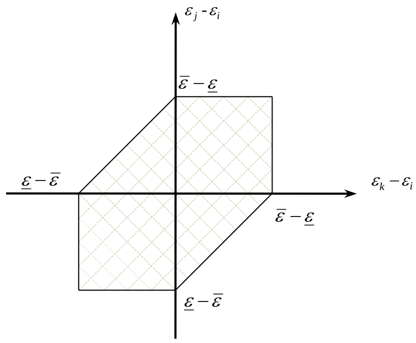


Figure1.Support of $(\varepsilon_j - \varepsilon_i, \varepsilon_k - \varepsilon_i)$

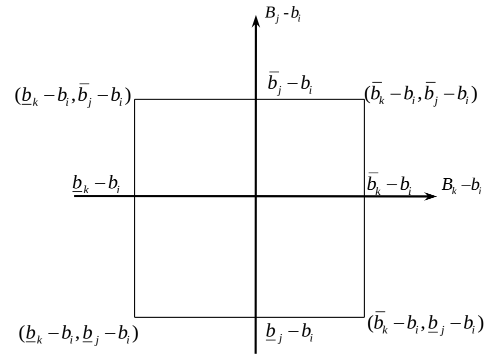


Figure2.Support of $(B_j - b_i, B_k - b_i)$ conditional on $B_i = b_i$

We now present a heuristic argument on how conditions in Remark 1 can occur in a simple case without the secret reserve price R and the entry stage (the addition of the secrete reserve price and the entry stage only complicates the algebra without adding to the intuition). The idea is to show that the bidding strategy is continuous in the length of the support of seller-specific component of buyer's tastes, $\bar{\varepsilon} - \underline{\varepsilon}$. Suppose ϵ_i are i.i.d. uniform over $[\underline{\varepsilon}, \bar{\varepsilon}]$. Suppose sellers observe the set of competitors. Then

$$\max_b (b - c_i) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Pi_{j \in A \setminus i} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{1 - F_{B_j}(b - \Delta \varepsilon_{ij})}{\bar{\varepsilon} - \underline{\varepsilon}} d\varepsilon_i \frac{1}{\bar{\varepsilon} - \underline{\varepsilon}} d\varepsilon_j.$$

By changing variables between ε_j and $\varepsilon_j - \underline{\varepsilon}$ first, and then between $\varepsilon_j - \underline{\varepsilon}$ and $\tau_j \equiv \frac{\varepsilon_j - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$, we can rewrite the objective function as

$$(b - c_i) \int_0^1 \Pi_{j \in A \setminus i} \int_0^1 1 - F_{B_j}(b - (\bar{\varepsilon} - \underline{\varepsilon}) \Delta \tau_{ij}) d\tau_i d\tau_j,$$

which is continuous in the length of support of ϵ. Arguments based on the Theorem of Maximum suggest that the support of bids is also continuous in the length of the support of seller-specific component. Hence as long as there is enough variation in the support of private costs so that the support of bids without ϵ is sufficiently large, it can be shown that the condition in Remark 1 can hold when the support of ϵ is relatively small.

Additional details related to assumption $(A7)$ are given in Supplementary Appendix. In short, our results suggest that support conditions are satisfied if the variation in the private costs, quality differences, and observable characteristics are sufficiently large relative to the supports of ϵ, α, and β.

5 Non-parametric Inference of Quality Rankings

This section translates the identification ideas presented in the preceding sections into an estimation methodology. While in population the qualities are transitive, the estimated quality group may not be so in small samples, if we use only individually pairwise tests to assign each seller to a quality group. This means that we need to estimate the whole group structure simultaneously. On the other hand, with a reasonably large number of sellers and with more than two quality groups, estimation of a group structure would involve too many pairwise comparisons. This paper suggests a feasible algorithm that classifies the sellers sequentially so that the transitivity of quality groups is maintained and the computational cost is substantially reduced.

5.1 Estimation of Classifications with Known Number of Groups

We consider the case where X_i takes values from a finite set $(x_1, \cdots, x_\Lambda)$. This implies that the set of the permanent sellers can be partitioned into Λ groups, $N_1, \cdots, N_\Lambda$, such that for each $\lambda = 1, \cdots, \Lambda$, and any $i, j \in N_\lambda$, $x_i = x_j$. The analysis in this section is performed conditional on (z, λ). For brevity, we omit conditioning on (z, λ) in the exposition below. Define K_0 to be the number of distinct quality levels among the permanent sellers.

For ease of exposition, we first present the case with the number of quality levels K_0 equal to 2 so that $q_i \in \{\bar{q}_h, \bar{q}_l\}$ for a pair of unknown numbers $\bar{q}_h$ and $\bar{q}_l$. We explain how the algorithm generalizes to the case with $K_0 > 2$ later. For each $\lambda = 1, \cdots, \Lambda$, let $N_{h,\lambda} \subset N_\lambda$ be the collection of high quality sellers and $N_{l,\lambda} \subset N_\lambda$ be the collection of low quality sellers. We estimate an ordered partition $(N_{h,\lambda}, N_{l,\lambda})$ of N_λ in three steps. First, for each $i \in N_\lambda$, we estimate two ordered partitions: one partition consists of the group of the sellers with higher or equal quality than that of i (denoted by $N_{1,\lambda}(i)$) and the rest (denoted by $N_\lambda \backslash N_{1,\lambda}(i)$), and the other partition consists of the group of sellers with lower or equal quality than that of i (denoted by $N_{2,\lambda}(i)$) and the rest. Second, among the two ordered partitions, we choose the one that is mostly likely to coincide with $(N_{h,\lambda}, N_{l,\lambda})$. Third, we choose i such that the estimated partition associated with this i is most strongly supported by the data.

Our method relies on the estimates of winning probabilities in Proposition 1. Let $W_{i,l}^p \in \{0, 1\}$ be an indicator taking 1 if the i-th seller that is permanent wins at auction l and 0 otherwise. Define $\hat{\delta}_{ij}(b) \equiv \hat{r}_{ij}(b) - \hat{r}_{ji}(b)$, where

$$\hat{r}_{ij}(b) \equiv \frac{\sum_{l=1}^{L} W_{i,l}^p K_h(B_{i,l} - b) 1\{j \notin A_l\}}{\sum_{l=1}^{L} K_h(B_{i,l} - b) 1\{j \notin A_l\}},$$

where $K_h(v) = K(v/h)/h$ for a univariate kernel function K. Then we construct test statistics: $\hat{\tau}_{ij}^+ = \int \max\{\hat{\delta}_{ij}(b), 0\} db$, $\hat{\tau}_{ij}^- = \int \max\{-\hat{\delta}_{ij}(b), 0\} db$, and $\hat{\tau}_{ij}^0 = \int |\hat{\delta}_{ij}(b)| db$. We confine the integral domains to $\Omega_i \cap \Omega_j$, and this restriction is omitted from the notation.

We use a bootstrap method to estimate the finite sample distributions of the test statistics. We first construct $\hat{r}_{ij,s}^*(b)$ and $\hat{\delta}_{ij,s}^*(b)$ using the s-th bootstrap sample, $s = 1, \cdots, B$, and consider the re-centered bootstrap test statistics:

$$\begin{aligned}
\hat{\tau}^{*+}_{ij,s} &= \int \max\{\hat{\delta}^{*}_{ij,s}(b) - \hat{\delta}_{ij}(b), 0\} db \\
\hat{\tau}^{*-}_{ij,s} &= \int \max\{-\hat{\delta}^{*}_{ij,s}(b) + \hat{\delta}_{ij}(b), 0\} db, \text{ and} \\
\hat{\tau}^{*0}_{ij,s} &= \int |\hat{\delta}^{*}_{ij,s}(b) - \hat{\delta}_{ij}(b)| db.
\end{aligned}$$

Using the bootstrap test statistics, we define the bootstrap p-values as follows:

$$p^*_z(i,j) = \frac{1}{B}\sum_{s=1}^{B} 1\left\{\hat{\tau}^{*z}_{ij,s} > \hat{\tau}^{z}_{ij}\right\} \text{ with } z \in \{+,-,0\}.$$

We proceed in three steps as outlined above.

Step 1: Define

$$\begin{aligned}
\hat{N}_{1,\lambda}(i) &= \{j \in N_\lambda \backslash \{i\} : p^*_+(i,j) \le p^*_-(i,j)\} \text{ and} \\
\hat{N}_{2,\lambda}(i) &= \{j \in N_\lambda \backslash \{i\} : p^*_+(i,j) > p^*_-(i,j)\}.
\end{aligned}$$

Step 2: We determine now whether seller i has the same quality as those of $N_{1,\lambda}(i)$ or $N_{2,\lambda}(i)$, i.e., high quality or low quality. If both $\hat{N}_{1,\lambda}(i)$ and $\hat{N}_{2,\lambda}(i)$ are non-empty,[20] we classify seller i as follows:

$$\begin{aligned}
\text{Take } \hat{N}_{l,\lambda}(i) &= \hat{N}_{1,\lambda}(i) \text{ and } \hat{N}_{h,\lambda}(i) = \hat{N}_{2,\lambda}(i) \cup \{i\} \text{ if } \min_{j \in \hat{N}_{1,\lambda}(i)} \log p^*_0(i,j) < \min_{j \in \hat{N}_{2,\lambda}(i)} \log p^*_0(i,j) \\
\text{Take } \hat{N}_{l,\lambda}(i) &= \hat{N}_{1,\lambda}(i) \cup \{i\} \text{ and } \hat{N}_{h,\lambda}(i) = \hat{N}_{2,\lambda}(i) \text{ if } \min_{j \in \hat{N}_{1,\lambda}(i)} \log p^*_0(i,j) \ge \min_{j \in \hat{N}_{2,\lambda}(i)} \log p^*_0(i,j).
\end{aligned}$$

That is, we put seller i into the high-quality group, if the evidence against the hypothesis that seller i has the same quality as the sellers in group $\hat{N}_{1,\lambda}(i)$ is stronger than the evidence against the hypothesis that seller i has the same quality as the sellers in group $\hat{N}_{2,\lambda}(i)$.

[20] If $\hat{I}_{1,\lambda}(i)$ is empty, we pick some level $\alpha = 0.05$:

$$\begin{aligned}
\text{Take } \hat{I}_{l,\lambda}(i) &= \varnothing \text{ and } \hat{I}_{h,\lambda}(i) = \hat{I}_{2,\lambda}(i) \cup \{i\} \text{ if } \min_{j \in \hat{I}_{2,\lambda}(i)} p^*_0(i,j) \ge \alpha. \\
\text{Take } \hat{I}_{l,\lambda}(i) &= \{i\} \text{ and } \hat{I}_{h,\lambda}(i) = \hat{I}_{2,\lambda}(i) \text{ if } \min_{j \in \hat{I}_{2,\lambda}(i)} p^*_0(i,j) < \alpha.
\end{aligned}$$

We proceed in a similar way if $\hat{I}_{2,\lambda}(i)$ is empty.

Step 3: For each $i \in N_\lambda$, we compute the following index:

$$s_\lambda^*(i) = \begin{cases} \frac{1}{|\hat{N}_{l,\lambda}(i)|} \sum_{j \in \hat{N}_{l,\lambda}(i)} \log p_+^*(i,j) \text{ if } i \in \hat{N}_{h,\lambda}(i) \\ \frac{1}{|\hat{N}_{h,\lambda}(i)|} \sum_{j \in \hat{N}_{h,\lambda}(i)} \log p_-^*(i,j) \text{ if } i \in \hat{N}_{l,\lambda}(i) \end{cases},$$

where $|\hat{N}_{l,\lambda}(i)|$ denotes the cardinality of the set $\hat{N}_{l,\lambda}(i)$. The quantity $s_\lambda^*(i)$ indicates the weakness of the likelihood that i is classified into her right quality group. Then choose i^* that minimizes $s_\lambda^*(i)$ over $i \in N_\lambda$, and let $\hat{N}_{h,\lambda} = \hat{N}_{h,\lambda}(i^*)$ and $\hat{N}_{l,\lambda} = \hat{N}_{l,\lambda}(i^*)$. We take $\hat{C}_\lambda = (\hat{N}_{h,\lambda}(i^*), \hat{N}_{l,\lambda}(i^*))$ as an estimated classification of players into two quality groups.[21]

The constructed classification can be justified as follows. We introduce the metric of classification discrepancy: for any two different classifications $C_{A,\lambda} = (N_{h,\lambda}^A, N_{l,\lambda}^A)$ and $C_{B,\lambda} = (N_{h,\lambda}^B, N_{l,\lambda}^B)$, $N_\lambda = N_{h,\lambda}^A \cup N_{l,\lambda}^A = N_{h,\lambda}^B \cup N_{l,\lambda}^B$, we define

$$\delta\left(C_{A,\lambda}, C_{B,\lambda}\right) = \max\left\{\#(N_{h,\lambda}^A \Delta N_{h,\lambda}^B), \#(N_{l,\lambda}^A \Delta N_{l,\lambda}^B)\right\},$$

where $N_{h,\lambda}^A \Delta N_{h,\lambda}^B$ denotes the set difference between $N_{l,\lambda}^A$ and $N_{l,\lambda}^B$.

Let $N_{h,\lambda} = \{i \in N_\lambda : q_i = \bar{q}_h\}$ and $N_{l,\lambda} = \{i \in N_\lambda : q_i = \bar{q}_l\}$ and write $C_\lambda = (N_{h,\lambda}, N_{l,\lambda})$. Also, we write $\hat{C}_\lambda = (\hat{N}_{h,\lambda}, \hat{N}_{l,\lambda})$. We show that the estimated classification $\hat{C}_\lambda$ is *consistent* under regularity conditions.

Theorem 1: *As* $L \to \infty$,

$$P\left\{\delta(\hat{C}_\lambda, C_\lambda) \geq 1\right\} \to 0.$$

The generalization of the procedure to the case of $K_0 > 2$ with K_0 known can proceed as follows. First, we split N_λ into $\hat{N}_{h,\lambda}$ and $\hat{N}_{l,\lambda}$ using the algorithm for $K_0 = 2$. Then we find a minimum value (denoted by $\hat{p}_h$) of $\log p_0^*(i,j)$ among the pairs (i,j) such that $i \neq j$, and $i, j \in \hat{N}_{h,\lambda}$, and a minimum value (denoted by $\hat{p}_l$) of $\log p_0^*(i,j)$ among the pairs (i,j) such that $i \neq j$, and $i, j \in \hat{N}_{l,\lambda}$. If $\hat{p}_h < \hat{p}_l$, we split $\hat{N}_{h,\lambda}$ into $\hat{N}_{hh,\lambda}$ and $\hat{N}_{hl,\lambda}$ using the same algorithm for $K_0 = 2$, and otherwise, we split $\hat{N}_{l,\lambda}$ into $\hat{N}_{lh,\lambda}$ and $\hat{N}_{ll,\lambda}$ using the same algorithm for $K_0 = 2$. We repeat the procedure. For example, suppose that we have classifications $\hat{N}_{1,\lambda}, \cdots, \hat{N}_{k-1,\lambda}$ obtained. For each $r = 1, \cdots, k-1$, we compute the minimum value (say, $\hat{p}_r$) of $\log p_0^*(i,j)$ among the pairs (i,j) such that $i \neq j$ and $i, j \in \hat{N}_{r,\lambda}$, and then select its minimum (say, $\hat{p}_{r^*}$) over $r = 1, \cdots, k-1$. We split $\hat{N}_{r^*,\lambda}$ into $\hat{N}_{r^*h,\lambda}$ and $\hat{N}_{r^*l,\lambda}$ using the algorithm for $K_0 = 2$ to obtain a classification of N_λ into k groups. We continue until the groups become as many as K_0.

[21] There may be alternative ways to obtain estimators of the quality partition. One way is to fix i and apply hypothesis testing to the null hypothesis that $q_i \leq q_j$. This essentially boils down to comparing the p-value $p_z^*(i,j)$ with a certain level of the test. One unattractive feature of this alternative procedure is that the result can be different depending on whether the null hypothesis is taken to be $q_i \leq q_j$ or $q_i \geq q_j$. This is because the conventional hypothesis testing procedure treats the null hypothesis and the alternative hypothesis asymmetrically. Hence in this paper, instead of comparing a p-value with a fixed level of the test, we compare a p-value with an alternative p-value, to capture the symmetry of the comparison.

5.2 Consistent Selection of the Number of Groups

The methodology outlined above assumes that we know the exact number of groups. To accommodate the situation with real life data without knowledge of the number of the groups, we offer a method of consistent selection of the number of groups. We suggest that the number of groups should be selected to minimize the criterion function that balances a measure of goodness-of-fit that captures a misspecification bias versus a penalty term that penalizes overfitting. The goodness-of-fit measure is based on the variance test approach.

Given an estimated classification $N = \cup_{k=1}^{K} \hat{N}_k$ with K groups, let

$$\hat{V}_k(K) = \left| \min_{i,j \in \hat{N}_k} \log p_0^*(i,j) \right|,$$

for each $k = 1, \cdots, K$.

Suppose that K_0 is the true number of groups. Let $g(L) \to \infty$ be such that $g(L)/\sqrt{L} \to 0$ as $L \to \infty$. Then, define

$$\hat{Q}(K) \equiv \frac{1}{K} \sum_{k=1}^{K} \hat{V}_k(K) + Kg(L).$$

We select K as follows:

$$\hat{K} = \mathrm{argmin}_{1 \leq K \leq N} \hat{Q}(K).$$

The following theorem shows that this selection procedure is a consistent one.

Theorem 2: $P\{\hat{K} = K_0\} \to 1$ as $L \to \infty$.

The consistency result stems from two auxiliary facts. First, $\frac{1}{K}\sum_{k=1}^{K} \hat{V}_k(K) = O_P(1)$, as $L \to \infty$, if $K \geq K_0$. This measures the asymptotic behavior of the goodness-of-fit measure when the classification is weakly finer than the true classification. Since $Kg(L) \to \infty$ as $L \to \infty$, the minimization of $\hat{Q}(K)$ over K leans toward a lower choice of K that is closer to K_0. On the other hand, if the classification is strictly coarser than the true classification, i.e., $K < K_0$, the quantity $\frac{1}{K}\sum_{k=1}^{K} \hat{V}_k(K)$ diverges at a rate faster than $g(L)$, as $L \to \infty$. In this case, the minimization of $\hat{Q}(K)$ over K excludes K such that $K < K_0$ for large samples. Thus we obtain the consistency of $\hat{K}$.

We describe methods of constructing confidence sets for the estimated classifications as well as report the results of Monte Carlo study investigating the algorithm's small sample performance in Supplemental Appendix.

6 Estimation of Structural Elements

6.1 Overview

While our estimation approach is guided by the nonparametric identification results obtained in Section 4, the complexity of our model makes full nonparametric estimation impractical. Instead, we make parametric assumptions about the distribution of buyers' tastes, α, β and ϵ, and estimate the parameters of these distributions using generalized method of moments estimation. In a parametric setting we have to confront one additional challenge: in order to match the moments or distributions observed in the data, we need to integrate out the relationships between the bidding and participation decisions and the qualities of transitory sellers, as they are not directly observed in the data. A traditional approach to this problem would be to re-solve the bidding and participation game for every parameter guess. In our setting, however, such an approach is computationally intractable. Instead, we jointly estimate the parameters of the distributions of the buyers' tastes, the quality levels, the distribution of transitory sellers' bids and the participation strategies conditional on the transitory seller qualities, noting that such equilibrium objects as the bid distribution of transitory sellers and participation strategies can be non-parametrically identified alongside our model's primitives (buyers' tastes and quality levels) given available data.

6.2 Notation

We introduce some notation. For each x in the common support $\mathcal{X}$ of x_i, let $\mathbb{Q}_x \equiv \{q_{1,x}, \cdots, q_{K,x}\}$ be the set of possible quality levels for a seller $i \in \mathcal{N}$ with $x_i = x$. With each $(x, q) \in \mathcal{X} \times \mathbb{Q}_x$ are associated sets of sellers indices, $A^p_{x,q,l} \equiv \{i \in A^p_l : (x_i, q_i) = (x, q)\}$, $N^p_{x,q,l} \equiv \{i \in N^p_l : (x_i, q_i) = (x, q)\}$, $A^t_{x,l} \equiv \{i \in A^t_l : x_i = x\}$ and $N^t_{x,l} \equiv \{i \in N^t_l : x_i = x\}$. It is convenient for exposition to arrange observations in a certain order. More specifically, the observations for permanent and transitory sellers are allocated into separate vectors. We enumerate observations for actual entrants first then for non-entrants, and group the observations for permanent sellers according to (x, q)-characteristics, and those for transitory sellers according to x-characteristics. Thus we write $B^t_{j,l}$ to denote the j-th transitory seller's bid at auction l, $B^p_{j,l}$ the j-th permanent seller's bid at auction l, $Q^t_{j,l}$ the j-th transitory seller's quality at auction l, and $W^p_{j,l} \in \{1, 0\}$ taking the value of one if and only if the j-th permanent seller wins at the l-th auction. Similarly, we define $x^t_{j,l}$, $x^p_{j,l}$, and $q^p_{j,l}$. After the rearrangement, the competitive nature of auction l is summarized by

$$\mathbf{I}_l \equiv \bigcup_{(x,q) \in \mathcal{X} \times \mathbb{Q}_x} \{|A^p_{x,q,l}|, |N^p_{x,q,l}|, |A^t_{x,l}|, |N^t_{x,l}|\},$$

where $|A|$ for any set A denotes its cardinality. For each auction l, we define $\mathbf{B}_l = [\mathbf{B}^{p\prime}_l, \mathbf{B}^{t\prime}_l]'$, where $\mathbf{B}^t_l$ and $\mathbf{B}^p_l$ are random vectors with their j-th entries given by $B^t_{j,l}$ and $B^p_{j,l}$ respectively. We also define $\mathbf{Q}^t_{N,l}$ and $\mathbf{Q}^t_{A,l}$ to be both random vectors of entries $Q_{j,l}$ with $j = 1, \cdots, |N^t_l|$ and with $j = 1, \cdots, |A^t_l|$ respectively. We denote the set of values for $\mathbf{Q}^t_{N,l}$ by $\{\bar{\mathbf{q}}_{N,1}, \cdots, \bar{\mathbf{q}}_{N,\bar{K}_{N,l}}\}$ with $\bar{q}_{N,k} = (q_{N,1,k} \cdots q_{N,|N^t_l|,k})$. Similarly, the set of values for $\mathbf{Q}^t_{A,l}$ is denoted by $\{\bar{\mathbf{q}}_{A,1}, \cdots, \bar{\mathbf{q}}_{A,\bar{K}_{A,l}}\}$

with $\bar{q}_{A,k} = (q_{A,1,k} \cdots q_{A,|A_l^t|,k})$. These sets change across auctions because the dimensions of $\mathbf{Q}_{N,l}^t$ and $\mathbf{Q}_{A,l}^t$ change.

6.3 Preliminary Results

In accordance with the parametric estimation approach, we assume that ϵ_{il} and (α, β) are distributed according to $F(\epsilon|\theta_1)$ and $F(\alpha, \beta; \theta_2)$, distributions known up to a set of parameters (θ_1, θ_2), so that the vector of parameters to be estimated is given by $\theta = (\theta_1, \theta_2, (\mathbb{Q}_x : x \in \mathcal{X}))$ along with the parameters involved in the parametrization of $f(\mathbf{B}_i|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ and $P(i \in A_{x,l}^t|\, \mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$.

We begin by deriving a representation of permanent seller's winning probability conditional on the vector of bids and auction competitive structure as observed by the econometrician. Unlike the economerician, a buyer observes all the relevant characteristics for all actual competitors. Let $e_{x,q,k,l}^p(\mathbf{B}_l, \mathbf{I}_l; \theta, j) \equiv P\{W_{j,l}^p = 1|\mathbf{B}_l, \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k,l}, \mathbf{I}_l\}$ be the probability that seller j (with (x,q)-characteristics) wins conditional on a full competitive structure of the auction, including information on transitory actual bidders' vector of qualities. More specifically,

$$e_{x,q,k,l}^p(\mathbf{B}_l, \mathbf{I}_l; \theta, j) = \int P\left(\alpha_l q + \beta_l x - B_{j,l} \geq \alpha_l q_i + \beta_l x_i - B_{i,l}\,\forall\, i \neq j|\, \alpha_l,\, \beta_l\right) dF_{\alpha,\beta}(\alpha_l,\, \beta_l).$$

Also let $p_{k,l} = P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k,l}|\mathbf{B}_l, \mathbf{I}_l\}$ be the probability that the vector of transitory actual bidders' qualities is $\bar{\mathbf{q}}_{k,l}$ conditional on the vector of bids $\mathbf{B}_l$, and information about the auction's competitive structure as summarized in $\mathbf{I}_l$. Then by the law of total probability, we obtain

$$P\{W_{j,l}^p = 1|\mathbf{B}_l, \mathbf{I}_l\} = \sum\nolimits_{k=1}^{\bar{K}_{A,l}} e_{x,q,k,l}^p(\mathbf{B}_l, \mathbf{I}_l; \theta, j)\, p_{k,l}. \tag{10}$$

Using the modeling assumptions, we can reformulate the expression in (10) as shown in the proposition below. (The proof is lengthy and given in the Appendix.) For this, we introduce some definitions. Let

$$\begin{aligned}
\mathbf{I}_{l,1} &\equiv \{|N_{x,q,l}^p|, |N_{x,l}^t| : (x,q) \in \mathcal{X} \times \mathbb{Q}_x\}, \\
\mathbf{I}_{l,2}^p &\equiv \{|A_{x,q,l}^p| : x \in \mathcal{X},\, q \in \mathbb{Q}_x\} \text{ and } \mathbf{I}_{l,2}^t \equiv \{|A_{x,l}^t| : x \in \mathcal{X}\},
\end{aligned}$$

so that $\mathbf{I}_l = \mathbf{I}_{l,1} \cup \mathbf{I}_{l,2}^p \cup \mathbf{I}_{l,2}^t$. We also let

$$\omega_{k,l} \equiv \prod_{i \in \bar{A}_l^t} P(\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}|\, \bar{\mathbf{x}}_{i,l}^t),$$

and

$$g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t) \equiv \prod_{x \in \mathcal{X}} \prod_{i \in \bar{A}_{x,l}^t} f(\mathbf{B}_i|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1}) P(i \in A_{x,l}^t|\, \mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1}).$$

We are prepared to state the proposition.

Proposition 4 *Under (A1)-(A5), for each $x \in \mathcal{X}$, $q \in \mathbb{Q}_x$, and for the j-th permanent seller with (x,q)-characteristic who participated in auction l,* [22]

$$P\{W^p_{j,l} = 1 | \mathbf{B}_l, \mathbf{I}_l\} = \sum\nolimits_{k=1}^{\bar{K}_{A,l}} e^p_{x,q,k,l}\left(\mathbf{B}_l; \theta, j\right) \frac{\omega_{k,l} g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})}{\sum_{d=1}^{\bar{K}_{A,l}} \omega_{d,l} g_d(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})}. \tag{11}$$

The quantities $g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})$ involve $f(\cdot|\mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$, i.e., the density of a transitory seller's bids in equilibrium conditional on this seller's quality, and $P(i \in A^t_{x,l} |\, \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$, i.e., the probability of transitory seller i's participation in the auction conditional on his quality. As mentioned earlier, we estimate these equilibrium objects jointly with the parameters of buyer's taste distribution and quality levels. In doing so, we do not need to recover these objects separately. Since in our setting the distribution of signals is the same for permanent and transitory bidders, we can use permanent bidder's optimization problem, bid distribution, and participation frequency to recover the distributions of signals. This requires knowing only $g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})$ and not separately $f(\mathbf{B}_i|\mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ and $P(i \in A^t_{x,l} |\, \mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$.

We next argue that $g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})$ are non-parametrically identified in our environment alongside the distribution of buyers' tastes and quality levels. First, note that under (A1)-(A5), the probability weights $\omega_{k,l}$ in (11) are directly identified from observable distributions. Next, note that since sellers' information and buyers' tastes are independent, the identities and bids of permanent entrants $\mathbf{I}^p_{l,2}$ do not enter $g_k(.)$. Hence the variations in $\mathbf{B}^p_l$ and $\mathbf{I}^p_{l,2}$ could be used to identify the objects of interest.

Recall from the nonparametric identification section that the variations in permanent sellers' bids under specific configurations of $\mathbf{I}^p_{l,2}$ is used to identify the distributions of buyers' tastes and quality levels. A simple counting exercise shows that the identification of these objects uses up to $1 + \sum_x K_x + dim\{x\}$[23] configurations of the set of permanent actual bidders. The total number of possible configurations for the set of permanent actual bidders far exceeds this requirement. Therefore, some actual bidder set configurations remain unexploited after the distribution of buyers' tastes and quality levels are non-parametrically identified. Notice next that if these objects are known, $e^p_{x,q,k,l}\left(\mathbf{B}_l; \theta, j\right)$ are known as well. Further, the expression in (11) can be re-written as

$$\sum\nolimits_{k=1}^{\bar{K}_{A,l}} \omega_{k,l}(e^p_{x,q,k,l}\left(\mathbf{B}_l; \theta, j\right) - P(W^p_{j,l} = 1 | \mathbf{B}_l, \mathbf{I}_l)) g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2}) = 0, \tag{12}$$

which is a system of linear functional equations where $g_k(\mathbf{B}^t_l, \mathbf{I}_{l,1}, \mathbf{I}^t_{l,2})$ and $[\omega_{k,l}(e^p_{x,q,k,l}\left(\mathbf{B}_l; \theta, j\right) - P(W^p_{j,l} = 1 | \mathbf{B}_l, \mathbf{I}_l))]$ play the roles of the unknown functions and known coefficients respectively.

[22] In fact, $\mathbf{E}[W^p_{x,q,l} | \mathbf{B}_l, \mathbf{I}_l] = \mathbf{E}[W^p_{j,l} | \mathbf{B}_l, \mathbf{I}_l]$ for all j such that $j \in A^p_{x,q,l}$ by symmetry. This formulation facilitates its sample analogue when we replace the sample version of the moment conditions for estimation.

[23] We need one configuration involving symmetric bidders to identify the distribution of ϵ, and $\sum_x K_x$ configurations to identify the distribution of α and the differences between quality levels. Finally, we need $dim\{x\}$ configurations to identify the distribution of β.

The remaining configurations of permanent actual bidder sets as well as different values of B_l^p combined with these configurations could be used to form a sufficient number of equations so that this linear system could be solved to recover the $\{g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)\}_{k=1,\cdots,K}$. Of course, formally we need to verify that the determinant of the matrix of the coefficients should be non-zero. However, given the high non-linearity of our environment both in $\mathbf{I}_{l,2}^p$ and in B_l^p such a property is likely to hold in general. This argument also establishes that the data contain enough information to identify $g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$ nonparametrically when the distribution of buyers' tastes is assumed to belong to a parametric family of distributions. These results support the validity of our joint estimation approach. While the empirical study in this paper adopts a parametric specification of $g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$, one may instead specify it as a nonparametric function and proceeds with a semiparametric estimation method similar to Ai and Chen (2003). Details in this direction are explained in the Supplemental Appendix.

In some applications, $f(\mathbf{B}_i^t|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ and $P(i \in A_{x,l}^t|\, \mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$ may be of independent interest. While working with these objects allows one to make use of further restrictions such as

$$\begin{aligned}
\sum_k P(i &\in A_{x,l}^t|\, \mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})\omega_{kl} = P(i \in A_{x,l}^t|\, \mathbf{I}_{l,1}) \text{ or} \\
\sum_k f(\mathbf{B}_i|\mathbf{Q}_{A,i,l}^t &= \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k,l}|\mathbf{I}_l) = f(\mathbf{B}_i^t|\, \mathbf{I}_{l,1}),
\end{aligned}$$

one needs to invoke additional restrictions to separate $f(\mathbf{B}_i^t|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ from $P(i \in A_{x,l}^t|\, \mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$ within $g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$. For this, one could use the expected profit condition that summarizes the optimal participation behavior of different groups of transitory potential bidders. Or one could alternatively use some exclusion restrictions if they are plausible. We discuss more on this issue in the empirical section of our paper.

6.4 Generalized Method of Moments Estimation

We formulate the moment conditions which are primarily based on the probability that permanent seller wins conditional on the information available to the econometrician as summarized by the expression in (11) and in accordance with the identification argument in the previous subsection.

More specifically, for any vector valued map $h_{x,q,1}^p : \mathbf{R}^{|A|} \to \mathbf{R}^{d_{h1}}$, and for each $x \in \mathcal{X}$, $q \in \mathbb{Q}_x$ we can write the moment condition as:

$$\begin{aligned}
&\mathbf{E}\left[h_{x,q,1}^p(\mathbf{B}_l, \mathbf{I}_l) W_{x,q,l}^p|\mathbf{I}_l\right] \\
= &\sum_{k=1}^{\bar{K}_{A,l}} \mathbf{E}\left[h_{x,q,1}^p(\mathbf{B}_l, \mathbf{I}_l) e_{x,q,k,l}^p\left(\mathbf{B}_l;\theta\right) \frac{\omega_{k,l} g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t;\theta)}{\sum_{d=1}^{\bar{K}_{A,l}} \omega_{d,l} g_d(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t;\theta)}|\mathbf{I}_l\right].
\end{aligned}$$

where, $g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t; \theta)$ is a parametric approximation of $g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$,

$$\begin{aligned} W^p_{x,q,l} &= \frac{1}{|A^p_{x,q,l}|} \sum_{j \in A^p_{x,q,l}} W^p_{j,l} \text{ and} \\ e^p_{x,q,k,l}(\mathbf{B}_l; \theta) &= \frac{1}{|A^p_{x,q,l}|} \sum_{j \in A^p_{x,q,l}} e_{x,q,k,l}(\mathbf{B}_l; \theta, j). \end{aligned}$$

This can be further re-written as:

$$\begin{aligned} &\mathbf{E}\left[\mathbf{h}^p_{x,q,1}(\mathbf{B}_l, \mathbf{I}_l)\left(W^p_{x,q,l} - e^p_{x,q,l}(\theta, g)\right)|\mathbf{I}_l\right] = 0 \text{ with} \\ &e^p_{x,q,l}(\theta, g) = \sum_{k=1}^{\bar{K}_l} e^p_{k,x,q,l}(\mathbf{B}_l; \theta) \frac{\omega_{k,l} g_k(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t; \theta)}{\sum_{d=1}^{\bar{K}_{A,l}} \omega_{d,l} g_d(\mathbf{B}_l^t, \mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t; \theta)}. \end{aligned} \tag{13}$$

As we discussed in the previous subsection, it may be useful, especially in the parametric setting, to separately specify and estimate $f(\mathbf{B}_i^t|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})$ and $P(i \in A_{x,l}^t|\, \mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k},\, \mathbf{I}_{l,1})$ functions. In such a case, restrictions associated with (a) transitory sellers' bid distribution, (b) the transitory sellers' probability of participation, as well as (c) restrictions summarizing optimal participation behavior may be additionally imposed. More details are given in Supplementary Appendix.

To construct a sample version of the moment conditions, we first obtain a consistent estimator $\hat{\omega}_{k,l}$ of $\omega_{k,l}$ via the sample analog principle, and construct

$$\hat{\rho}_{x,q,l}(\theta) = \mathbf{h}^p_{x,q,1}(\mathbf{B}_l^p, \mathbf{I}_l)\left(W^p_{x,q,l} - \hat{e}^p_{x,q,l}(\theta)\right),$$

where $\hat{e}^p_{x,q,s}(\theta)$ is equal to $e^p_{x,q,l}(\theta)$ except that $\omega_{k,l}$ is replaced by $\hat{\omega}_{k,l}$. We define $\hat{\rho}_l(\theta)$ to be a column vector with $\hat{\rho}_{x,q,l}(\theta)$ stacked up with (x, q) running in $\mathcal{X} \times \mathbb{Q}_1$. Hence the dimension of $\hat{\rho}_l(\theta)$ is $d_{h,p} \times |\mathcal{X} \times \mathbb{Q}_1|$, where $|\mathcal{X} \times \mathbb{Q}_1|$ is the cardinality of the set $\mathcal{X} \times \mathbb{Q}_1$. Then define a general method of moment estimator as follows:

$$\hat{\theta}_{GMM} = \text{argmin}_{\theta \in \Theta} \hat{\mathbf{Q}}_{GMM}(\theta),$$

where

$$\begin{aligned} \hat{\mathbf{Q}}_{GMM}(\theta) &= \left(\frac{1}{L}\sum_{l=1}^{L} \hat{\rho}_l(\theta)\right) \hat{\Sigma}^{-1} \left(\frac{1}{L}\sum_{l=1}^{L} \hat{\rho}_l(\theta)\right), \text{ and} \\ \hat{\Sigma} &= \frac{1}{L}\sum_{l=1}^{L} \hat{\rho}_l(\bar{\theta})\hat{\rho}_l(\bar{\theta})', \end{aligned}$$

and $\bar{\theta}$ is the first step estimator of θ_0 which is a minimizer of $\hat{\mathbf{Q}}_{GMM}(\theta)$ only with $\hat{\Sigma}^{-1}$ replaced by the identity matrix. Under regularity conditions, the estimator is known to be asymptotically normal with a positive definite covariance matrix. Note that the estimation error due to $\hat{\omega}_{k,l}$

does not affect the asymptotic variance matrix because the components $P\{\mathbf{Q}^t_{A,i,l} = \bar{\mathbf{q}}_{A,i,k}|I_l\}$ of $\omega_{k,l}$ take values only from a finite set and hence has a convergence rate that is arbitrarily fast as $L \to \infty$.

7 Empirical Analysis

7.1 Data Description

Our data include information on close to 600,000 projects that involve the participation of around 50,000 different sellers. For every project, we observe the type of work, the approximate size of the project, the time requirements, and the location of the buyer. We also observe all bids submitted, including bids rejected through violation of the reserve price, the identity of the winner, and measures of the winner's subsequent performance.

The projects fall into several broad classes such as platform programming, databases, graphics programming and website design. The work is then further divided into finer categories within these classes. For example, one of the recurrent requirements is the specification that a particular programming language should be used. We focus on graphics-related programming projects in our analysis. The projects in this set involve programming computer games, computer-generated animation, and media-related programming. Our decision was mostly motivated by sample size considerations. However, this is also a highly specialized segment of the market. The related work is very sophisticated and is done exclusively by hard-core professionals. This, therefore, is an environment where the seller's quality is likely to matter. On the other hand, this environment perhaps would be characterized by lower variation in provider qualities as opposed to the less skilled types of projects.

Table 4 provides some descriptive statistics for projects in our data set. Each row of the table summarizes a marginal distribution of the correspondent variable. The table shows that a sizable number of the projects are very small (below \$100). On the other hand, some of the projects are quite big (above \$1000). We focus on the medium to medium-large size projects (between \$100 to \$700).[24] The projects are fairly short: the deadline for the majority of the projects is between one to three weeks. Median number of sellers submitting bids for a project is six while median number of permanent bidders is five. However, about 10% of projects receive more than 18 bids (12 from permanent bidders). The projects with a large number of bids tend to be small.

The table also summarizes the characteristics of permanent sellers. It shows that a median permanent seller has completed 100 projects, while 10% of sellers completed 250 or more projects. The distribution of the average reputation scores appears to be quite tight. A median permanent seller has an average score of 9.87, while less than 25% have an average score below 9.7 or above 9.95. Similarly, a median permanent seller was never involved in an arbitration or had a delay.

[24]We can offer some anecdotal insight that helps to put the size into the right perspective. One of the authors used this market to procure programming services: the project that costs \$200 in this on-line market was quoted at \$800 in the off-line programming market in Philadelphia.

Table 4: Data Summary Statistics

	25%	50%	75%	90%
Project Characteristics				
Size	$150	$250	$500	$1000
Duration	5	10	14	21
Number of Bidders	4	6	11	18
Number of Permanent Bidders	3	5	10	12
Permanent Sellers' Characteristics				
Experience	75	100	150	250
Average Score	9.7	9.87	9.95	10
Arbitrations	0	0	0	1
Delays	0	0	0	1
Number of Projects	32,679			

The results in this table are based on full sample of projects with graphics-related programming. Duration of project is measured in days.

However, less than 10% of permanent sellers were involved in at least one arbitration or had at least one delay.

We group sellers into country groups by geographic proximity and similarity of language and economic conditions. We end up with seven country groups: North America (USA and Canada), Latin America, Western Europe, Eastern Europe, Middle East and Africa, South East Asia, Australia (grouped with New Zealand). In our data North America, Eastern Europe and South or East Asia account for the majority of submitted bids.

We now turn to the discussion of the estimation results. We first summarize estimates from our classification procedure, then we discuss the parametric estimates of the buyers' tastes and sellers' quality levels as well as the bidding and participation strategies of transitory bidders.

7.2 Empirical Results: Classification

In this section we summarize the results of the group structure estimation. Classification algorithm is applied to the set of permanent participants specializing in graphics-related programming. For each seller we discard the first year of his tenure and only use observations that correspond to the later years of his career with on-line market.

We assume that the buyer cares about the seller's quality, price, and the seller's covariates such as reliability and country affiliation. We specifically distinguish between the seller's quality and reliability. In our environment quality reflects the seller's ability to handle complex and not fully specified jobs and his ability to deliver a product of superior quality. In contrast, reliability measures the likelihood that he completes the job if engaged, that he is on-time, maintains regular communication with the buyer, is responsive to buyer's requests, etc. We believe that the number and value of reputation scores reflects seller's reliability.[25,26] The seller's country affiliation can proxy for things such as convenience of working with a given seller related to time difference, the likelihood of language proficiency, and work culture.

All permanent sellers have a high number of ratings, therefore, we assume that the exact number of ratings is not important. We divide all the sellers into three cells according to the average reputation score: (cell 1) average reputation score less than 9.7, (cell 2) average reputation score above 9.7 and below 9.9, (cell 3) average reputation score above 9.9. This results approximately in an allocation of 30%, 30%, and 40% across cells.

The classification index is constructed for the pair of sellers on the basis of projects where they both belong to the set of potential bidders. Our data do not contain information on the set of potential sellers for a specific project. In our analysis we assume that the set of potential sellers for project l consists of all sellers who were active in the market (i.e., submitted bids or sent messages to buyers) during the week when project l was posted and who are qualified for the type of work indicated for project l (i.e., they bid for similar projects in the past). The pair-wise nature of our index does not have strong implications for our sample. We are able to compute an index for each pair of sellers within each of our cells. We have also experimented with alternative definitions of the sets of potential sellers. The results of the classification remain stable even with different definitions.

We follow the steps described in the Section 5. That is, we start by estimating a group structure for a range of the number of groups. We then apply a criterion function to select the structure with the number of groups most supported by the data. For this structure we then compute confidence sets. Table 9 in Supplemental Appendix demonstrates steps 1 and 2 for the group of Eastern European sellers with a medium level of average reputation score.

Table 5 reports the estimated group structures with corresponding confidence sets for cells of North American, Eastern European and East Asian sellers. We estimate multiple quality groups in each cell and the confidence sets associated with each group structure are quite tight. It is difficult to draw any substantive conclusions about quality distribution on the basis of these results since classification into groups is ordinary and does not allow for comparison of levels across countries or reputation scores. We note here that even the cells that correspond to a very narrow range of reputation scores (such as medium or high reputation scores) allow for a non-trivial number of quality groups. Also, mass allocation between quality groups differs across cells. We defer the more interesting substantive inference to the section on the results of the parametric estimation.

[25] The number of scores is highly correlated with the number of projects completed. It, therefore, serves as a proof of sellers's reliability. On the other hand, the value of the reputation score may include information about the quality of work but also reflects whether buyer was satisfied with experience of working with this seller.

[26] We have also verified robustness of our results by repeating the analysis while including the number of arbitrations and delays as additional measures of reliability. The results remain virtually unchanged.

Table 5: Estimated Quality Groups by Supplier Covariates

Country Group	Average Score	Total Number of Suppliers	Q_1	Q_2	Q_3
North America	low	12	4	8	
			(6)	(10)	
North America	medium	13	4	9	
			(6)	(11)	
North America	high	17	12	5	
			(13)	(6)	
Eastern Europe	low	18	6	12	
			(8)	(14)	
Eastern Europe	medium	52	33	12	7
			(37)	(14)	(9)
Eastern Europe	high	83	6	65	12
			(7)	(69)	(15)
East Asia	low	91	62	18	11
			(68)	(22)	(13)
East Asia	medium	66	6	53	7
			(8)	(57)	(9)
East Asia	high	58	50	8	
			(53)	(11)	

This table shows the estimated group structure and a consistently selected number of groups for each cell determined by covariate values. Column 3 indicates the total number of the suppliers in the cell. Columns 4-6 report the size of the estimated quality group. The size of the corresponding confidence set with 90% coverage is reported in parenthesis.

7.3 Empirical Results: Parametric Estimation

In this section we present the results of the parametric analysis. We begin by summarizing our specification and the exact set of moments used in estimation. Next, we discuss the estimates of the objects of interest: parameters of buyers' tastes distribution, quality distributions for a range of covariate values, as well as sellers' bidding strategies and recovered cost distributions.

7.3.1 Parametric Specifications and Moment Conditions

As we stated in the previous section, we assume that buyers' utility from selecting a specific seller depends on the seller's quality, price quote, and country group affiliation as well as performance-related indicators such as the number of scores, and the average reputation score. We modify the utility specification for the purpose of estimation. More specifically, we divide the expression for the utility function by the quality coefficient α. This obtains a utility function specification that is often used in the estimation of differentiated product models:[27]

$$\tilde{u}_{li} = q_i(x) + x_i\tilde{\beta}_l - \tilde{\alpha}_l b_{li} + \tilde{\epsilon}_{li}.$$

Here $q_i(x)$ plays the role of a product-level unobservable that was first introduced into the differentiated products studies by Berry, Levinsohn and Pakes (1995), and Nevo(2001). Further, we assume that utility errors, $\tilde{\epsilon}$ follow the Extreme Value Type I distribution with standard error σ_ϵ, while taste parameters α and β are assumed to be distributed according to the normal distributions $N(\mu_\alpha,\, \sigma_\alpha^2)$, and $N(\beta_0,\, \Sigma_\beta)$, respectively.[28] We impose the normalization assumptions implied by our identification argument. That is, we normalize the expected value of ϵ to be equal to zero, the expected value of α to be equal to one, and one of the quality levels (quality level 1 of the low average score group, the South and East Asian country group) to be equal to zero. We, therefore, aim to estimate the vector of parameters $\theta = \{\sigma_\epsilon, \sigma_\alpha, \beta_0, \Sigma_\beta, \{q_x\}\}$ where $\{q_x\}$ is the vector of quality levels that correspond to the quality groups recovered in the previous section.

We assume that transitory and permanents sellers' bid distributions are well approximated by normal distributions $N(\mu_{B^t}, \sigma^2_{B^t})$ and $N(\mu_{B^p}, \sigma^2_{B^p})$,[29] respectively. The means of the bid distribution depend on the seller's quality group, number of reputation scores (projects completed), and average reputation score, and on the number of potential competitors by quality group. We allow the effect of the average reputation score to vary flexibly with the number of scores. Notice that the bid distribution of transitory sellers depends both on the current average score and the long-run average score through the group structure. This is because the long-run average score correctly reflects the seller's true reliability. However, it is not observed in the data for transitory sellers. Therefore, the buyer has to base his expectation of the long-run average reputation score on contemporaneously available measures when awarding the project. This, in turn, implies that transitory bidders would incorporate their current average scores into their bids. We explain how we link the transitory seller's long-run average score group to his current performance below.

Similarly, we approximate permanent and transitory bidders' respective probabilities of participation by normal distribution functions that depend on linear indices of the seller's quality group, number of reputation scores (projects completed), and average reputation score, and on the number of potential competitors by quality group. As in the case with the bid distribution, we allow the effect of the average reputation score to vary flexibly with the number of scores.

We have estimated the quality groups for permanent sellers conditional on country affiliation and long-run average reputation score. The majority of transitory sellers complete only one or two projects. Hence, their long-run average reputation scores are not observed in the data. We assume that buyers use public information to form beliefs about the probability that a beginning seller with a given number and sum of scores and a given quality level belongs to a particular long-run average score group. We recover these beliefs non-parametrically using beginning of

[27]We could be worried about such re-parameterization in the case when zero belongs to the support of α. However, this would only mean that infinity belongs to the supports of $\tilde{\alpha} = \frac{1}{\alpha}$, $\tilde{\beta} = \frac{\beta}{\alpha}$, and $\tilde{\epsilon} = \frac{\epsilon}{\alpha}$, the case that can be easily accommodated.

[28]Strictly speaking, the distribution of α should have been chosen to have a non-negative support. However, we estimate the standard error of this distribution to be quite small so that this assumption does not make any practical difference. The same comment applies to our assumption on the distribution of bids below.

[29]See the comment for the distribution of α above.

career and long-run data on permanent bidders. We use these beliefs to form the expected utility that the buyer derives from transitory sellers.

The estimation is based on two types of moment conditions discussed in Section 6. The first type of moments relates to the probability that permanent bidder wins under a variety of permanent actual bidder set configurations. The second type of moments matches transitory and permanent sellers' empirical distribution of bids and participation frequencies to their theoretical counterparts. The full details about moment conditions used in estimation can be found in Supplemental Appendix.

7.3.2 Quality and Other Attributes as Determinants of Buyer's Choice

Table 6 reports the estimated coefficients of the buyers' utility function and quality levels. All estimates have the expected signs and small standard errors. In the estimation the prices are normalized by the project size; therefore, all coefficients represent the percentage mark-up over the project size that an average buyer would be willing to pay for the unitary increase in the corresponding covariate.

Notice that the estimated variance of ϵ is quite small, which indicates that the seller' covariates indeed play an important role in our environment in comparison to stochastic or unexplained factors. The price coefficient α has a comparable variance. Nevertheless, the price component plays a more important role than ϵ since it is additionally multiplied by price.

The last panel of the table reports the estimated quality levels across covariate cells. The estimated levels have the expected sign and are increasing according to group ranking. The differences across quality levels are substantial in magnitude. In addition, the model with quality is capable of explaining 70% of buyers' choices in comparison to the 25% that the model without quality could explain. These things indicate that quality plays an important role in our environment.

Next, we observe that the quality levels are consistent across covariate cells. There appears to be roughly three quality levels present in this market, with the lowest normalized to be around zero, the medium quality level estimated to be somewhere in the range 0.1-0.3, and the highest quality level is between 0.45-0.68. The exact levels differ across country groups with Eastern Europe characterized by the highest values for each quality level and North America characterized by the lowest "high" quality levels.

Having established that the quality levels are very similar across covariate groups, we can conclude based on the results from the previous section that there exist important differences in the distribution of quality mass across covariate levels. In particular, North America is missing a middle quality level, whereas the lowest average score cell for Middle Europe and the highest average score cell for South and East Asia are missing the lowest quality levels. Similarly, the medium score cell for Eastern Europe allocates the most mass to the lowest and medium quality levels, whereas the highest score cell allocates the most mass to the medium and high quality levels. We observe similar regularities in the case of South and East Asia. Hence, the distribution of qualities varies significantly with covariate values. That finding underscores the importance of using our methodology, which allows for such dependence, as opposed to a mixture methodology that would have to impose the restriction that the distribution of unobserved heterogeneity is orthogonal to other variables that enter utility function.

Country and long-run average reputation score appear to have independent effects on the buyer's utility. These effects, however, are rather small relative to the differences in quality levels. For example, an average buyer would be willing to pay almost 7% more of the project size, $(0.523 - 0.451 = 0.072)$, to obtain the service of a high-quality North American seller with a high reputation score rather than a high-quality North American seller with a low reputation score. Similarly, an average buyer would be willing to pay 14% more of the project size, $(0.693 - 0.532) = 0.141$, to hire a medium score, high-quality supplier from Eastern Europe rather than a medium score, high-quality supplier from South or East Asia.

We estimate that the number of reputation scores and an average reputation score matter for beginning or transitory bidders in a statistically significant way. For example, at any quality level, having no reputation scores bears a negative premium of close to 2%. On the other hand, having a positive but small number of scores erodes this negative premium to zero. The average reputation score does not appear to be important when the number of scores is really small. However, the difference between 9 points and 10 is rewarded with a 5% premium if the number of scores is moderate. This is comparable to the 7% premium documented above for the case of a long-run average reputation score that corresponds to the number of scores much larger than 10.

7.3.3 Pricing Strategies, Cost Distributions and Quality Heterogeneity

Tables 7, and 10 in Supplemental Appendix report the estimated coefficients for bid distributions and participation probabilities. These coefficients are difficult to interpret without the context of a pricing game. We use them to comment on the performance of our estimation procedure and model fit.

Our estimates indicate a statistically significant relationship between the transitory sellers' mean bids, participation probabilities and quality levels. Further, the estimated coefficients for the permanent sellers' bid distributions and participation probabilities are very similar in sign and magnitude to the coefficients from the transitory sellers' bid distribution and participation probabilities. Recall that the transitory sellers' bid distribution is estimated jointly with the utility function parameters from observed buyers' choices via the set of moments that exploit the structure of our model and proposed identification strategy. In particular, there is no direct link between the transitory seller's bids and his quality level. Our estimates, therefore, support our assumption that the quality of the transitory bidder is observable to buyers and that transitory bidders are not inherently different from permanent bidders in any way.

We rely on first order condition from the permanent bidder optimization problem to compute the inverse bid functions, $\xi(b|(q,x))$, for sellers with various affiliations. More specifically, we compute inverse bid function as

$$\xi(b_i|(q,x)_i) = b_i - \frac{P(i\,\text{wins} \mid b_i; \sigma^E_{-i}, \sigma^B_{-i})}{\frac{\partial}{\partial b} P(i\,\text{wins} \mid b; \sigma^E_{-i}, \sigma^B_{-i})|_{b=b_i}}.$$

The details can be found in Supplemental Appendix. The distributions of the seller's costs are then recovered by combining the bid distributions and the inverse bid functions:

$$F_C(\xi(b|(q,x))|(q,x)) = F_B(b|(q,x)).$$

Figure 2 depicts the estimated permanent sellers' bidding functions for North America, Eastern Europe, and South and East Asia respectively. Similarly, figure 3 shows the estimated densities of the cost distributions across country groups and across average reputation score levels. The estimated bid functions are increasing in costs, which is consistent with the theoretical predictions for the environment with private values. The graphs imply that the mark-up over sellers' cost changes very little with cost level and, in fact, for some groups increases as costs reach the upper end of the support. This feature arises because the buyer's choice is based in part on a purely stochastic (from the seller's point of view) component, ϵ. As the seller's costs increase, his probability of winning increasingly depends on the realization of the ϵ component, which in turn makes his bidding strategy less aggressive. This regularity is further clarified in figure 5 shown in Supplemental Appendix. This graph shows that, in our environment, the probability of winning and the derivative of probability of winning decrease at the same rate over the large part of the support and the derivative of the probability of winning decreases faster near the end of the support. This is in contrast to the standard auction environment where the probability of winning usually decreases slower than the derivative throughout the support. As should be expected, this "gambling effect" appears to be most pronounced in the pricing strategies of lower quality levels. In general, stochasticity plays an important role in our environment: sellers are uncertain about buyers' tastes as well as their actual competition. This accounts for the relatively large mark-ups we document in our environment.

The depicted cost distributions are based on the estimated bid distributions and inverse bid functions for permanent bidders. The estimated project cost distributions are typically "increasing" in sellers' quality. More specifically, the cost distribution of the high-quality group is always shifted to the right relative to the distribution of the medium-quality group. However, the low-quality group often has costs that are comparable to or even higher than the costs of the high-quality group. This indicates substantial costs heterogeneity unrelated to quality that characterize the participants in this market.

Notice further that the estimated project cost distributions appear to have substantially lower variances relative to the variance of the bid distributions. Thus, our model is capable of rationalizing the highly variable pricing environment through reasonably tight cost distributions. The "gambling" property of the bid functions described above explains this effect. Indeed, convexity or increasing mark-up near the end of the support induces high variance in sellers' prices and also explains the presence of really high bids in this environment. Thus, again our modeling choice for buyers' preferences appear to work well in this environment.

Table 6: Buyers' Tastes and Quality levels

Variable			Coefficient	Std.Error
$\log(\sigma_\epsilon)$			-0.381**	0.03
$\log(\sigma_\alpha)$			-0.346**	0.01
no ratings			-0.018**	0.004
$0 <$ ratings ≤ 3			-0.008**	0.002
$3 <$ ratings ≤ 10			-0.005	0.004
Average Score* $D_{0<ratings\leq 3}$			-0.006	0.005
Average Score* $D_{3<ratings\leq 10}$			0.054**	0.003
North America,	low score,	Q=1	-0.016**	0.007
North America,	low score,	Q=2	0.451**	0.006
North America,	medium score,	Q=1	-0.016**	0.008
North America,	medium score,	Q=2	0.484**	0.005
North America,	high score,	Q=1	-0.016**	0.003
North America,	high score,	Q=2	0.523**	0.004
Eastern Europe,	low score,	Q=1	0.271**	0.002
Eastern Europe,	low score,	Q=2	0.613**	0.003
Eastern Europe,	medium score,	Q=1	-0.101**	0.007
Eastern Europe,	medium score,	Q=2	0.259**	0.005
Eastern Europe,	medium score,	Q=3	0.683**	0.008
Eastern Europe,	high score,	Q=1	-0.102**	0.004
Eastern Europe,	high score,	Q=2	0.279**	0.005
Eastern Europe,	high score,	Q=3	0.674**	0.004
South and East Asia,	low score,	Q=1	0.000	
South and East Asia,	low score,	Q=2	0.096**	0.007
South and East Asia,	low score,	Q=3	0.426**	0.008
South and East Asia,	medium score,	Q=1	-0.029**	0.004
South and East Asia,	medium score,	Q=2	0.092**	0.004
South and East Asia,	medium score,	Q=3	0.532**	0.003
South and East Asia,	high score,	Q=1	0.102**	0.005
South and East Asia,	high score,	Q=2	0.563**	0.005

The results are based on the dataset consisting of $11,300$ projects. The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

Table 7: Participation Decision and Bid Distribution

	Score	Q	I(T)	II(T)	I(P)	II(P)
Mean						
Constant			0.511** (0.013)	-2.205** (0.011)	0.615** (0.043)	-2.155**(0.008)
No Ratings			-0.097** (0.022)	-0.48** (0.023)		
$0 < \text{Ratings} \leq 3$			0.021** (0.003)	0.004 (0.011)		
$3 < \text{Ratings} \leq 10$			0.051** (0.002)	0.003 (0.009)		
Number of Ratings					0.068 (0.052)	0.006 (0.005)
Average Score 1			-0.006 (0.004)	-0.006 (0.011)		
Average Score 2			0.012** (0.003)	0.012** (0.002)		
North America,	Low,	1	-0.218** (0.023)	0.231** (0.021)	-0.307** (0.046)	0.031 (0.027)
North America,	Low,	2	-0.173** (0.018)	-0.042 (0.022)	-0.271** (0.044)	-0.053** (0.025)
North America,	Medium,	1	-0.004 (0.043)	0.251** (0.012)	0.086** (0.043)	0.288** (0.023)
North America,	Medium,	2	-0.038** (0.022)	-0.139** (0.023)	-0.062 (0.046)	-0.105** (0.026)
North America,	High,	1	-0.173** (0.012)	0.134** (0.023)	-0.214** (0.041)	0.165** (0.017)
North America,	High,	2	-0.108** (0.021)	-0.265** (0.031)	-0.166** (0.039)	-0.193** (0.031)
Eastern Europe,	Low,	1	-0.026** (0.017)	0.232** (0.021)	-0.083* (0.043)	0.205** (0.017)
Eastern Europe,	Low,	2	-0.062* (0.022)	-0.194** (0.013)	-0.176** (0.038)	-0.108** (0.021)
Eastern Europe,	Medium,	1	-0.199** (0.035)	0.034** (0.015)	-0.246** (0.044)	0.013 (0.012)
Eastern Europe,	Medium,	2	-0.192** (0.024)	-0.245** (0.021)	-0.226** (0.048)	-0.198** (0.022)
Eastern Europe,	Medium,	3	-0.128** (0.032)	-0.257** (0.017)	-0.167** (0.051)	-0.232** (0.029)
Eastern Europe,	High,	1	-0.191** (0.024)	-0.131** (0.023)	-0.248** (0.038)	-0.257 (0.034)
Eastern Europe,	High,	2	-0.178** (0.043)	-0.091** (0.031)	-0.249** (0.051)	-0.099** (0.012)
Eastern Europe,	High,	3	-0.132** (0.022)	-0.221** (0.011)	-0.172** (0.038)	-0.206** (0.029)
South-East Asia,	Low,	2	-0.246** (0.034)	-0.231** (0.012)	-0.226** (0.041)	-0.204** (0.021)
South-East Asia,	Low,	3	-0.359** (0.044)	-0.075**(0.011)	-0.434** (0.043)	-0.051 (0.033)
South-East Asia,	Medium,	1	-0.108* (0.063)	-0.075** (0.021)	-0.196** (0.044)	-0.117** (0.017)
South-East Asia,	Medium,	2	-0.183** (0.028)	-0.071** (0.031)	-0.231** (0.038)	0.033** (0.010)
South-East Asia,	Medium,	3	-0.253** (0.035)	-0.074** (0.023)	-0.374** (0-.045	-0.059** (0.027)
South-East Asia,	High,	1	-0.112** (0.037)	-0.195** (0.011)	-0.178** (0.038)	-0.105** (0.012)
South-East Asia,	High,	2	-0.095** (0.024)	-0.299** (0.014)	-0.065** (0.053)	-0.281** (0.034)
Std Error			0.232** (0.012)		0.292** (0.015)	

This table reports the effects of the covariates and the group premiums on sellers' bid distribution and participation decisions. Columns I(T), II(T), and I(P), II(P) report estimated coefficients for the bid distribution and probability of participation of the transitory and permanent sellers respectively. "Average Score 1" and "Average Score 2" denote interactions of the current average score variable with the indicators for $0 \leq Ratings \leq 3$ and $3 \leq Ratings \leq 10$. The results are based on the data set consisting $11,300$ projects. The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

We assess the magnitude of the entry costs using a simple model of entry such that (a) entry cost constitutes the seller's private information, (b) entry cost is orthogonal to the seller's cost of completing the project, (c) the cost of completing the project is not observed at participation decision.[30] Under this model, the observed probability of participation satisfies the equation

$$F_S(E[\pi(q,x)]) = \Pr(i \in A(x,q)),$$

where $F_S(.)$ denotes the distribution of the entry costs and $\pi(q,x)$ is an ex-ante expected profit.

We estimate the mean and standard deviation of entry costs distribution by fitting the truncated normal distribution (truncated at 0) to the set of points implied by the ex-ante expected profit and the probability of participation values for various covariate cells and quality groups. The estimated value for the mean and standard deviation of the entry costs are 0.082 and 0.081 respectively. That is entry costs roughly correspond to 6% of the project cost on average. This number is slightly higher than documented in other markets.[31] The relatively large entry costs estimated in this market may reflect the fact that active bidding for a project involves substantial interaction with the buyer and possibly preparation of supplementary materials.

8 Conclusion

In this paper we proposed an empirical methodology that could be used to study many markets for services. It is applicable in the environment where heterogeneous buyers take into account seller characteristics (foremost their quality) in addition to price when choosing service providers, while sellers are small, distinct and characterized by a large turnover rate. Our methodology overcomes one of the most important hurdles that inhibited the study of these markets in the existing literature – the lack of reliable data on sellers' quality.

The environment we consider combines features of discrete choice (differentiated products) and auction settings. We build on the insights offered by these two literatures to develop a novel identification strategy as well as an implementable econometric procedure that recovers the distribution of buyers' tastes, the distribution of sellers' qualities conditional on other seller characteristics, as well as the distribution of sellers' costs conditional on quality and other attributes.

Our empirical findings confirm the economic significance of quality differences in our market. In fact, these differences dominate other types of seller heterogeneity. Allowing for the variation in sellers' quality and buyers' tastes for quality significantly improves the fit of the model. Recovering the distribution of qualities conditional on sellers' performance-related characteristics provides interesting insights into the availability of information in the online markets as well as the role of performance measures, such as the "reputation scores" collected in these markets. Finally, the recovered distribution of costs conditional on sellers' characteristics including quality

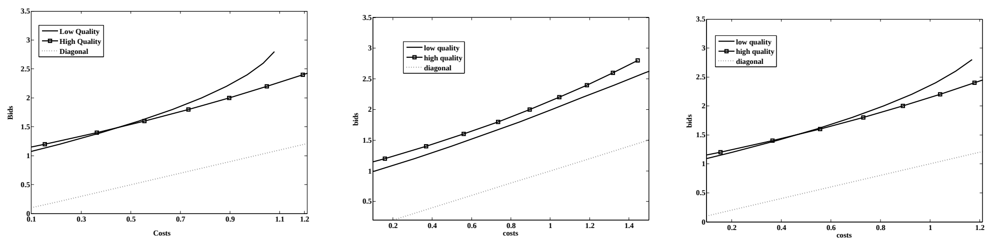
[30] The details of similar models can be found in Krasnokutskaya ans Seim (2011) and Li and Zheng (2009).

[31] Studies of the US highway procurement market have estimated entry costs to be around $2-5\%$ of the engineer's estimate.

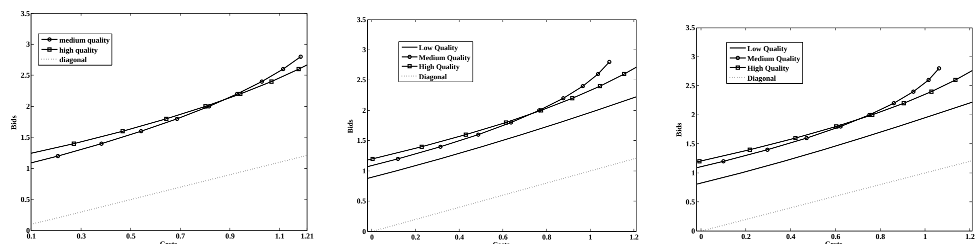
Figure 2: Bid Functions

Low Score | Medium Score | High Score

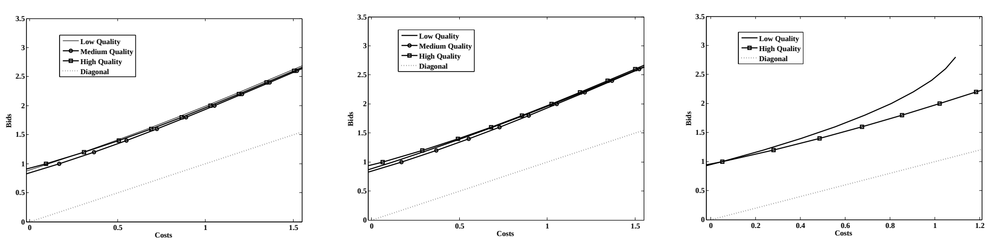
North America



Eastern Europe



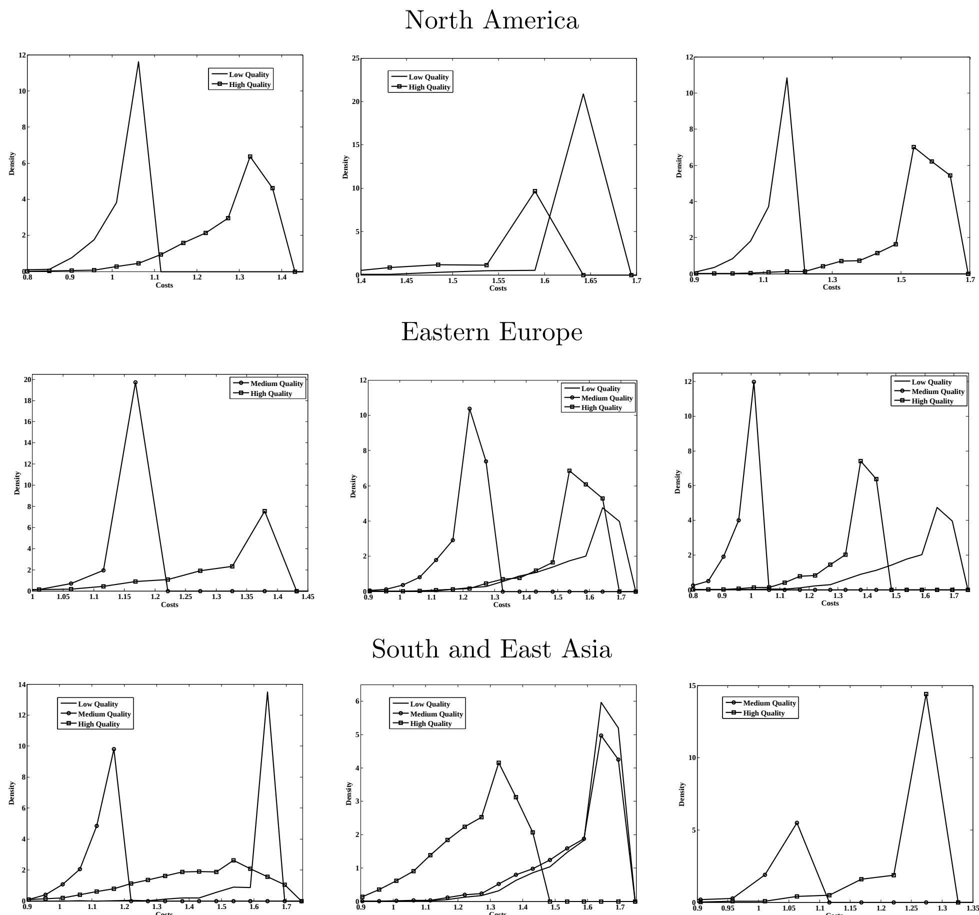
South and East Asia



The figure shows the equilibrium bidding strategies of permanent sellers recovered from the first order conditions of bidders' optimization program. The convexity at the upper end of the costs' support arises due to presence of stochastic component in buyers' tastes.

Figure 3: Density of Project's Cost Distribution

Low Score Medium Score High Score



The figure shows the permanent sellers' distribution of costs recovered by combining estimated bidding strategies and bid distributions.

builds a foundation for a better understanding of the composition of participants attracted to online markets as well as the cost of delivering quality services.

This paper represents the first step in the analysis of multi-attribute auctions with unobserved quality. Consequently, it made a number of simplifying assumptions that we expect to refine in subsequent research. We expect the basic insights of our identification and implementation strategies to carry over to those environments. In particular, we assume that sellers are completely uninformed about buyers' preferences, or, in more technical terms, we assume no unobserved auction (buyer) heterogeneity. Such an assumption may be too strong in some settings. We have some preliminary results that indicate that this assumption could be relaxed under certain conditions. Further, we assume that the buyer is perfectly informed about the seller's quality. We realize that a number of alternative informational assumptions may appear to be applicable in similar environments. We carefully thought about this aspect of our analysis, and we believe that the assumption we currently use in the paper is the most appropriate in our market. This assumption is supported by our empirical results, which show that the distribution of transitory sellers' bids depends on sellers' qualities in a statistically significant way. However, we also have some insights into how the model could be identified under alternative informational assumptions. We plan to pursue these issues in future research.

Finally, we find that reputation scores and to some degree the number of scores (or accumulated experience) are valued by the buyer. This potentially introduces dynamic considerations into the pricing and participation behavior of the sellers who are new in the market. We do not investigate these issues in our analysis. This does not impact the validity of our results. We exploit sellers' optimal decision making only in the last step when we use the seller's problem to recover the distribution of costs. In our analysis we rely on the problem of permanent sellers, who we believe are less concerned with the dynamic implications of their behavior. However, the reputation-building issues are of independent interest as an enforcement mechanism used in many Internet markets. We hope that future research will explore these issues in more detail.

In summary, we believe that our methodology opens the possibility of analyzing various aspects of service markets: from optimal pricing and optimal procurement to product design and studying the market mechanism that eliminates moral hazard concerns in this environment. Given the importance of service markets in modern economies, this seems to be an important research agenda.

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Appendix

Proofs on Non-parametric Identification of Quality Rankings

Consider a set of potential sellers N partitioned into $N^p \cup N^p$. Let A_Q and A_U denote subsets of entrants that meet (or fail to meet) reserve requirements respectively. (That is, $B_k > R$ for all $k \in A_U$ and $B_k \leq R$ for all $k \in A_Q$.) For any $\{i,j\} \subseteq N$, define a collection of mutually exclusive sets $\mathcal{A}_{i,j} \equiv \{(a,a') : a \neq \varnothing,\ a \cap a' = \varnothing$ and $a \cup a' \subseteq N \backslash \{i,j\}\}$. For any $(a,a') \in \mathcal{A}_{i,j}$, define

$$\mathcal{P}_i(b; a, a') \equiv P(i\,\text{wins}|\, B_i = b,\ A_Q = i \cup a,\ A_U = a')$$

for any $b \in \Omega_i$.

Lemma A1 *Suppose A1-A4 hold, and consider any i, j with $x_i = x_j$ and $\Omega_i \cap \Omega_j \neq \varnothing$. Then (a) for any $b \in \Omega_i \cap \Omega_j$ and $(a,a') \in \mathcal{A}_{i,j}$,*

$$\begin{aligned}
\Delta q_{ij} &> 0 \Rightarrow \mathcal{P}_i(b;a,a') \geq \mathcal{P}_j(b;a,a') \\
\Delta q_{ij} &= 0 \Rightarrow \mathcal{P}_i(b;a,a') = \mathcal{P}_j(b;a,a') \\
\Delta q_{ij} &< 0 \Rightarrow \mathcal{P}_i(b;a,a') \leq \mathcal{P}_j(b;a,a').
\end{aligned}$$

(b) For any $a^* \subseteq N \backslash \{i,j\}$ s.t. $x_k = x_i$ and "either $q_k = q_i$ or $q_k = q_j$" for all $k \in a^*$,

$$sign(\Delta q_{ij}) = sign(\mathcal{P}_i(b;a^*,a') - \mathcal{P}_j(b;a^*,a'))$$

for all $b \in \Omega_i \cap \Omega_j$ and any a' with $(a^*, a') \in \mathcal{A}_{i,j}$.

Proof of Lemma A1. *Proof of (a).* By A1-A3, sellers' entry and bidding strategies do not depend on realizations of $(\alpha, \beta, \epsilon)$ or R. Therefore, for any given $a \subseteq N\backslash\{i,j\}$, the random vector $(\Delta\epsilon_{ki}, B_k, \Delta x_{ik}\beta)_{k\in a}$ and the event "$B_i = b$, $A_Q = \{i\} \cup a$, $A_U = a'$" are independent from α. Let $\mathcal{E}(a, a')$ be shorthand for the event that "the set of entrants is $a \cup a'$ with $B_k \leq R$ for all $k \in a$, and $B_{k'} > R$ for all $k' \in a'$." By definition, for all such a, a',

$$\begin{aligned} & \mathcal{P}_i(b; a, a') \equiv P(i \,\text{wins}|\, B_i = b, A_Q = \{i\} \cup a, A_U = a') \\ = \;& P(\Delta\epsilon_{ki} - B_k - \Delta X_{ik}\beta - \alpha\Delta_{ik} \leq -b \;\forall k \in a|\, B_i = b, R \geq b, \mathcal{E}(a, a')) \\ = \;& \int_{\Omega_\alpha} P(\Delta\epsilon_{ki} - B_k - \Delta X_{ik}\beta \leq \alpha\Delta q_{ik} - b \;\forall k \in a|\, R \geq b, \mathcal{E}(a, a'))dF(\alpha|R \geq b, \mathcal{E}(a, a')), \end{aligned} \tag{14}$$

where $F(\alpha|.)$ is the conditional distribution of α. By similar arguments,

$$\mathcal{P}_j(b; a, a') = \int_{\Omega_\alpha} P(\Delta\epsilon_{kj} - B_k - \Delta X_{jk}\beta \leq \alpha\Delta q_{jk} - b \;\forall k \in a|\, R \geq b, \mathcal{E}(a, a'))dF(\alpha|R \geq b, \mathcal{E}(a, a')), \tag{15}$$

Note $(x_k)_{k\in a}$ is a vector of constant given a. By part (i) in A4 and that $x_i = x_j$, the distribution of $(\Delta\epsilon_{ki}, B_k, \Delta x_{ik}\beta)_{k\in a}$ is identical to that of $(\Delta\epsilon_{kj}, B_k, \Delta x_{jk}\beta)_{k\in a}$ conditional on "$R \geq b$, $A_Q = i \cup a$, $A_U = a'$". It follows that

$$\begin{aligned} \Delta q_{ij} &> 0 \Rightarrow \mathcal{P}_i(b; a) \geq \mathcal{P}_j(b; a) \\ \Delta q_{ij} &= 0 \Rightarrow \mathcal{P}_i(b; a) = \mathcal{P}_j(b; a) \\ \Delta q_{ij} &< 0 \Rightarrow \mathcal{P}_i(b; a) \leq \mathcal{P}_j(b; a), \end{aligned}$$

for all $b \in \Omega_i \cap \Omega_j$ and all a, a' such that $(a, a') \in \mathcal{A}_{i,j}$.

Proof of (b). It is sufficient to show that the weak inequalities above hold strictly for all $b \in \Omega_i \cap \Omega_j$ and any a^* that satisfies the conditions in part (b). By definition of a^*,

$$\begin{aligned} & \mathcal{P}_i(b; a^*, a') - \mathcal{P}_j(b; a^*, a') \\ = \;& \int_{\Omega_\alpha} \left(\begin{array}{l} P(\Delta\epsilon_{ki} - (B_k - b) \leq \alpha\Delta q_{ik} \;\forall k \in a^*|\, R \geq b, \mathcal{E}(a^*, a')) \\ -P(\Delta\epsilon_{kj} - (B_k - b) \leq \alpha\Delta q_{jk} \;\forall k \in a^*|\, R \geq b, \mathcal{E}(a^*, a')) \end{array} \right) dF(\alpha|R \geq b, \mathcal{E}(a^*, a')) \end{aligned}$$

for all $b \in \Omega_i \cap \Omega_j$ and a' such that $(a^*, a') \in \mathcal{A}_{i,j}$. Under our assumptions, for a given a^*, $(B_i)_{i\in a^*}$ are independent from $(\epsilon_i)_{i\in a^*}$ and α. Under A4, $(\Delta\epsilon_{ki})_{k\in a^*}$ is continuously distributed with positive densities conditional on α, and the support $[\underline{\varepsilon} - \overline{\varepsilon}, \overline{\varepsilon} - \underline{\varepsilon}]^{\#\{a^*\}}$ contains 0 in its interior. Likewise for $(\Delta\epsilon_{kj})_{k\in a^*}$. For any b in the interior of $\Omega_i \cap \Omega_j$, there is a positive probability that $(B_k - b)_{k\in a^*}$ is close enough to 0 and α is small enough so that $\mathcal{P}_i(b; a^*, a') >$ (and $=, <$) $\mathcal{P}_j(b; a^*, a')$ for all a' with $(a^*, a') \in \mathcal{A}_{i,j}$ whenever $\Delta q_{ij} >$ (and $=, <$ respectively) 0. *Q.E.D.*

Proof of Proposition 1. By definition,

$$r_{i,j}(b) \equiv P(i\,\text{wins}|\, B_i = b,\, i \in A,\, j \notin A) = P(i\,\text{wins}|\, B_i = b, R \geq b, i \in A, j \notin A)P(R \geq b)$$

$$= P(R \geq b) \sum_{(a,a') \in \mathcal{A}_{i,j}} \begin{array}{l} P(i\,\text{wins}|\, B_i = b, A_Q = i \cup a, A_U = a') \\ \times\, P(A_Q = i \cup a,\, A_U = a'|\, B_i = b, R \geq b, i \in A,\, j \notin A) \end{array},$$

where the first equality follows from the law of total probability, and the independence between R and $(S_i, C_i)_{i \in N}$, which implies $P(R \geq b|\, B_i = b,\, i \in A,\, j \notin A) = P(R \geq b)$. Likewise,

$$\begin{aligned} r_{j,i}(b) &\equiv P(j\,\text{wins}|\, B_j = b,\, j \in A,\, i \notin A) \\ &= P(j\,\text{wins}|\, B_j = b, B_j \leq R, j \in A, i \notin A)P(R \geq b) \\ &= P(R \geq b) \sum_{(a,a') \in \mathcal{A}_{i,j}} \begin{array}{l} P(j\,\text{wins}|\, B_j = b, A_Q = j \cup a, A_U = a') \\ \times\, P(A_Q = j \cup a,\, A_U = a'|\, B_j = b, R \geq b, j \in A,\, i \notin A) \end{array}. \end{aligned}$$

It follows from Lemma A1 that $P(i\,\text{wins}|\, B_i = b, A_Q = i \cup a, A_U = a') \geq$ (or $=$, $\leq$) $P(j\,\text{wins}|\, B_j = b$, $A_Q = j \cup a$, $A_U = a')$ whenever $\Delta_{ij} > 0$ (or $= 0$, < 0 respectively) for all $(a, a') \in \mathcal{A}_{i,j}$. Weak inequalities hold strictly for any $(a, a') \in \mathcal{A}_{i,j}$ such that $x_k = x_i$ and "either $q_k = q_i$ or $q_k = q_j$" for all $k \in a$. Such a pair $(a$, $a')$ exists in $\mathcal{A}_{i,j}$ given the conditions for the Proposition. Recall under our assumptions $P\{D_k = 1\} > 0$ for all $k \in N$; R is independent from $(S_i, C_i)_{i \in N}$; and (S_i, C_i) are also independent across $i \in N$. Hence there is a positive probability that "$D_k = 1\ \forall k \in a \cup a'$ *and* $D_k = 0\ \forall k \notin (i \cup a \cup a')$" given "$B_i = b$, $R \geq b$, $i \in A$, $j \notin A$" for any $(a, a') \subseteq \mathcal{A}_{i,j}$. Consequently, $P(A_Q = i \cup a, A_U = a'|\, B_i = b, R \geq b, i \in A,\, j \notin A)$ must be strictly positive for some $(a, a') \subseteq \mathcal{A}_{i,j}$ such that $x_k = x_i$ and "either $q_k = q_i$ or $q_k = q_j$" for all $k \in a$. The same argument applies as we switch the role of i and j. Finally note that under A1-A4, $P(A_Q = i \cup a,\, A_U = a'|\, B_i = b,\, R \geq b$, $i \in A,\, j \notin A)$ is identical to $P(A_Q = j \cup a,\, A_U = a'|\, B_j = b,\, R \geq b,\, j \in A,\, i \notin A)$ for all $(a, a') \in \mathcal{A}_{i,j}$. Hence $sign(r_{i,j}(b) - r_{j,i}(b)) = sign(\Delta q_{ij})$. *Q.E.D.*

Proof of Proposition 2. We first show that (6) is well-defined and can be recovered as in (7). Recall $\underline{b}_m < \bar{c}$ for all $m \in N$ in any PSBNE, and the reserve price R is independent from $(\alpha, \beta, \epsilon, S, C)$. Hence the events $D_{i,j,k}$, $G_{i,j,k}$ and $R \geq b_i$ happen with positive probability, provided $P(D_i = 1) > 0$ for all $i \in N$ and the support condition in R1 holds. Let $i, j, k \in N^p$. Consider any (b_i, b_j, b_k) and $A_Q = a$ such that $\{i, j, k\} \subset a$ (with $a \subseteq N^t \cup N^p$). Under (A1)-(A3),

$$\begin{aligned} & P\left(i\,\text{wins}|\, A_Q = a \backslash \{j, k\},\ b_i,\right) \\ = \ & P(u_i \geq u_m\ \forall m \in a \backslash \{i, j, k\}|\, A_Q = a,\, b_{i,j,k}) \end{aligned} \tag{16}$$

for all b_i, b_j, b_k on the support. Thus (6) can be written as:

$$\frac{P(i\,\text{wins}|\, A_Q = a,\ b_{i,j,k})}{P(i\,\text{wins}|\, A_Q = a \backslash \{j, k\},\, b_i)}$$

where the numerator is directly observable from the data as the seller qualification are reported. Under (A4'),

(ϵ_j, ϵ_k) is independent from $(\alpha, \beta, \epsilon_{-j,k}, S, C, R)$ and $\epsilon_j \perp \epsilon_k$. This implies that the joint distribution of $(\epsilon_i, \epsilon_j, \epsilon_k)$ is independent from the event "$A_Q = a,\, b_{i,j,k}$". Furthermore, conditional on "$A_Q = a,\, u_i \geq \max_{m \in a \setminus \{i,j,k\}} u_m$, $B_i = b_i$", the distribution of ϵ_i is independent from (b_j, b_k). Hence (6) takes the form of $P(\epsilon_i - \epsilon_j \geq \Delta b_{ij}$, $\epsilon_i - \epsilon_k \geq \Delta b_{ik} | \, \omega,\, B_i = b_i)$, where $F_{\epsilon_i, \epsilon_j, \epsilon_k | \omega, B_i = b_i} = F_{\epsilon_i | \omega, B_i = b_i} F_{\epsilon_j} F_{\epsilon_k}$ and $\omega \equiv$"$A_Q = a,\, u_i \geq \max_{m \in a \setminus \{i,j,k\}} u_m$." Under A6, this joint distribution of $(\epsilon_i - \epsilon_j, \epsilon_i - \epsilon_k)$ conditional on ω and $B_i = b_i$ can be recovered over the complete support by varying B_j and B_k. The Kotlarski Theorem then applies to show that F_{ϵ_i} is identified up to a location normalization. *Q.E.D.*

Proofs on the Estimation of Quality Classifications

Let $\tau_{ij}^{+} = \int \max\{\delta_{ij}(b), 0\} db$, $\tau_{ij}^{-} = \int \max\{-\delta_{ij}(b), 0\} db$, and $\tau_{ij}^{0} = \int |\delta_{ij}(b)| db$, where $\delta_{ij}(b) = r_i(b) - r_j(b)$. We confine the integral domains to $\Omega_i \cap \Omega_j$, and this restriction is omitted from the notation. From Theorem 2 of Lee, Song, and Whang (2012), one can show that under regularity conditions there exist fixed positive numbers σ_{ij}^{+}, σ_{ij}^{-}, and σ_{ij}^{0} such that whenver $\delta_{ij}(b) = 0$ for all $b \in \mathbf{R}$ (i.e., under the least favorable configuration),

$$\frac{\sqrt{L}\{\hat{\tau}_{ij}^{z} - \mathbf{E}\hat{\tau}_{ij}^{z}\}}{\sigma_{ij}^{z}} \to_d \quad N(0,1), \tag{17}$$

as $L \to \infty$ for $z \in \{+, -, 0\}$. Here L denotes the number of projects used for comparison.

As for the regularity conditions, we require two high-level conditions. First, the convergence in distribution (17) is satisfied under the least favorable configuration. The second condition is that the bootstrap tests are consistent against fixed alternatives. Detailed low level conditions for the convergence in distribution in (17) can be found in Lee, Song, and Whang (2012).

It is worth noting that the form of the test statistic is slightly different from that in Lee, Song, and Whang (2012) because $\hat{\delta}_{ij}(b)$ is the difference between the two kernel estimators. However, the observations pertaining to the i-th seller and the observations pertaining to the j-th seller are independent, if i and j are different. Using this particular property, we can apply the Poissonization method by focusing on the Poissonized statistic and representing it as an independent sum of Poisson random variables. We omit the details.

Proof of Theorem 1: First, observe that

$$\begin{aligned} \text{if } i &\in N_{l,\lambda},\ P\left\{\#\left(\hat{N}_{1,\lambda}(i) \Delta N_{1,\lambda}(i)\right) \geq 1\right\} \to 0 \text{ and} \\ \text{if } i &\in N_{h,\lambda},\ P\left\{\#\left(\hat{N}_{2,\lambda}(i) \Delta N_{2,\lambda}(i)\right) \geq 1\right\} \to 0. \end{aligned}$$

This is a simple consequence of the consistency of the bootstrap tests. Also, the consistency of the bootstrap test implies that for any $i \in N_{h,\lambda}$, we have that for any $j \in \hat{N}_{2,\lambda}(i)$, $p_0^*(i,j) \to_P 0$ as $L \to \infty$ but for any $j \in \hat{N}_{1,\lambda}(i)$, $p_0^*(i,j)$ is stochastically bounded as $L \to \infty$. Therefore, i is classified in the high-quality group with probability approaching one. That is, $P\{i \in \hat{N}_{h,\lambda}\} \to 1$. Hence for each $i \in N_{h,\lambda}$,

$$\begin{aligned} & P\left\{\delta((\hat{N}_{h,\lambda}(i), \hat{N}_{l,\lambda}(i)), C_\lambda) \geq 1\right\} \\ = \quad & P\left\{\delta((\hat{N}_{2,\lambda}(i) \cup \{i\}, \hat{N}_{1,\lambda}(i)), C_\lambda) \geq 1\right\} + o(1) \to 0. \end{aligned}$$

Also, similarly, for any $i \in N_{l,\lambda}$,

$$P\left\{\delta((\hat{N}_{h,\lambda}(i), \hat{N}_{l,\lambda}(i)), C_\lambda) \geq 1\right\} \to 0.$$

Since N_λ is a fixed finite set as $L \to \infty$, we have

$$P\left\{\delta((\hat{N}_{h,\lambda}(i), \hat{N}_{l,\lambda}(i)), C_\lambda) \geq 1 \text{ for some } i \in N_\lambda\right\} \to 0.$$

Therefore,

$$\begin{aligned} P\left\{\delta(\hat{C}_\lambda, C_\lambda) \geq 1\right\} &= P\left\{\delta((\hat{N}_{h,\lambda}(i^*), \hat{N}_{l,\lambda}(i^*)), C_\lambda) \geq 1\right\} \\ &\leq P\left\{\delta((\hat{N}_{h,\lambda}(i), \hat{N}_{l,\lambda}(i)), C_\lambda) \geq 1 \text{ for some } i \in N_\lambda\right\} \to 0, \end{aligned}$$

giving us the desired result. ■

Lemma A1: (i) If $K \geq K_0$, $\frac{1}{K}\sum_{k=1}^K \hat{V}_k(K) = O_P(1)$, as $L \to \infty$.

(ii) If $K < K_0$, for any $M > 0$, as $L \to \infty$,

$$P\left\{\frac{\frac{1}{K}\sum_{k=1}^K \hat{V}_k(K)}{g(L)} > M\right\} \to 1.$$

Proof: (i) The first statement of Lemma A1 can be proved in three steps. First we reclassify the true group structure into a finer one with K groups. Let this new group structure be $\{N_{k,1} : k = 1, \cdots, K\}$. Second, invoking Theorem 1 above, we show that the quantity $\frac{1}{K}\sum_{k=1}^K \hat{V}_k(K)$ is asymptotically equivalent to the same quantity (denoted by $\frac{1}{K}\sum_{k=1}^K \tilde{V}_k(K)$) only with $\hat{N}_k$ replaced by $N_{k,1}$. Finally, we show that $\frac{1}{K}\sum_{k=1}^K \tilde{V}_k(K) = O_P(1)$. To see the latter convergence rate, first, note that

$$\frac{\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}}{\sigma_{ij}^0} \to_d \mathbb{Z}$$

under the least favorable configuration, where $\mathbb{Z}$ is a standard normal random variable. This can be proved as in the proof of Theorem 1 of Lee, Song, and Whang (2012). Hence using the asymptotic validity of the bootstrap test, we find that

$$\begin{aligned} |\log p_0^*(i,j)| &= \left|\log\left(1 - \Phi(\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}/\sigma_{ij}^0)\right)\right| + o_P(1) \\ &\leq |\log(1 - \Phi(\mathbb{Z}))| + o_P(1). \end{aligned}$$

The inequality becomes an equality when we are under the least favorable configuration.

Suppose that $K \geq K_0$ and $P\{i, j \in \hat{N}_k\} \to 1$. Then $i, j \in N_k$, i.e., i and j belong to the same quality group. Hence by Proposition 1 of this paper, and $\delta_{ij}(b) = 0$ for all $b \in \Omega_i \cap \Omega_j$. Since we confine the integral domain to $\Omega_i \cap \Omega_j$, it follows that we are under the least favorable configuration. From the previous arguments, this yields the result that $|\log p_0^*(i,j)| = O_P(1)$.

(ii) Suppose that $K < K_0$. Then for some $k = 1, \cdots, K$, and for some $i, j \in N_k$, $\tau_{ij}^0 > 0$. By invoking the smoothness conditions for the winning probabilities and using the proof of Theorem 4 of Lee, Song, and Whang (2012), it can be shown that under the $\sqrt{L}$-converging local alternatives, there exists $c_{ij} > 0$, for any $m > 0$,

$$P\left\{\frac{\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}}{\sigma_{ij}^0} > m\right\} = P\left\{\mathbb{Z} + c_{ij} > m\right\} + o(1). \tag{18}$$

Using similar arguments, we can show that under the fixed local alternatives,

$$P\left\{\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}/\sigma_{ij}^0 > m\right\} = P\left\{\mathbb{Z} + \sqrt{L}c_{ij} > m\right\} + o(1).$$

Therefore, for any $M > 0$,

$$\begin{aligned} & P\left\{(1/g(L))\left|\log p_0^*(i,j)\right| > M\right\} \\ \geq\ & P\{(1/\sqrt{L})|\log(1 - \Phi(\sqrt{L}\{\hat{\tau}_{ij}^0 - \mathbf{E}\hat{\tau}_{ij}^0\}/\sigma_{ij}^0))| > Mg(L)/\sqrt{L}\} + o(1) \\ \geq\ & P\{(1/\sqrt{L})|\log(1 - \Phi(\mathbb{Z} + \sqrt{L}c_{ij}))| > Mg(L)/\sqrt{L}\} + o_P(1) \end{aligned}$$

as $L \to \infty$. Since $g(L)/\sqrt{L} \to 0$, it follows from (18) that the last probability converges to 1. ■

Proof of Theorem 2: For all $K > K_0$, we have

$$\begin{aligned} \hat{Q}(K_0) - \hat{Q}(K) &= \frac{1}{K_0}\sum_{k=1}^{K_0}\hat{V}_k(K_0) - \frac{1}{K}\sum_{k=1}^{K}\hat{V}_k(K) + (K_0 - K)g(L) \\ &\to -\infty, \end{aligned}$$

because $\frac{1}{K_0}\sum_{k=1}^{K_0}\hat{V}_k(K_0) - \frac{1}{K}\sum_{k=1}^{K}\hat{V}_k(K) = O_P(1)$ as $L \to \infty$.

And for all $K < K_0$, we have

$$\begin{aligned} \hat{Q}(K_0) - \hat{Q}(K) &= \frac{1}{K_0}\sum_{k=1}^{K_0}\hat{V}_k(K_0) - \frac{1}{K}\sum_{k=1}^{K}\hat{V}_k(K) + (K_0 - K)g(L) \\ &\to -\infty, \end{aligned}$$

because $\frac{1}{K}\sum_{k=1}^{K}\hat{V}_k(K) \to \infty$ faster than the rate $(K_0 - K)g(L) \to \infty$ as $L \to \infty$.

Therefore, $P\left\{\hat{Q}(K_0) - \hat{Q}(K) < 0\right\} \to 1$. Hence we find that $P\{\hat{K} = K_0\} \to 1$ as $L \to \infty$. ■

The Appendix on the Estimation of Structural Elements

For the purpose of the derivations below it is convenient to introduce mapping $\pi(\cdot; N, A) : \{1, \cdots, |N|\} \to N$. This mapping plays the following role. Sometimes we need to consider a scenario where a subset of potential bidders different from the one realized in the data would choose to participate in the auction. In considering such a case we would re-arrange the observations in such a way that the observations for this hypothetical set of actual bidders are listed first and the observations for the remaining potential bidders would be listed after them. The mapping $\pi_l(j; N, A)$ reflects the original (data set) position of the observation that would be listed in position j under this re-arrangement. In our analysis the order in which observations are listed within the set of entering or non-entering bidders is not important. Therefore, when re-arranging observations we do not consider all possible permutations (orderings) of the hypothetical set of actual bidders. Instead, we re-allocate them to the front of the vector without changing the order in which they were listed originally.

We use $\bar{N}^p_{x,q,l}$, $\bar{A}^p_{x,q,l}$, $\bar{N}^t_{x,l}, \bar{A}^t_{x,l}$ to denote the realizations of respective random sets as they are recorded in the data. Notice that, $\pi(j; \bar{N}^p_l, \bar{A}^p_l) = j$ and $\pi(j; \bar{N}^t_l, \bar{A}^t_l) = j$. For simplicity, we write $\pi^t_l(j) = \pi(j; A^t, N^t_l)$ and $\pi^p_l(j) = \pi(j; A^p, N^p_l)$ whenever it is clear which A and N sets are used.

Proof of Proposition 4: Notice that we consider the probability of a two-part event: (1) that a given vector of qualities characterizes a subset of potential bidders, (2) potential bidders characterized by these qualities enter.

First, as for $p_{k,l}$, note that by the Bayes rule, we can write

$$\begin{aligned} p_{k,l} &= P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k} | \mathbf{B}_l, \mathbf{I}_l\} = \frac{f(\mathbf{B}_l | \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_l) P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_l\}}{f(\mathbf{B}_l | \, \mathbf{I}_l)} \\ &= \frac{f(\mathbf{B}^t_l | \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_l) P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_l\}}{f(\mathbf{B}^t_l | \, \mathbf{I}_l)} = \frac{f(\mathbf{B}^t_l | \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \, \mathbf{I}_{l,1}) P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_l\}}{f(\mathbf{B}^t_l | \mathbf{I}_l)}. \end{aligned} \tag{19}$$

The first equality holds because the bids of permanent sellers are independent of bids of the transitory sellers and do not depend on the qualities of the transitory sellers, $f(\mathbf{B}^p_l | \, \mathbf{I}_l) = f(\mathbf{B}^p_l | \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_l)$.

We denote terms in this expression by

$$(I) = f(\mathbf{B}^t_l | \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_{l,1}), \; (II) = f(\mathbf{B}^t_l | \, \mathbf{I}_{l,1}), \; (III) = P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_l\}.$$

Next, we work with these terms one by one.

Term (I) Notice that $\mathbf{B}^t_l$ are independent conditional on $\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}$, and $\mathbf{I}_{l,1}$, therefore

$$(I) = \prod_{x \in \mathcal{X}} \prod_{j \in A^t_{x,l}} f(\mathbf{B}^t_{j,l} | \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_{l,1}) = \prod_{x \in \mathcal{X}} \prod_{j \in A^t_{x,l}} f(\mathbf{B}^t_{j,l} | \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1}). \tag{20}$$

The last equality holds because the transitory seller knows his quality but not the quality of his transitory competitors.

Term (II) Applying the rule of total probability we obtain

$$(II) = \sum_{d=1}^{\bar{K}_A} f(\mathbf{B}_l^t | \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d}, \mathbf{I}_l) P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d} | \, \mathbf{I}_l\} \tag{21}$$

$$\sum_{d=1}^{\bar{K}_A} f(\mathbf{B}_l^t | \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d}, \mathbf{I}_{l,1}) P\{\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d} | \, \mathbf{I}_l\}. \tag{22}$$

We will return to this expression after we tackle term (III).

Term (III) Our goal here is to relate an event in (III) to transitory bidders' participation (entry) decisions, and to express (III) in terms of the participation probabilities of the transitory bidders. First, we consider

$$(III) = P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_{l,1}, \mathbf{I}_{l,2} = \bar{I}_{l,2}),$$

where $\bar{I}_{l,2} = (m_{x,q}^p, m_x^t : x \in \mathcal{X} \text{ and } q \in \mathbb{Q}_x)$. Then observe that this conditional probability is equal to

$$\begin{aligned} P(\mathbf{Q}_{A,l}^t &= \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_{l,1}, \, |A_{x,q,l}^p| = m_{x,q}^p, \, |A_{x,l}^t| = m_x^t \text{ for all } x \text{ and } q) \\ &= \frac{P(|A_{x,q,l}^p| = m_{x,q}^p, \, |A_{x,l}^t| = m_x^t \text{ for all } x \text{ and } q | \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \, \mathbf{I}_{l,1}) P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \mathbf{I}_{l,1})}{P(|A_{x,q,l}^p| = m_{x,q}^p, \, |A_{x,l}^t| = m_x^t \text{ for all } x \text{ and } q | \, \mathbf{I}_{l,1})} \\ &= \frac{P(|A_{x,l}^t| = m_x^t \text{ for all } x, \, \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_{l,1})}{\sum_{d=1}^{\bar{K}_A} P(|A_{x,l}^t| = m_x^t \text{ for all } x, \, \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,d} | \, \mathbf{I}_{l,1})}. \end{aligned} \tag{23}$$

The second equality holds because the events $|A_{x,q,l}^p| = m_{x,q}^p,$ for all $(x, \, q)$ and $|A_{x,l}^t| = m_x^t,$ for all x are independent conditional on $\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \, \mathbf{I}_{l,1}$, and the event $|A_{x,q,l}^p| = m_{x,q}^p,$ for all $(x, \, q)$ is independent of $\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}$ conditional on $\mathbf{I}_{l,1}$. We next work on the expression $P(|A_{x,l}^t| = m_x^t \text{ for all } x, \, \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \bar{\mathbf{x}}_{A,l}^t, \mathbf{I}_{l,1})$ in the numerator of equation (23). We then return to equations (23) and (19) to conclude our derivation. We let $\mathbf{Q}_{N,l}^t = (Q_{N,j,l}^t)_{j \in N_l^t}$ and $\mathbb{Q}_l^N$ be the set of values that $\mathbf{Q}_{N,l}^t$ takes. Then

$$\begin{aligned} &P(|A_{x,l}^t| = m_x^t \text{ for all } x, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_{l,1}) = \\ &= \sum_{\tilde{\mathbf{q}} \in \mathbb{Q}_l^N} P(|A_{x,l}^t| = m_x^t \text{ for all } x, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}, \, \mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}} | \, \bar{\mathbf{x}}_{A,l}^t, \mathbf{I}_{l,1}) \\ &= \sum_{\tilde{\mathbf{q}} \in \mathbb{Q}_l^N} \prod_{x \in \mathcal{X}} P\left\{|A_{x,l}^t| = m_x^t, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \, \mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}}, \, \mathbf{I}_{l,1}\right\} P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}} | \, \mathbf{I}_{l,1}). \end{aligned} \tag{24}$$

Further notice that $P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}} | \, \bar{\mathbf{x}}_l^t, \, \mathbf{I}_{l,1}) = P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}} | \, \bar{\mathbf{x}}_l^t) = \prod_{j \in N_l^t} P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j | \, \bar{\mathbf{x}}_{j,l}^t)$. The probability $P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j | \, \bar{\mathbf{x}}_{j,l}^t)$ is primitive in our environment, which characterizes the distribution of sellers' qualities within $x-$cell. We now show how the expression for $P\left\{|A_{x,l}^t| = m_x^t, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \, \mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}}, \, \mathbf{I}_{l,1}\right\}$ can be modified and then return to equation (24). Recall that $\pi_l^t(j)$ links elements from some set $\Omega_x \subset \{1, ..., |N_l^t|\}$ to a vector $\{1,...,|A_l^t|\}$. Then for a given $\bar{\mathbf{q}}_{A,k}$ and $\tilde{\mathbf{q}}$ we obtain

$$\sum_{\Omega_x \subset \bar{N}_{x,l}^t} P\left\{\begin{array}{c} j \in A_{x,l}^t,\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\ \text{for all } \pi_l^t(j) \in \Omega_x \text{ and} \\ j \in N_{x,l}^t - A_{x,l}^t \text{ for all } \pi_l^t(j) \in N_{x,l}^t - \Omega_x \end{array} |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\, \bar{\mathbf{x}}_l^t,\, \mathbf{I}_{l,1}\right\}$$

$$= \sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j)\in\Omega_x} P\left\{j \in A_{x,l}^t,\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k} |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\, \bar{\mathbf{x}}_l^t,\, \mathbf{I}_{l,1}\right\} \times$$
$$\prod_{\pi_l^t(i)\in\bar{N}_{x,l}^t-\Omega_x} P\left\{i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\, \bar{\mathbf{x}}_l^t,\, \mathbf{I}_{l,1}\right\},$$

where the sum over all sets Ω_x that are consistent with the restrictions imposed on the set of entrants, i.e, $\Omega_x \subset \bar{N}_{x,l}^t$ such that $|\Omega_x| = m_x^t$, $\tilde{q}_{\pi_l^t(j)} = \bar{\mathbf{q}}_{A,j,k}$ for all j such that $\pi_l^t(j) \in \Omega_x$. Next,

$$= \sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j)\in\Omega_x} P\left\{j \in A_{x,l}^t,\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k} |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\, \mathbf{I}_{l,1}\right\} \times$$
$$\prod_{\pi_l^t(i)\in\bar{N}_{x,l}^t-\Omega_x} P\left\{i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,s,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(s)} \text{ for all } s \in N_l^t,\, \mathbf{I}_{l,1}\right\}$$
$$= \sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j)\in\Omega_x} P\left\{j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\} \times$$
$$\prod_{\pi_l^t(i)\in\bar{N}_{x,l}^t-\Omega_x} P\left\{i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\, \mathbf{I}_{l,1}\right\}$$

Notice that for every Ω_x the set of qualities within Ω_x and $N_{x,l}^t - \Omega_x$ is the same. Therefore, the expression above can be written

$$\sum_{\Omega_x \subset \bar{N}_{x,l}^t} \prod_{\pi_l^t(j)\in\Omega_x} P\left\{j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1}\right\} \times \tag{25}$$
$$\prod_{\pi_l^t(i)\in\bar{N}_{x,l}^t-\Omega_x} P\left\{i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\, \mathbf{I}_{l,1}\right\}$$
$$= |\Omega^x| \prod_{\pi_l^t(j)\in\Omega_x^0} P\left\{j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \bar{\mathbf{x}}_{\pi_l^t(j),l}^t,\, \mathbf{I}_{l,1}\right\}$$
$$\prod_{\pi_l^t(i)\in\bar{N}_{x,l}^t-\Omega_x^0} P\left\{i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\, \mathbf{I}_{l,1}\right\}$$

Here, $|\Omega^x|$ denotes the cardinality of set $\Omega^x = \{\Omega_x : \Omega_x \subset \bar{N}_{x,l}^t$, such that $|\Omega_x| = m_x^t$ and $\tilde{q}_{\pi_l^t(j)} = \bar{\mathbf{q}}_{A,j,k}$, for all $\pi_l^t(j) \in \Omega_x\}$, with Ω_x^0 representing one specific member of Ω^x. For example, we can set $\Omega_x^0 = \bar{A}_{x,l}^t$.

Returning with expression (25) to equation (24) obtains

$$
\begin{aligned}
&\sum_{\tilde{\mathbf{q}}\in\mathbb{Q}_l^N} P(|A_{x,l}^t| = m_x^t \text{ for all } x, \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} |\, \mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}},\, \mathbf{I}_{l,1}) P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}} |\, \mathbf{I}_{l,1}) \\
= &\sum_{\tilde{\mathbf{q}}\in\mathbb{Q}_l^N} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{\pi_l^t(j)\in\Omega_x^0} P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1} \right\} \times \\
&\prod_{\pi_l^t(i)\in\bar{N}_{x,l}^t - \Omega_x^0} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\, \mathbf{I}_{l,1} \right\} P(\mathbf{Q}_{N,l}^t = \tilde{\mathbf{q}} |\, \mathbf{I}_{l,1}) \\
= &\sum_{\tilde{\mathbf{q}}\in\mathbb{Q}_l^N} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{j\in\bar{A}_{x,l}^t} P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1} \right\} \times \\
&\prod_{\pi_l^t(i)\in\bar{N}_{x,l}^t - \bar{A}_{x,l}^t} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_{\pi_l^t(i)},\, \mathbf{I}_{l,1} \right\} \prod_{j\in N_l^t} P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j |\, \bar{\mathbf{x}}_{j,l}^t)
\end{aligned}
$$

Note that in the last summation over $\tilde{\mathbf{q}} \in \mathbb{Q}_l^N$, part of the vectors in $\tilde{\mathbf{q}}$ such that $(\tilde{q}_j)_{j\in\bar{A}_l^t}$, and hence the summation is essentially over vectors in $\mathbb{Q}_l^{N-\bar{A}_l^t}$ which is the set of values for $\mathbf{Q}_{N-\bar{A}_l^t}^t \equiv (Q_{N_l^t-\bar{A}_l^t,j,l}^t)_{j\in N_l^t-\bar{A}_l^t}$. Thus we write the last sum as

$$
\begin{aligned}
&\sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{j\in\bar{A}_{x,l}^t} P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1} \right\} \prod_{j\in\bar{A}_l^t} P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j |\, \bar{\mathbf{x}}_{j,l}^t) \qquad (26) \\
&\times \prod_{i\in\bar{N}_{x,l}^t-\bar{A}_{x,l}^t} P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1} \right\} \prod_{j\in\bar{N}_l^t-\bar{A}_l^t} P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j |\, \bar{\mathbf{x}}_{j,l}^t) \\
= &\prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}_{x,l}^t} \left\{ P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1} \right\} P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j |\, \bar{\mathbf{x}}_{j,l}^t) \right\} \\
&\times \sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{i\in\bar{N}_{x,l}^t-\bar{A}_{x,l}^t} \left\{ P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1} \right\} P(\mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i |\, \bar{\mathbf{x}}_{i,l}^t) \right\}.
\end{aligned}
$$

The expression in (26) is derived for an arbitrary $\bar{\mathbf{q}}_{\mathbf{A},\mathbf{k}}$. Therefore, we substitute it into both the numerator and the denominator of (23). Notice that the dimensionalities of actual and potential bidders' x-sets are the same in the numerator and the denominator and that is why both expressions contain the common factor:

$$
\sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} \prod_{x\in\mathcal{X}} |\Omega^x| \prod_{i\in\bar{N}_{x,l}^t-\bar{A}_{x,l}^t} \left\{ P\left\{ i \in N_{x,l}^t - A_{x,l}^t |\, \mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1} \right\} P(\mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i |\, \bar{\mathbf{x}}_{i,l}^t) \right\}
$$

Therefore, after canceling out this factor, the expression in (23) transforms into

$$
P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k} | \mathbf{I}_{l,1}, \mathbf{I}_{l,2} = \bar{I}_{l,2}) = \frac{\prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}_{x,l}^t} \left\{ P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1} \right\} P(\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k} |\, \bar{\mathbf{x}}_{j,l}^t) \right\}}{\sum_{d=1}^{\bar{K}_l} \prod_{x\in\mathcal{X}} \prod_{j\in\bar{A}_{x,l}^t} \left\{ P\left\{ j \in A_{x,l}^t |\, \mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,d},\, \mathbf{I}_{l,1} \right\} P(\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,d} |\, \bar{\mathbf{x}}_{j,l}^t) \right\}}.
$$

Having obtained an expression for (III), we now return to (II).

Denote $\omega^t_{A,k,l} = \prod_{j \in \bar{A}^t_l} P(\mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k} | \, \bar{\mathbf{x}}^t_{j,l})$.

$$(II) = \sum_{k=1}^{\bar{K}_A} f(\mathbf{B}^t_l | \mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k}, \mathbf{I}_{l,1}) P\{\mathbf{Q}^t_{A,l} = \bar{\mathbf{q}}_{A,k} | \, \mathbf{I}_l\} = \tag{27}$$
$$\sum_{k=1}^{\bar{K}_A} \frac{\omega^t_{A,k,l} \prod_{x \in \mathcal{X}} \prod_{j \in \bar{A}^t_{x,l}} f(\mathbf{B}_j | \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1}) P\left\{ j \in A^t_{x,l} | \, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}, \, \mathbf{I}_{l,1} \right\}}{\sum_{d=1}^{\bar{K}_l} \omega^t_{A,d,l} \prod_{x \in \mathcal{X}} \prod_{j \in \bar{A}^t_{x,l}} P\left\{ j \in A^t_{x,l} | \, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d}, \, \mathbf{I}_{l,1} \right\}}$$

Finally, combining (I), (II), and (III) obtains $p_{k,l} =$

$$\frac{\omega^t_{A,k,l} \prod_{x \in \mathcal{X}} \prod_{j \in \bar{A}^t_{x,l}} f(\mathbf{B}_j | \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1}) P\left\{ j \in A^t_{x,l} | \, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,k}, \, \mathbf{I}_{l,1} \right\}}{\sum_{d=1}^{\bar{K}_A} \omega^t_{A,d,l} \prod_{x \in \mathcal{X}} \prod_{j \in \bar{A}^t_{x,l}} f(\mathbf{B}_j | \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d}, \mathbf{I}_{l,1}) P\left\{ j \in A^t_{x,l} | \, \mathbf{Q}^t_{A,j,l} = \bar{\mathbf{q}}_{A,j,d}, \, \mathbf{I}_{l,1} \right\}}.$$

Supplemental Appendix

Nonparametric Identification

Buyers' Tastes for Observed Characteristics

To see how the distribution of β (buyers' tastes for observed characteristics) can be recovered, consider an auction involving $M+1$ permanent sellers, who all bid below the reserve price. Let their quality levels be identified. Then

$$\begin{aligned} & P\left(i \text{ wins } | A = A_Q = a, \, (b_i)_{i \in a}\right) \\ = \; & P\left(\Delta\epsilon_{ji} + \alpha\Delta q_{ji} + \Delta x_{ji}\beta \leq \Delta b_{ji} \; \forall j \in a \backslash \{i\}\right) \end{aligned}$$

where $\#\{a\} = M+1$. Provided the joint support of $(\Delta B_{ji})_{j \in a \backslash i}$ is large enough to cover the joint support of $(\Delta\epsilon_{ji} + \alpha\Delta q_{ji} + \Delta x_{ji}\beta)_{j \in a \backslash i}$, we can recover the distribution of the later. With F_{ϵ_i} and F_α identified as above, and under our assumption that $(\epsilon_i)_{i \in a}$ are i.i.d. and jointly independent from α, the distribution of $\Delta\epsilon_{ji} + \alpha\Delta q_{ji}$ is known. With β assumed to be independent from α and $(\epsilon_i)_{i \in N}$, we can identify the distribution of $(\Delta x_{ji}\beta)_{j \in a \backslash i}$ by taking the ratio of the characteristic functions of $(\Delta\epsilon_{ji} + \alpha\Delta q_{ji} + \Delta x_{ji}\beta)_{j \in a \backslash i}$ and $(\Delta\epsilon_{ji} + \alpha\Delta q_{ji})_{j \in a \backslash i}$. As long as the matrix $(\Delta x_{ji})_{j \in a \backslash i}$ is full-rank, then the joint density of β is identified as the product of the joint density of $(\Delta x_{ji}\beta)_{j \in a \backslash i}$ and the absolute value of the determinant of the square matrix $(\Delta x_{ji})_{j \in a \backslash i}$ under the standard change-of-variable techniques. We formalize this idea for the rest of this subsection.

(*A8*) For some i and $a \subseteq N^p$ with $i \in a$ and $\#\{a\} = M+1$ such that (i) $(\Delta x_{ji})_{j \in a \backslash i}$ has a full rank; (ii) the support of $(\Delta\epsilon_{ji} + \alpha\Delta q_{ji} + \Delta x_{ji}\beta)_{j \in a \backslash i}$ is contained in the support of $(\Delta B_{ji})_{j \in a \backslash i}$; and (iii) $(\Delta\epsilon_{ji} + \alpha\Delta q_{ji})_{j \in a \backslash i}$ have non-vanishing characteristic functions.

Both (i) and (iii) in A8 are mild conditions. Sufficient conditions for (ii) in A8 can be derived using arguments similar to those in Section 4.4.

Proposition 5 *Suppose the conditions A1-3,A4',A6-8 hold. Then the distribution of β is identified.*

Distribution of Private Project Costs

In this section we discuss how the distribution of project's costs can be identified. We consider a simple case when bidders's entry costs are independent of the project's costs. The general result obtains by combining steps presented below with the identification strategy proposed by Li and Gentry (2011).

The identification of the distribution of project's costs[32] in a simple case of signals independence follows an argument similar to Guerre, Perrigne, and Vuong (2000). To see this, note that the quality levels for permanent sellers, the distribution of quality levels of transitory sellers, and buyer tastes can be considered known since they are identified in preceding sub-sections.

The inverse bidding strategy can be recovered as follows. The first-order condition for bidder i choosing price b_0 in equilibrium is:

$$\begin{aligned} & (b_0 - c_i)\, \tfrac{\partial}{\partial b} P\left\{ \max\nolimits_{j \in A_Q \backslash i} \left(\alpha Q_j - \alpha q_i + \Delta x_{ji}\beta + \Delta\epsilon_{ji} - B_j \right) \leq -b \right\} |_{b=b_0} \\ = \;& P\left\{ \max\nolimits_{j \in A_Q \backslash i} \left(\alpha Q_j - \alpha q_i + \Delta x_{ji}\beta + \Delta\epsilon_{ji} - B_j \right) \leq -b_0 \right\}. \end{aligned} \tag{28}$$

Thus, it suffices to show that the distribution on the right-hand side can be identified. It would imply that the derivative on the left-hand side would also be identified, in which case the inverse bidding strategy (and consequently the distribution of private cost c_i) would also be recovered for every (x, q) group of sellers. Note the right-hand side is

$$\sum\nolimits_{\{a \subseteq N\}} P\left\{ \max\nolimits_{j \in a \backslash i} \left(\alpha Q_j - \alpha q_i + \Delta x_{ji}\beta + \Delta\epsilon_{ji} - B_j \right) \leq -b_i \middle|\ A_Q = a \right\} P\left\{ A_Q = a \right\}.$$

For those sellers that are in $(A_Q \backslash i) \cap N^t$, Q is a multinomial random variable whose distribution is recovered due to (A5). With seller qualification ($1\{B_i \leq R\}$) observed in data, $P\{A_Q = a\}$ in a symmetric PSBNE can be recovered. Next, note with a fixed, the distribution of $\max_{j \in a \backslash i} \left(\alpha Q_j - \alpha q_i + \Delta x_{ji}\beta + \Delta\epsilon_{ji} - B_j \right)$ can be recovered since $F_\alpha, F_{\epsilon_i}, F_\beta$ and the distribution of B_j are identified.

Further Discussion of Support Conditions

To better illustrate how support conditions in A7 can be satisfied, we focus on a simpler case with $A = \{i, j, l\}$, where i, j, l have identical observed characteristics in x; both i, j are permanent with $q_i > q_j$ and $q_i = q^1$ (highest

[32]The identification of the distribution of entry costs is standard. It can be found, for example, in **?**.

level possible); and l is transitory. Recall that, in this case, Q_l is a random variable correlated with B_l. In this simple case, the support condition we need is that the support of $(B_i - B_j, B_i - B_l)$ conditional on $B_l = b_l$ covers the unconditional support of $(\alpha\Delta q_{ij} + \Delta\epsilon_{ij}\ ,\ \alpha\Delta q_{i,l} + \Delta\epsilon_{i,l})$. Let Q_L denote the lowest quality level possible, and let $\bar{\alpha}$ denote the supremum of the support of α.

Remark 2 *Suppose $\bar{c}_m = \bar{c}$ for all $m \in N$ and $\bar{r} = \bar{c}$. If for some $i, j \in N^p$ with $q_i = q^1$ and $q_i > q_j$.*

$$\begin{aligned} &\underline{b}_j < \underline{b}_i\ ;\ \bar{b}_i - \underline{b}_i > 2(\bar{\varepsilon} - \underline{\varepsilon}) + \bar{\alpha}\Delta q_{iL}\ ; \\ &\bar{b}_j - \underline{b}_j > 2(\bar{\varepsilon} - \underline{\varepsilon}) + \bar{\alpha}(\Delta q_{iL} + \Delta q_{ij}) \end{aligned} \tag{29}$$

and there exists l with $x_i = x_j = x_l$, then (A7) holds.

It is not hard to visualize the restrictions posed by A7, and why the conditions in Remark 2 are sufficient for A7. Figure 4(a) depicts the joint support of $(B_i - B_j, B_i - B_l)$ conditional on $B_l = b_l$. Figure 4(b) gives the joint support of $(\alpha\Delta q_{i,j}, \alpha\Delta q_{i,l})$, and Figure 4(c) gives a superset of the joint support of $(\alpha\Delta q_{i,j} + \Delta\epsilon_{i,j}$, $\alpha\Delta q_{i,l} + \Delta\epsilon_{i,l})$. Note because ϵ and α, C are independent, Figure 4(c) is simply the addition of Figure 4(b) and Figure 1. For A7 to hold, we need the shape in Figure 4(c) to be covered by the support in Figure 4(a). By comparing the coordinates of the vertices in these two figures, we can derive the following set of sufficient conditions for A7:

$$\begin{aligned} &\underline{b}_i - b_l < \underline{\varepsilon} - \bar{\varepsilon}\ ;\ b_l - \bar{b}_j < \underline{\varepsilon} - \bar{\varepsilon}\ ;\ \bar{\varepsilon} - \underline{\varepsilon} + \bar{\alpha}\Delta q_{i,L} < \bar{b}_i - b_l; \\ &\bar{\varepsilon} - \underline{\varepsilon} + \bar{\alpha}\Delta q_{i,j} < b_l - \underline{b}_j\ ;\ \underline{\varepsilon} - \bar{\varepsilon} > \bar{\alpha}\Delta q_{i,j} + \underline{b}_j - b_l\ ;\ \bar{b}_i - \bar{b}_j < \bar{\alpha}\Delta q_{i,j}; \\ &\bar{\alpha}\Delta q_{i,L} < \underline{\varepsilon} - \bar{\varepsilon} + \bar{\alpha}\Delta q_{i,j} + \bar{b}_i - b_l. \end{aligned}$$

For this system of inequalities to hold for some b_l with $q^l < q^1$, it is sufficient to have the system of inequalities in Remark 2 above.

To get the intuition of how the condition in Remark 2 can hold, consider a simplified model with two types of sellers and without ϵ. (We have learned from the previous paragraph that the support of bids can be large relative to that of ϵ. Therefore adding ϵ into the model will not pose addtional problems for showing the support conditions.) For now, consider a model in which all potential sellers are permanent.

The basic argument in this case is that we need the support of bids for the permanent high-quality seller i to be sufficiently large in a hypothetical model where the permanent potential seller j and other sellers with lower qualities have been removed from the set of potential sellers. Denote the support by $[\underline{b}_i^0, \bar{b}_i^0]$. Now consider the model where lower quality sellers remain among the potential sellers. This support will be shortened as high-quality sellers take into account the positive probability that after entry decisions, they may find themselves competing with low-quality sellers only. Call this new support $[\underline{b}_i, \bar{b}_i]$. Due to the existence of secret reserve prices (whose upper bound of support is $\bar{c}$), it can be shown that the upper bound of the support of the bids must be identical in both cases, i.e., $\bar{b}_i^0 = b_i^0 = \bar{c}$. The lower bound of the support is increased but the magnitude of increment must be bounded between $\underline{\alpha}\Delta q_{i,j}$ and $\bar{\alpha}\Delta q_{i,j}$. Thus, as long as $[\underline{b}_i^0, \bar{b}_i^0]$ is longer than $\bar{\alpha}\Delta q_{i,j} + \bar{\alpha}\Delta q_{H,L}$,

Figure 4: Support Conditions

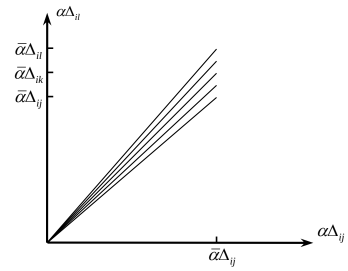


Figure.4a

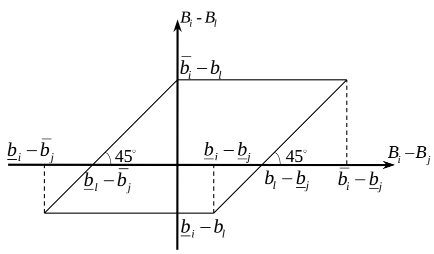


Figure.3

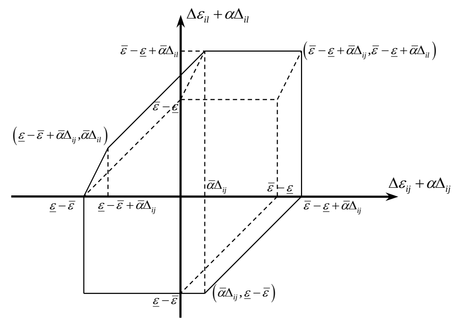


Figure.4b

then the support $[\underline{b}_i, \bar{b}_i]$ will satisfy the first inequality in (29). That the second inequality in (29) can hold can be shown using similar heuristic arguments. In the more general model involving transitory sellers, this intuitive argument still applies, with the only difference being that seller i's strategies in both cases (with permanent lower-quality sellers removed or included among the set of potential sellers) now also depend on the quality of other permanent sellers and the identity of the remaining transitory sellers.

Nonparametric Estimation of Quality Group Structure

Confidence Set for Classification

Suppose that we have K_0 groups. Since we can estimate K_0 consistently, we assume we know it. We fix $k = 1, \cdots, K_0$ and construct a confidence set for the k-th quality group. In other words, we would like to construct a random set $\hat{C}_k \subset N$ such that

$$\text{liminf}_{L\to\infty} P\{N_k \subset \hat{C}_k\} \geq 1 - \alpha.$$

We need to devise a way to approximate the finite sample probabilities like $P\{N_k \subset \hat{C}^k\}$. Since we do not know the cross-sectional dependence structure among the sellers, we use a bootstrap procedure that preserves this dependence structure from the original sample. We first estimate $\hat{N}_{k,\lambda}$ as prescribed above and also obtain $\hat{\tau}^0_{ij}$. Given the estimate $\hat{N}_{k,\lambda}$, we construct a sequence of sets as follows:

Step 1: Find $i_1 \in N \backslash \hat{N}_k$ that minimizes $\min_{j \in \hat{N}_k} \hat{\tau}^0_{i_1,j}$, and construct $\hat{C}_k(1) = \hat{N}_k \cup \{i_1\}$.

Step 2: Find $i_2 \in N \backslash \hat{C}_k(1)$ that minimizes $\min_{j \in \hat{C}_k(1)} \hat{\tau}^0_{i_2,j}$, and construct $\hat{C}_k(2) = \hat{C}_k(1) \cup \{i_2\}$.

Step m: Find $i_m \in N \backslash \hat{C}_k(m-1)$ that minimizes $\min_{j \in \hat{C}_k(m-1)} \hat{\tau}^0_{i_m,j}$ and construct $\hat{C}_k(m) = \hat{C}_k(m-1) \cup \{i_m\}$.

Repeat Step m up to $|N|$.

Now, for each bootstrap iteration $s = 1, \cdots, B$, we construct the sets $\hat{N}^*_{k,s}$ and $\{\hat{C}^*_{k,s}(m)\}$ following the steps described above but using the bootstrap sample. (Note that this bootstrap sample is independent of the bootstrap sample used to construct $p^*_0(i,j)$.)

Then, we compute the following:

$$\hat{\pi}^k(m) \equiv \frac{1}{B} \sum_{s=1}^{B} 1 \left\{ \hat{N}_k \subset \hat{C}^*_{k,s}(m) \right\}.$$

Note that the sequence of sets $\hat{C}^*_{k,s}(m)$ increases in m. Hence the number $\hat{\pi}^k(m)$ should also increase in m. An $(1-\alpha)\%$ level confidence set is given by $\hat{C}_k(m)$ with m, such that

$$\hat{\pi}^k(m-1) < 1 - \alpha \leq \hat{\pi}^k(m).$$

If there exists no such $m \leq m_0$ that satisfies this inequality, then set $m = m_0$.

Monte Carlo Analysis

Here we explore properties of our classification algorithm in simulation analysis.

We choose the distributions of project and entry costs to be the same across quality levels and given by truncated normals with $N(1.5, 0.2^2)$ and $N(0.08, 0.02^2)$ correspondingly. Further, we assume that the distribution of the reserve price coincides with the bid distribution of high-quality bidders. The bidders are assumed to be heterogeneous with respect to quality only. Buyers' tastes, therefore, are represented by the distributions of α and ϵ. We fix the distribution of α to be truncated normal $N(0.4, 0.2^2)$ with support $[0, 1]$. The distribution of ϵ is also chosen to be truncated normal with mean 0 and variance σ^2_ϵ. We vary σ_ϵ in experiments below to explore the sensitivity of our methodology to the noise in buyers' tastes. We truncate the support of epsilon at $[-\sigma_\epsilon, \sigma_\epsilon]$. Finally, we assume that the set of suppliers consists of 30 programmers and is split equally between high- and low-quality suppliers.

We use the modified projection algorithm from Paarsch, Hubbard (2009) and Bajari (2000) to solve for participation and bidding strategies of our game. The data are generated through repetition of the following steps:

1. At each round, 10 randomly selected bidders from the set above are declared to be potential bidders.
2. For each potential bidder we draw an entry and project cost. We, then, apply participation and bidding strategies to these draws to determine whether a potential bidder enters the set of active participants and if he does what bids he submits.
3. Next, we take draws from the distributions of α, ϵ, and of the reserve price. The winner of the project is determined by evaluating submitted bids using the reserve price and buyer's tastes.
4. The data record the set of potential bidders with their qualities, outcomes of participation and entry decisions as well as the buyer's choice.

Table 8: Results of Simulation Study

		Probability of Correct Classification					
		number of bids=300		number of bids=200		number of bids=100	
d_Q	σ_ϵ	Q_H	Q_L	Q_H	Q_L	Q_H	Q_L
0.3	$0.2\sigma_c$	0.9773	0.9901	0.9613	0.9547	0.9314	0.9013
0.3	$0.5\sigma_c$	0.9645	0.9858	0.9477	0.9512	0.9223	0.8998
0.3	$1.5\sigma_c$	0.9619	0.9782	0.9457	0.9401	0.9207	0.8941
0.1	$0.2\sigma_c$	0.9632	0.9774	0.9329	0.9503	0.9164	0.8904
0.1	$0.5\sigma_c$	0.9551	0.9743	0.9263	0.9421	0.9034	0.8815
0.1	$1.5\sigma_c$	0.9518	0.9701	0.9227	0.9397	0.8927	0.8623

This table reports results of the simulation study of the sensitivity of the classification procedure to the quality differences, the magnitude of the preference noise, and the data set size. The latter is measured in the average number of bids per supplier. The difference in quality levels is measured relative to the project costs spread, i.e., $d_Q = \frac{Q_H - Q_L}{\bar{c} - \underline{c}}$. The variance of the preference noise is measured relative to the project cost variance, i.e., $\sigma_\epsilon = d_\epsilon \sigma_c$.

We use the simulated data to investigate the sensitivity of our methodology to the magnitude of the quality differences, the noise in buyer's preferences, and the number of available observations. For the first two experiments we tie the quality differences and the noise magnitude to the variance in the private project costs. That is, we consider (a) high-quality differences with $\Delta Q = 0.3(\bar{c} - \underline{c})$ and (b) low-quality differences with $\Delta Q = 0.3(\bar{c} - \underline{c})$. Similarly, we consider (c) low preference noise with $\sigma_\epsilon = 0.2\sigma_c$, (d) medium preference noise with $\sigma_\epsilon = 0.5\sigma_c$, (e) high preference noise with $\sigma_\epsilon = 1.2\sigma_c$. Finally, we explore how the performance of our procedure changes with sample size. Our procedure is performed at the individual level, therefore, we explore the performance of our procedure as a function of the average number of bids per supplier.

We run the simulation experiments as follows. For every set of parameters, we apply our procedure to 500 data sets simulated according to steps (1)-(4) described above. We then compute for every supplier the fraction of the data sets in which his type was correctly recovered. We report the average of these fractions across bidders of the same quality level in table 8.

The results of the simulation analysis show that the classification procedure performs quite well.[33] In particular, it is not very sensitive to the magnitude of the preference noise. We would expect the preference noise to impede recovery of the quality level since it disguises the link between the probability of winning and the quality of participant. It would be natural to expect that the procedure should impose higher data requirements in the presence of more noise. However, the endogeneity of prices successfully compensates for the noise in buyers' preferences at least for moderate levels of noise. As the magnitude of the noise grows, bidding functions become flatter, thus ensuring that more observations fall in the neighborhood of a specific price level.

As expected, the performance of the procedure does depend on the importance of the quality differences. The estimation is more precise when quality differences are large and grows less precise as quality differences diminish. Finally, the procedure is sensitive to the size of the data set. As the number of bids drops from 300 bids per supplier to 100 bids per supplier the probability of correct classification drops from 0.96 to 0.92 for high-quality suppliers and from 0.98 to 0.89 for low-quality suppliers. The classification of low-quality suppliers is affected to a larger degree since due to the lower probability of participation, the number of bids they submit is substantially below the average.

Extension: Semiparametric Estimation

The estimation method in the previous section employs parametrization of nonparametric functions g_k. The functions g_k involve the density of the transitory sellers' bids and participation probabilities. Since the bids and participations are equilibrium objects, one might prefer to use a more flexible specification for the nonparametric functions g_k. In this section, we explain how this extension can be done in practice.

Recall our definition $e^p_{x,q,l}(\theta, g)$ in (13) and define

$$\hat{\rho}_{x,q,s}(\theta, g) = \mathbf{h}^p_{x,q,1}(\mathbf{B}^p_l, \mathbf{I}_l) \left(W^p_{x,q,l} - \hat{e}^p_{x,q,l}(\theta, g) \right),$$

making its dependence on the nonparametric function g explicit. Let

$$\hat{\mathbf{m}}_{x,q,l}(\theta, g) = \frac{\sum_{s=1}^{L} \hat{\rho}_{x,q,s}(\theta, g) 1\{\mathbf{I}_s = \mathbf{I}_l\}}{\sum_{s=1}^{L} 1\{\mathbf{I}_s = \mathbf{I}_l\}}$$

and define $\hat{\mathbf{m}}_l(\theta, g)$ to be a column vector with $\hat{\mathbf{m}}_{x,q,l}(\theta, g)$ stacked up with (x, q) running in $\mathcal{X} \times \mathbb{Q}_1$. Hence the dimension of $\hat{\mathbf{m}}_l(\theta, g)$ is $d_{h,p} \times |\mathcal{X} \times \mathbb{Q}_1|$, where $|\mathcal{X} \times \mathbb{Q}_1|$ is the cardinality of the set $\mathcal{X} \times \mathbb{Q}_1$.

In the semiparametric estimation, we regard the nonparametric function g as an infinite dimensional nuisance

[33] The procedure performs best when prices are scaled to lie in the [0,1] interval.

parameter, and employ the sieve minimum distance estimation method of Ai and Chen (2003). As argued previously, the functions g are nonparametrically identified under our set-up. Now let $\mathcal{G}_L$ be the space of finite dimensional sieves whose dimension increases as the number of auctions L grows. Details about the appropriate sieves are found in the appendix. We construct the estimator as follows:

$$(\hat{\theta}_{CMD}, \hat{g}) = \text{argmin}_{(\theta,g)\in\Theta\times\mathcal{G}_L} \hat{\mathbf{Q}}_{CMD}(\theta, g),$$

where

$$\hat{\mathbf{Q}}_{CMD}(\theta, g) = \frac{1}{L}\sum_{l=1}^{L} \hat{\mathbf{m}}_l(\theta, g)\hat{\Sigma}_l^{-1}\hat{\mathbf{m}}_l(\theta, g),$$

and $\hat{\Sigma}_l$ is a consistent estimator of a nonsingular matrix Σ_l. Ai and Chen (2003) showed that under regularity conditions, we have

$$\sqrt{n}\left(\hat{\theta}_{CMD} - \theta_0\right) \to_d N(0, V),$$

where V is a nonsingular covariance matrix.

We turn to the form of the covariance matrix formula which follows Ai and Chen (2003). For each $i = 1, \cdots, N$, we let for $\alpha = (\theta, g) \in \mathcal{A}_L = \Theta \times \mathcal{G}_L$,

$$\begin{aligned}
\mathbf{r}^{(1)}_{x,q,l}(\alpha) &= \mathbf{E}\left[\mathbf{h}^p_{x,q,1}(\mathbf{B}_l, \mathbf{I}_l)\{W^p_{x,q,l} - e^p_{x,q,l}(\alpha)\}|\mathbf{I}_l\right] \\
\mathbf{r}^{(2)}_{x,q,l}(\alpha) &= \mathbf{E}\left[\mathbf{h}^t_{x,q,2}(\mathbf{B}_l, \mathbf{I}_l)\{W^t_{x,l} - e^t_{x,q,l}(\alpha)\}|\mathbf{I}_l\right]
\end{aligned}$$

and $\mathbf{r}^{(1)}_l(\alpha) = [\mathbf{r}^{(1)}_{x,q,l}(\alpha)']'_{q\in\mathbb{Q}_x, x\in\mathcal{X}}$ and $\mathbf{r}^{(2)}_l(\alpha) = [\mathbf{r}^{(2)}_{x,q,l}(\alpha)']'_{q\in\mathbb{Q}_x, x\in\mathcal{X}}$. In other words, $\mathbf{r}^{(1)}_l(\alpha)$ is a column vector with vectors $\mathbf{r}^{(1)}_{x,q,l}(\alpha)$, $q \in \mathbb{Q}_x, x \in \mathcal{X}$, stacked up and $\mathbf{r}^{(2)}_l(\alpha) = [\mathbf{r}^{(2)}_{x,q,l}(\alpha)']'_{q\in\mathbb{Q}_x, x\in\mathcal{X}}$ is a column vector with vectors $\mathbf{r}^{(2)}_{x,q,l}(\alpha)$, $q \in \mathbb{Q}_x, x \in \mathcal{X}$, stacked up. We define

$$\mathbf{m}_l(\alpha) = \begin{bmatrix} \mathbf{r}^{(1)}_l(\alpha) \\ \mathbf{r}^{(2)}_l(\alpha) \end{bmatrix}.$$

For each $j = 1, \cdots, d_\theta$, we define for $w \in \mathcal{G}$,

$$\frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}[w - g_0] = \frac{\partial \mathbf{m}_l(\theta_0, \tau w + (1-\tau)g_0)}{\partial \tau}|_{\tau=0}.$$

The left-hand side term represents the pathwise derivative of $\mathbf{m}_l(\theta_0, g_0)$ along the direction $w - g_0$. Let w_j^* be the solution to the minimization problem:

$$\inf_{w_j\in\mathcal{G}} \left(\frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial \theta_j} - \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}[w_j - g_0]\right)' \Sigma_l \left(\frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial \theta_j} - \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}[w_j - g_0]\right),$$

where

$$\Sigma_l = \mathbf{E}\left[[\mathbf{r}^{(1)}_l(\alpha), \mathbf{r}^{(2)}_l(\alpha)][\mathbf{r}^{(1)}_l(\alpha)', \mathbf{r}^{(2)}_l(\alpha)']'|\mathbf{I}_l\right].$$

Then we take

$$D_{w^*,l} = \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial \theta'} - \frac{\partial \mathbf{m}_l(\theta_0, g_0)}{\partial g}\left[w_1^* - g_0, \cdots, w_{d_\theta}^* - g_0\right].$$

The asymptotic covariance matrix formula becomes

$$V = \mathbf{E}\left[D_{w^*,l}\Sigma_l^{-1}D_{w^*,l}\right].$$

The estimation of the asymptotic covariance matrix formula can be done in a straightforward way. For example, we may follow Section 5 of Ai and Chen (2003), except that instead of using the series estimation to obtain the sample analogue of the conditional expectation $\mathbf{E}[\cdot|\mathbf{I}_l]$, we use the usual sample analogue of the conditional expectation with discrete conditional variables because $\mathbf{I}_l$ is a discrete random vector. Details are omitted.

Choice of the Sieve Space

Now let us consider the choice of the sieve space $\mathcal{G}_L$. Let $\bar{K} > 0$ be an integer such that for all $L \geq 1$, $\max_{l=1,\cdots,L} \bar{K}_l \leq \bar{K}$. Now let us discuss the construction of the sieve space $\mathcal{G}_L$. For each $k = 1,\cdots,\bar{K}$, and realized value of $(\mathbf{I}_{l,1}, \mathbf{I}_{l,2}^t)$, the function $g_k(\cdot,\mathbf{I}_{l,1},\mathbf{I}_{l,2}^t)$ is of particular form. First we take $\mathcal{G}_L = \mathcal{G}_{1,L}\times\cdots\times\mathcal{G}_{\bar{K},L}$, where $\mathcal{G}_{k,L}$'s are constructed as follows. For $i = 1,\cdots,N$, L, and $k = 1,\cdots,\bar{K}$, let $\mathcal{G}_{k,L}$ and $\mathcal{F}_{k,L}$ be sieve spaces, where for each $g_{k,L} \in \mathcal{G}_{k,L}$, $g_{k,L} : \mathcal{I} \to [0,1]$ and for each $f_{k,L} \in \mathcal{F}_{k,L}$, $f_{k,L} : \mathbf{R}\times\mathcal{I} \to [0,\infty)$ and for each $\mathbf{I}_{l,1} \in \mathcal{I}$, $\int f_{k,L}(b,\mathbf{I}_{l,1})db = 1$. Then we construct a sieve space $\mathcal{G}_{k,L}$ as the collection of maps $g_{k,L}(b,\mathbf{I}_{l,1})$, where

$$g_{k,L}(b,\mathbf{I}_{l,1}) = \prod_{x\in\mathcal{X}}\prod_{j=1}^{|\bar{A}_{x,l}^t|} f_{k,j,L}(b,\mathbf{I}_{l,1})v_{k,j,L}^x(\mathbf{I}_{l,1}),$$

where $v_{k,j,L}^x \in \mathcal{V}_{k,j,L}^x$, and $f_{k,j,L} \in \mathcal{F}_{k,j,L}$. For $\mathcal{V}_{k,j,L}^x$, we choose a sieve space $\mathcal{M}_{k,j,L}^x$ of real-valued functions and define

$$\mathcal{V}_{k,j,L}^x = \left\{\frac{\exp(m(\cdot))}{1+\exp(m(\cdot))} : m \in \mathcal{M}_{k,j,L}^x\right\}.$$

As for $\mathcal{M}_{k,j,L}^x$, we can take polynomial series.

As for $\mathcal{F}_{k,L} = \times_{j=1}^{\bar{K}}\mathcal{F}_{k,j,L}$, we use a Hermite polynomial sieve, where

$$\mathcal{F}_{k,j,L} = \left\{\begin{array}{c} f_{k,K_{L,k}}(x;\sigma_k,\varepsilon_{0,k},r_{0,k},\mathbf{a}_k) : \varepsilon_0 > 0, \sigma > 0,\ r_0, a_{s,k} \in \mathbf{R},\ s = 1,\cdots,K_{L,k} \\ \int f_{k,K_{L,k}}(x;\sigma_k,\varepsilon_{0,k},r_{0,k},\mathbf{a}_k)dx = 1 \end{array}\right\},$$

and $f_{k,K_{L,k}}(x;\sigma_k,\varepsilon_{0,k},r_{0,k},\mathbf{a}_k)$ is defined to be

$$\frac{1}{\sqrt{2\pi}\sigma_k}\left(\varepsilon_{0,k} + \left\{\sum_{s=1}^{K_{L,k}} a_{s,k}\left(\frac{x-r_{0,k}}{\sigma_k}\right)^s\right\}^2\right)\exp\left(-\frac{(x-r_{0,k})^2}{2\sigma_k^2}\right).$$

Observe that

$$\int f_{k,K_{L,k}}(x;\sigma_k,\varepsilon_{0,k},r_{0,k},\mathbf{a}_k)dx = \varepsilon_{0,k} + \sum_{s=1}^{K_{L,k}}\sum_{t=1}^{K_{L,k}} a_{s,k}a_{t,k}\mathbf{E}\left(Z^{s+t}\right),$$

where Z is a standard normal random variable. The quantity $E(Z^{s+t})$ can be explicitly computed from the moment generating function of Z.

Empirical Analysis Section

Additional Restrictions Imposed in Estimation

(a) The restriction associated with transitory sellers' bid distribution:

$$f(\mathbf{B}_i^t|\,\mathbf{I}_l) = \sum_k f(\mathbf{B}_i^t|\mathbf{Q}_{A,i,l}^t = \bar{\mathbf{q}}_{A,i,k}, \mathbf{I}_{l,1})P(\mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k,l}|\mathbf{I}_l) = \tag{30}$$
$$\sum_{k=1}^{\bar{K}_A} \frac{\omega_{A,k,l}^t \prod_{j\in\bar{A}_l^t}\{f(\mathbf{B}_j|\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k}, \mathbf{I}_{l,1})P(j \in A_l^t|\,\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1})\}}{\sum_{d=1}^{\bar{K}_l} \omega_{A,d,l}^t \prod_{j\in\bar{A}_l^t} P(j \in A_l^t|\,\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,d},\, \mathbf{I}_{l,1})}.$$

Moment conditions associated with this restriction would relate the empirical moments of the $f(\mathbf{B}_i^t|\,\mathbf{I}_l)$ distribution to the theoretical moments computed using (30).

(b) The restriction associated with the transitory sellers' probability of participation:

$$P(|A_{x,l}^t| = m_{x,l}|\,\mathbf{I}_{l,1}) = \sum_{k=1}^{\bar{K}_{A,l}} P(|A_{x,l}^t| = m_{x,l} \text{ and } \mathbf{Q}_{A,l}^t = \bar{\mathbf{q}}_{A,k}|\,\mathbf{I}_{l,1}) = \tag{31}$$
$$\sum_{k=1}^{\bar{K}_{A,l}} \prod_{j\in\bar{A}_l^t} \left\{P(j \in A_l^t|\,\mathbf{Q}_{A,j,l}^t = \bar{\mathbf{q}}_{A,j,k},\, \mathbf{I}_{l,1})P(\mathbf{Q}_{N,j,l}^t = \tilde{\mathbf{q}}_j|\,\bar{\mathbf{x}}_{j,l}^t)\right\}$$
$$\times \sum_{\tilde{\mathbf{q}}_{N-A}\in\mathbb{Q}_l^{N-A}} |\Omega| \prod_{i\in\bar{N}_l^t-\bar{A}_l^t} \left\{P(i \in N_l^t - A_l^t|\,\mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i,\, \mathbf{I}_{l,1})P(\mathbf{Q}_{N,i,l}^t = \tilde{\mathbf{q}}_i|\,\bar{\mathbf{x}}_{i,l}^t)\right\}.$$

The derivation for this expression is provided in the proof of Proposition 5. Moment conditions associated with this restriction would relate the transitory sellers' empirical probability of participation and expected $x-$characteristics of entrants conditional on $\mathbf{I}_{l,1}$ to their theoretical counterparts using (31).

(c) The restriction related to the expected profit condition. This restriction summarizes optimal participation behavior. For example, in the environment where the cost of work is not observed at the time when the entry decision is made and where $C \perp S$, the participation behavior is summarized by the threshold strategy where potential bidders with S-draws below the ex-ante expected profit participate in the auctions and those with higher draws stay out. This implies that in equilibrium

$$P(j \in A_{x,l}^t|\,\mathbf{Q}_{j,l}^t = q_{k,x},\, \mathbf{I}_{l,1}) = F_S(\mathbb{E}[\pi^t(j, \mathbf{Q}_{A,j,l}^t = q_{k,x}, \mathbf{I}_{l,1})]), \tag{32}$$

where $F_S(.)$ is a distribution of entry costs S and $q_{k,x} \in \mathbb{Q}_x$.

Moments Used in Estimation

The estimation is based on two types of moment conditions discussed in Section 6. The first type of moments relates the probability that a permanent seller wins under a variety of permanent actual bidder set configurations. The second type of moments links transitory and permanents sellers' empirical distribution of bids and participation frequencies to their theoretical counterparts. The moments of the first type are further subdivided into three groups:

(a) Moments that are based on the permanent seller's probability of winning conditional on two or more qualified bidders belonging to the same quality group. In these moment conditions, we use the following $h^{cb}(b)$[34] functions: a constant (equal to one), the difference between the winning bid and another bid submitted by a bidder from the same quality group, or the squared difference between the winning bid and another bid submitted by a bidder from the same quality group respectively.

(b) Moments that are based on the permanent seller's probability of winning conditional on this seller's quality group, and one or more qualified bidders belonging to a different quality group. In these moment conditions, we use the following $h^{cb}(b)$ functions: a constant (equal to one), the winning bid, the squared winning bid, the difference between the winning bid and a bid submitted by seller from a different quality group, the squared difference between the winning bid and a bid submitted by a seller from a different quality group, respectively. We include moments for all possible pairs of different quality groups.

(c) Moments that are based on on the permanent seller's probability of winning conditional on this seller's quality group, and at least one transitory qualified bidder belonging to a specific country group. In these moment conditions, we use the following $h^{cb}(b)$ functions: the differences between the winning and transitory bidders' characteristics other than price, the winning bid, the product of the winning bid and the differences between the winning and transitory bidders' characteristics other than price, as well as the squared differences, the squared winning bid, and the squared product of the winning bid and differences.

The second type of moments matches the empirical mean and variance of the permanent and transitory bid distributions, as well as the covariance between the bid and the seller's other characteristics, the frequencies of transitory and permanent bidders submitting bids as well as the expected value of the actual bidders' characteristics conditional on a set of permanent potential bidders, country group for transitory bidders, and country, reputation score, and quality group for permanent bidders to their theoretical counterparts. We include a separate moment for each of the five most frequent configurations of the set of permanent actual bidders.

We estimate the distribution of transitory sellers' bids and their probability of participation separately, instead of working with the composite function $g_k(.)$ described in Section 6. That is why we include the second type of moments in addition to the probability-of-winning moments. More specifically, we rely on exclusion restrictions in order to separate the product of bid density and participation probability into individual components. We condition moments of type two on the country group of transitional sellers while restricting the coefficient that captures the effect of the number of scores or current average reputation score to be constant across countries. Therefore, the differences in the moments across country groups reveal the dependence of bidding or participation strategies on the bidder's own quality, since the distributions of qualities differ across country groups.

[34]For definition see section 6.

An alternative identification strategy relies on the expected profit conditions that summarize the optimal participation decision of transitory bidders. These conditions impose the restriction that in equilibrium only potential bidders with entry costs below the ax-ante expected profit value should participate. In our setting, re-computing the expected profit values at each iteration is very costly. That is why we opted for the exclusion restriction channel of identification. However, this alternative estimation approach is also feasible. We were able to obtain a set of coefficients using such an alternative estimation strategy. They are very similar to the set of estimates we report in the paper.

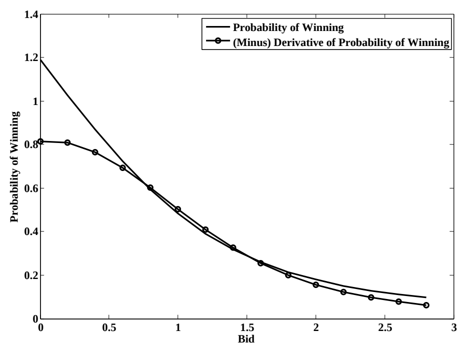
Additional Figures and Tables

Table 9: Estimated Quality Structure for a Given Number of Groups

	Number of Groups	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$
	1	52	45	33	2	2	2
	2	0	7	12	31	26	24
	3	0	0	7	12	5	2
	4	0	0	0	7	12	5
	5	0	0	0	0	7	12
	6	0	0	0	0	0	7
$\bar{V}$		9.21	2.61	1.77	0.85	0.73	0.31
$Q(K)$		10.03	4.22	4.11	4.24	4.81	5.22

This table shows the estimated quality group structures for the various numbers of quality groups for Eastern European suppliers with the medium levels of average reputation score. Rows 1-6 record the number of suppliers estimated to belong to a respective group. Rows 7 and 8 record the value of the $p-value$ component of the criterion function and the value of the criterion function. The results are based on the penalty function $g(L) = log(log(L))$. Results indicate that the number of groups most supported by the data is equal to three.

Figure 5: Probability of Winning



The graph is for Eastern European sellers, with the medium level of reputation score, and of medium quality.

Table 10: Participation Decision and Bid Distributions: Competitive Effects

			I(T)	II(T)	I(P)	II(P)
North America,	low,	1	0.021 (0.017)	0.021 (0.013)	0.003 (0.002)	0.011** (0.005)
North America,	low,	2	0.011 (0.012)	0.015 (0.012)	-0.005 (0.003)	0.011** (0.004)
North America,	medium,	1	-0.023** (0.011)	-0.089** (0.011)	-0.002 (0.002)	-0.004 (0.004)
North America,	medium,	2	-0.015* (0.008)	-0.031** (0.015)	-0.004** (0.002)	-0.007* (0.003)
North America,	high,	1	0.052 (0.031)	0.001 (0.0078)	-0.007** (0.002)	-0.012** (0.003)
North America,	high,	2	-0.026** (0.012)	-0.085** (0.021)	-0.008** (0.003)	-0.016** (0.004)
Eastern Europe,	low,	1	-0.005 (0.011)	0.001 (0.007)	-0.001 (0.002)	0.002 (0.005)
Eastern Europe,	low,	2	-0.007** (0.002)	-0.025** (0.012)	-0.005* (0.0026)	-0.009** (0.0026)
Eastern Europe,	medium,	1	-0.005 (0.003)	-0.007 (0.004)	-0.003* (0.001)	-0.007** (0.002)
Eastern Europe,	medium,	2	0.034 (0.031)	0.016 (0.012)	0.002 (0.004)	0.001 (0.003)
Eastern Europe,	medium,	3	-0.021** (0.011)	-0.016 (0.015)	0.001 (0.002)	-0.004 (0.004)
Eastern Europe,	high,	1	-0.011 (0.012)	-0.012 (0.017)	-0.002 (0.003)	-0.003 (0.005)
Eastern Europe,	high,	2	0.008 (0.005)	0.007 (0.004)	0.001 (0.005)	0.003 (0.002)
Eastern Europe,	high,	3	-0.007 (0.004)	-0.011 (0.019)	0.002 (0.003)	-0.0001 (0.003)
South-East Asia,	low,	1	0.001 (0.011)	-0.017* (0.003)	-0.002* (0.001)	-0.001 (0.001)
South-East Asia,	low,	2	-0.003 (0.004)	-0.004 (0.008)	-0.008** (0.003)	-0.019** (0.002)
South-East Asia,	low,	2	-0.023** (0.011)	-0.026** (0.011)	-0.001 (0.002)	0.001 (0.003)
South-East Asia,	medium,	1	-0.011 (0.012)	-0.013 (0.011)	-0.004 (0.003)	-0.001 (0.005)
South-East Asia,	medium,	2	-0.003 (0.011)	-0.002 (0.011)	-0.002 (0.002)	-0.004** (0.001)
South-East Asia,	medium,	3	0.023 (0.033)	0.015 (0.014)	-0.001 (0.002)	0.007 (0.005)
South-East Asia,	high,	1	-0.004 (0.003)	-0.007 (0.004)	-0.002* (0.001)	0.001 (0.001)
South-East Asia,	high,	2	-0.024** (0.012)	-0.039** (0.018)	-0.004* (0.002)	-0.007** (0.003)

This table reports the coefficients summarizing the impact of the various potential competitors on sellers' bid distribution and participation decisions. Columns I(T), II(T), and I(P), II(P) report estimated coefficients for the bid distribution and the probability of participation of transitory and permanent sellers respectively. The results are based on the data set consisting 11,300 projects. The quality level for South and East Asia, low score, $Q = 1$, is normalized to be equal to zero. The stars, $**$, indicate that a coefficient is significant at the 95% significance level.

Table 11: Costs Distributions: Mean and Standard Deviation

Variable			Mean	Std.Deviation
Project Costs				
North America,	low score,	Q=1	1.039	0.053
North America,	low score,	Q=2	1.275	0.087
North America,	medium score,	Q=1	1.627	0.039
North America,	medium score,	Q=2	1.551	0.073
North America,	high score,	Q=1	1.135	0.053
North America,	high score,	Q=2	1.535	0.099
Eastern Europe,	low score,	Q=1	1.169	0.037
Eastern Europe,	low score,	Q=2	1.328	0.079
Eastern Europe,	medium score,	Q=1	1.576	0.119
Eastern Europe,	medium score,	Q=2	1.202	0.062
Eastern Europe,	medium score,	Q=3	1.531	0.101
Eastern Europe,	high score,	Q=1	1.575	0.119
Eastern Europe,	high score,	Q=2	0.981	0.048
Eastern Europe,	high score,	Q=3	1.354	0.089
South and East Asia,	low score,	Q=1	1.621	0.056
South and East Asia,	low score,	Q=2	1.119	0.051
South and East Asia,	low score,	Q=3	1.417	0.169
South and East Asia,	medium score,	Q=1	1.609	0.107
South and East Asia,	medium score,	Q=2	1.589	0.121
South and East Asia,	medium score,	Q=3	1.255	0.124
South and East Asia,	high score,	Q=1	1.074	0.036
South and East Asia,	high score,	Q=2	1.235	0.047
Entry Costs			0.082	0.081

This table summarizes means and standard errors of the estimated distributions of permanent sellers' project costs that are displayed in Figure 2.