Implementation Cycles, Investment and Growth

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Abstract

We develop a model of "intrinsic" cycles, driven by the decentralized behaviour of entrepreneurs and firms making continuous, divisible improvements in their productivity. We show that when the introduction of productivity impovements is endogenous, implementation cycles arise even in the presence of reversible investment and consumption–smoothing. The implied cyclical equilibrium is unique within its class and shares several features in common with actual business cycles. In particular its predictions are qualitatively consistent with the joint behaviour of the investment rate and Tobin's Q during US recessions.

Key Words: Intangible investment, endogenous cycles and endogenous growth, Tobin's Q,
JEL: E0, E3, O3, O4

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1 Introduction

It is common in modern business cycle analysis to model fluctuations in aggregate investment and productive inputs as optimal responses to aggregate TFP shocks. While this approach has certainly proved useful and insightful, it continues to raise several conceptual questions. In particular, why treat investment in physical capital as endogenous, while implicitly treating those intangible investments that ultimately affect TFP as exogenous?[1] Moreover, why would these apparent productivity improvements, and the consequent changes in investment rates, take place in a clustered fashion across diverse sectors of the economy?[2] Even if one takes the view that aggregate fluctuations are often a result of demand–side factors, understanding co–movement across sectors is still a relevant concern. In this article, we develop a dynamic general equilibrium framework in which the rates of investment in both physical and knowledge capital are endogenously determined, and in which productivity improvements are optimally implemented in a clustered fashion across sectors.

A natural starting point for thinking about these issues is Shleifer's (1986) model of "implementation cycles". He shows that in the presence of imperfect competition, the implementation of a productivity improvement by one firm may increase the demand for others' products by raising aggregate demand. This induces producers, who anticipate short-lived profits due to imitation, to delay implementation of productivity improvements until others implement, thereby generating self–enforcing booms in aggregate activity. Unfortunately, though capable of generating sectoral co–movement in implementation, Shleifer's model cannot serve as a framework for understanding cyclical fluctuations in which investment plays an important role. This is because the sectoral co–movement that he establishes is not robust to the introduction of capital or, in fact, *any* storable commodity. Anticipating a boom, producers would produce early, store the output, and sell it in the boom, thereby undermining the cycle.[3]

Recently, Francois and Lloyd-Ellis (2003) show how a process of endogenous clustering can also arise due to the process of "creative destruction" familiar from Schumpeterian, endogenous growth models. Like imitation, potential obsolescence limits the longevity of profits and provides incentives to cluster implementation. However, when productive resources are needed to search for commercially viable ideas, allowing for the possibility of storage does not rule out clustering, and in fact yields a *unique* cyclical equilibrium. Moreover, because this costly search process

[1] Of course, this could be viewed as a measurement issue: the definition of investment could be broadened. But this would still ignore the effects of any spillovers usually associated with intangible investments.

[2] As Lucas (1987) reasons, while technological improvements may be important at the firm level, it is not immediately obvious why they would be important for economy–wide aggregate output fluctuations.

[3] His model is also subject to a number of other criticisms including the fact that there are no downturns in his model and that there is a continuum of multiple cyclical equilibria, making the predictions of the model rather imprecise. Moreover, while the timing of implementation booms is endogenous, the innovations themselves arise exogenously.

tends to be concentrated just before booms, it causes downturns in aggregate output (even if the measure of GDP includes this investment). Nevertheless, while a type of storage is allowed in this model, it is still not clear that the cycles are robust to the inclusion of reversible investment. The implementation delay central to generating aggregate fluctuations seems, at first blush, to be undermined by the consumption–smoothing possibilities afforded by capital accumulation. Since forward–looking households will partially consume anticipated increases in productivity in advance, consumption will not grow rapidly enough during booms to yield significant gains to delay.

In this paper we demonstrate that, in fact, while capital accumulation does allow consumption–smoothing, endogenous cycles persist because fluctuations in the rate of investment itself provide sufficient incentives for delayed implementation. We show that, when effort is required to introduce productivity improvements into production, a robust cycle with endogenous delay in multiple sectors exists, even in the presence of smoothly accumulable and reversible physical capital. Moreover, within this class of cyclical equilibria, there exist relatively weak sufficient conditions under which any such cyclical equilibrium is unique. Both the existence and uniqueness of the cyclical equilibrium in the presence of reversible investment are a consequence of allowing for endogenous search effort.

For firms to be willing to delay implementation, output (and profit) growth during booms must strictly exceed the rate at which they are discounted. However, the storage can only be ruled out if the discount rate (weakly) exceeds wage growth. In Shleifer (1986), output growth equals TFP growth equals wage growth, so both conditions cannot hold simultaneously. When ideas arise endogenously, as in Francois and Lloyd–Ellis (2003), the shift of skilled labour effort back into production during booms implies that output growth exceeds TFP and wage growth. Consequently, implementation delays can occur even if storage is allowed.[4] With reversible investment, the equilibrium discount factor is zero across the boom because households smooth their consumption. This implies that wages must evolve smoothly over the boom, otherwise producers would produce early and store. This is the case because, with physical capital in production, there are diminishing returns to labour, so that the effect of TFP on wages is exactly offset by the shift in labour effort back into production.

Our model endogenously generates a cyclical growth path for aggregates which share some qualitative features with the US economy. The investment rate is highly volatile, while consumption is relatively smooth. Both are pro–cyclial, as are TFP and labour productivity. Labor share is inherently counter–cyclical, reflecting the unmeasured withdrawal of skilled labor effort

[4] Moreover, because free entry into entrepreneurship also implies that the return on claims to firms must grow in proportion to the wage through the boom, the discount factor must exactly equal TFP growth. This pins down a unique equilibrium cycle.

form production during downturns.[5] Moreover, we demonstrate that the relationship between the investment rate and Tobin's Q — the ratio of the market value of firms' liabilities to the replacement cost of the capital stock — conforms qualitatively with some features of the data. In particular, increases in aggregate demand, resulting from the implementation of productivity improvements, are anticipated, so that Tobin's Q starts to rise prior to the boom (even in sectors that do not expect productivity improvements). However, since firms optimally choose to delay implementation, investment lags behind the increase in Q. Physical capital accumulation grows rapidly at the start of expansions and capital is accumulated continuously and smoothly, though at a declining rate, until its end. At this point, the economy enters a recessionary phase where output falls and capital accumulation declines precipitously, though still remaining positive. The anticipated fall in aggregate demand causes Tobin's Q to fall even while the economy is expanding, so that Q leads investment into the recession too.

In other models of endogenous cycles (e.g. Bental and Peled, 1996, Freeman Hong and Peled, 1999, Li, 2001, Walde, 2005), fluctuations come about through innovation booms in a single sector.[6] Although in such models it is relatively straightforward to accommodate capital accumulation, the single sector and (often) large, indivisible nature of innovations make them more suited to analysis of long–waves, or GPT type innovations.[7] Fluctuations at business cycle frequency exhibit striking sectoral co–movement in productivity, investment, output, and factor usage through the typical business cycle (see Christiano and Fitzgerald, 1998). To get at this sort of fluctuation it is necessary to understand the aggregate implications of the actions of "small" actors in multiple sectors making independent choices over the timing and level of investments in both physical and knowledge capital. The model developed here features multiple disparate sectors (and firms) each following privately optimal investment decisions, and thus better corresponds to standard business cycles frequencies.

The paper proceeds as follows: Section 2 sets up the basic model and Section 3 posits the cyclical behavior of entrepreneurs and capital owners, and describes the cyclical growth path. Section 4 characterizes the implied movement of key aggregates and prices through the posited cycle and derives necessary conditions for implied behavior to be optimal. Section 5 develops sufficient conditions for the uniqueness of a stationary cyclical equilibrium, demonstrates existence for a variety of parameter combinations and explores the model's implications for key aggregates. In Section 6, we discuss the key results relating to the relationship between the investment rate

[5] This stylized fact is emphasized by Rioss-Rull and Santaeulalia-Llopis (2006).

[6] Aghion and Howitt (1992) also consider the existence of cycles, but in a model without capital.

[7] Matsuyama's (1999, 2001) is one approach that does not neatly fit this scheme. The cycles that arise in his model do not depend on delay, and are thus robust to capital accumulation through the cycle. However, Matsuyama's framework is more suited to understanding longer–term movements in the *nature* of growth (e.g. productivity slowdown), rather than business cycle fluctuations. In particular, there is no phase of his cycle that could be called a recession: production and consumption never decline.

and Tobin's Q. Section 7 extends the basic model to allow for non–managerial, production labour. Section 8 offers some concluding remarks and an appendix provides all proofs.

2 The Model

2.1 Assumptions

There is no aggregate uncertainty. Time is continuous and indexed by $t \geq 0$. The economy is closed. The representative household has iso–elastic preferences

$$U(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma} d\tau \tag{1}$$

where ρ denotes the rate of time preference and σ represents the inverse of the elasticity of intertemporal substitution. The household maximizes (1) subject to the intertemporal budget constraint

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]} C(\tau) d\tau \leq Z(t) + \int_t^{\infty} e^{-[R(\tau)-R(t)]} \left[w(\tau) + \psi(\tau)\right] d\tau \tag{2}$$

where $w(t)$ denotes wage income, $Z(t)$ denotes the value of the household's stock of assets (firm shares and capital) at time t and $R(t)$ denotes the discount factor from time zero to t. The term $\psi(\tau)$ represents lump–sum transfers from the government (see below). The population is normalized to unity and each household is endowed with one unit of labour, which it supplies inelastically.

Final output is produced according to a Cobb–Douglas production function utilizing physical capital, $K(t)$, and a continuum of intermediates, x_i, indexed by $i \in [0, 1]$:

$$Y(t) = K(t)^{\alpha} X(t)^{1-\alpha}, \tag{3}$$

where $\alpha \in (0, 1)$ and

$$X(t) = \exp\left(\int_0^1 \ln x_i(t) di\right) \tag{4}$$

Final output can be used for consumption, $C(t)$, investment, $\dot{K}(t)$, or (potentially) stored, $S(t)$:

$$C(t) + \dot{K}(t) + \delta K(t) + S(t) \leq Y(t), \tag{5}$$

where δ denotes the rate of physical depreciation. Although we allow physical capital to be reversible in principle, in the equilibria we study negative investment never actually occurs.[8]

Output of intermediate i depends upon the state of technology in sector i, $A_i(t)$, and labour hours, $L_i(t)$, according to a simple linear technology:

$$x_i(t) = A_i(t) L_i(t) \tag{6}$$

[8] It is also possible to allow for some exogenous sources of TFP growth as explored in an earlier working paper version of the present paper (see Francois and Lloyd–Ellis, 2005).

Intermediates are completely used up in production, but can be produced and stored for later use. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later. At each date t, incumbents choose the price $p_i(t)$ for their product so as to maximize profits.

We assume that profits earned from intermediate production are taxed at a constant rate, ω. Revenue from this tax is redistributed back to households in a lump–sum fashion, so that the government's budget is balanced at every date t:

$$\psi(t) = \omega\pi(t). \tag{7}$$

Profit taxes are not a necessary part of a cyclical equilibrium. However, including them helps in terms of generating cycles with realistic features. Moreover, these tax effects can be thought of as a convenient representation of various other realistic factors, including implementation costs, imitation and labour market imperfections.[9] One can also interpret ω as the share of rents earned by the entrepreneur, with the remainder $(1-\omega)$ being received by an upstream party in the innovation process.[10]

Commercially viable productivity improvements are introduced into the economy via a process of "entrepreneurial search". Competitive entrepreneurs in each sector allocate labour effort to searching for ideas, and finance this by selling claims. The rate of success from search is $\mu h_i(t)$, where μ is a parameter, and h_i represents the labour effort allocated to search in sector i. At each date, entrepreneurs decide whether or not to allocate labour to search, and if they do so, how much. The aggregate labour effort allocated to search is given by

$$H(t) = \int_0^1 h_i(t)dt. \tag{8}$$

New ideas and innovations dominate old ones in terms of productivity by a factor e^{γ}, where $\gamma > 0$. This process is therefore formally identical to the innovation process in the quality–ladder model of Grossman and Helpman (1991). However, we explicitly do not interpret this activity as R&D. Although it is common to do so in the endogenous growth literature, this Poisson process is, in fact, a very bad description of R&D. Typically R&D is a knowledge-intensive (and often capital-intensive) activity, which involves accumulation of sector-specific knowledge. In sharp contrast, the search activity described here is a skill-intensive one, which we interpret as a form of entrepreneurship. In our view this entrepreneurial function is the central player in economic

[9] For example, in Francois and Lloyd–Ellis (2006a), the deadweight loss due to a worker–firm contracting problem acts very much like a tax on profits. In this case there is no revenue redistribution, ψ, but this makes no difference to the growth path since it is lump–sum.

[10] Francois and Lloyd–Ellis (2006b) explicity model innovation as a multi–stage process with basic R&D yielding undirected ideas, which are matched with particular applications by entrepreneurs.

activity, with R&D playing a supportive role that is not modeled here.[11] This activity could be undertaken by independent entrepreneurs, but in modern production it is often a role taken on by managers and other skilled workers *within* firms.

Successful manager/entrepreneurs must choose whether or not to implement commercially viable ideas immediately or delay until a later date. Once they implement, the associated knowledge becomes publicly available, and can be built upon by rivals. However, prior to implementation, the knowledge is privately held by the entrepreneur. Thus, in order to dominate an unimplemented technology, rivals must achieve two successes: firstly to reach the successful entrepreneur's level, and a second one to supersede it.[12] We let the indicator function $\chi_i(t)$ take on the value 1 if there exists a commercially viable innovation in sector i which has not yet been implemented, and 0 otherwise. The set of dates in which new ideas are implemented in sector i is denoted by Ω_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing a success at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable future date.

2.2 Definition of Equilibrium

Given initial state variables $\{A_i(0),\ \chi_i(0)\}_{i=0}^{1}$, $K(0)$ an equilibrium for this economy consists of:
(1) sequences $\left\{\hat{p}_i(t),\ \hat{x}_i(t).\ \hat{L}_i(t),\ \hat{h}_i(t),\ \hat{A}_i(t),\ \hat{\chi}_i(t),\ \hat{V}_i^I(t),\ \hat{V}_i^D(t)\right\}_{t\in[0,\infty)}$ for each intermediate sector i, and
(2) economy wide sequences $\left\{\hat{Y}(t),\ \hat{K}(t),\ \hat{R}(t),\ \hat{w}(t),\ \hat{q}(t),\ \hat{C}(t),\ \hat{S}(t)\right\}_{t\in[0,\infty)}$
which satisfy the following conditions:

• Households allocate consumption over time to maximize (1) subject to the budget constraint, (2). The first–order conditions of the household's optimization imply that

$$\hat{C}(t)^{\sigma} = \hat{C}(\tau)^{\sigma} e^{\widehat{R}(t)-\widehat{R}(\tau)-\rho(t-\tau)} \qquad \forall\ t,\tau, \tag{9}$$

and that the transversality condition holds

$$\lim_{\tau\to\infty} e^{-\hat{R}(\tau)}\hat{S}(\tau) = 0 \tag{10}$$

• Final goods producers choose capital and intermediates, x_i, to minimize costs given prices p_i,

[11] This view was shared by Schumpeter (1950, p.132): "...The function of entrepreneurs is to reform or revolutionize the pattern of production by exploiting an invention or, more generally, an untried technological possibility ... This function does not essentially consist in either inventing anything or otherwise creating the conditions which the enterprise exploits. It consists in getting things done". More recently, Comin (2004) estimates the contribution of R&D to US productivity growth to be very small. He notes that a larger contribution is likely to come from unpatented managerial and organizational innovations.

[12] Even for the case of intellectual property, Cohen, Nelson and Walsh (2000) show that firms make extensive use of secrecy in protecting productivity improvements. Secrecy likely plays a more prominent role for entrepreneurial innovations, which are the key here.

subject to (3). The derived demand for intermediate i is

$$x_i^d(t) = (1-\alpha)\frac{Y(t)}{p_i(t)}. \tag{11}$$

The conditional demand for capital is given by

$$K(t) = \frac{\alpha Y(t)}{q(t)} \tag{12}$$

• The unit elasticity of demand for intermediates implies that limit pricing at the unit cost of the previous incumbent is optimal. It follows that

$$p_i(t) = \frac{w(t)}{e^{-\gamma} A_i(t)} \qquad \forall t \tag{13}$$

The resulting instantaneous profit (before any taxes) earned in each sector is given by

$$\pi(t) = (1-e^{-\gamma})(1-\alpha)Y(t). \tag{14}$$

• Labor markets clear:

$$\int_0^1 \hat{L}_i(t)di + \hat{H}(t) = 1 \tag{15}$$

• Arbitrage trading in financial markets implies that, for all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{\hat{R}(s)-\hat{R}(t)}{s-t}$.
• Free entry into entrepreneurship:

$$\mu \max[\hat{V}_i^D(t), \hat{V}_i^I(t)] \leq \hat{w}(t), \quad \hat{h}_i(t) \geq 0 \qquad \text{with at least one equality.} \tag{16}$$

• At dates where there is implementation, entrepreneurs with commercially viable ideas must prefer to implement rather than delay until a later date

$$\hat{V}_i^I(t) \geq \hat{V}_i^D(t) \quad \forall\, t \in \hat{\Omega}_i. \tag{17}$$

• At dates where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } \hat{\chi}_i(t) &= 0, \\ \text{or if } \hat{\chi}_i(t) &= 1,\ \hat{V}_i^I(t) \leq \hat{V}_i^D(t) \quad \forall\, t \notin \hat{\Omega}_i. \end{aligned} \tag{18}$$

• Free entry of replacement capital.

3 The Cyclical Equilibrium Growth Path

Although there exists an acyclical equilibrium growth path that satisfies the conditions stated above, our focus here is on a cyclical equilibrium growth path. In this section, we posit a temporal pattern of innovation and implementation behavior by entrepreneurs. Section 4 then derives the implications of this for the evolution of aggregate variables, and a set of sufficient conditions under which the implied evolution of aggregate variables, and market clearing, yield optimal entrepreneurial behavior corresponding with the originally posited behavior.

Suppose then that implementation occurs at discrete dates denoted by T_ν where $v \in \{1, 2, ..., \infty\}$. We adopt the convention that the vth cycle starts at time T_{v-1} and ends at time T_ν. The posited behavior of entrepreneur/managers and investors over this cycle is illustrated in Figure 1. After implementation at date T_{v-1} an **expansion** is triggered by a productivity boom and continues through subsequent capital formation. During this phase, entrepreneurial search ceases and consequently all labour effort is used in production. At some time T_v^*, search commences and labour starts to be withdrawn from production. Commercially viable ideas are not implemented immediately but are withheld until time T_v. During this **contraction** phase, capital formation continues, but the rate of investment declines rapidly. As aggregate demand falls, labour continues to be released from production, so that search accelerates in anticipation of the subsequent implementation boom.

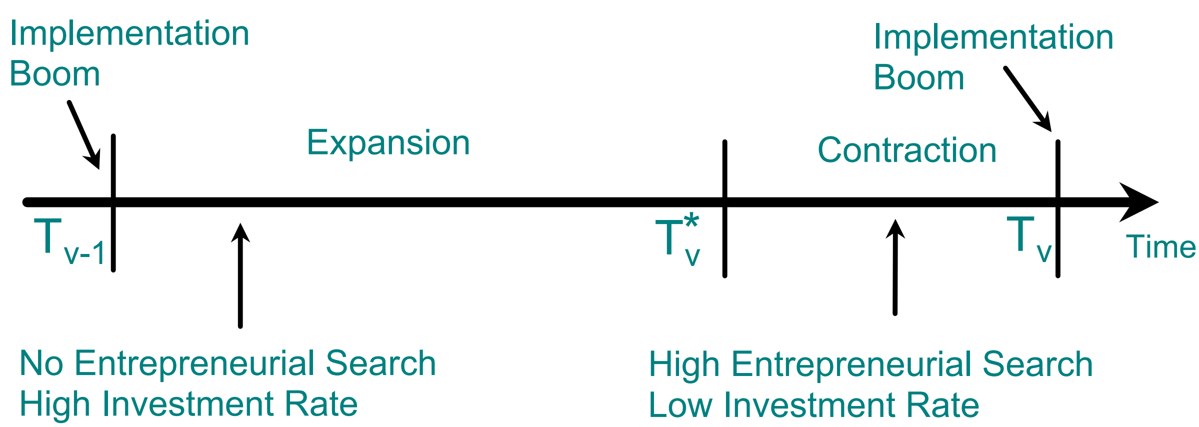

Figure 1: Search and Investment during the vth cycle

Let $P_i(s)$ denote the probability that, since time T_{v-1}, no commercially viable ideas have materialized in sector i by time s. It follows that the probability of there being no success by time T_v conditional on there having been none by time t, is given by $P_i(T_v)/P_i(t)$. Hence, the value of an incumbent firm in a sector where no new idea has arisen by time t during the vth cycle can be expressed as

$$V_{0,i}^{I}(t) = (1-\omega)\int_{t}^{T_v} e^{-[R(\tau)-R(t)]}\pi_i(\tau)d\tau + \frac{P_i(T_v)}{P_i(t)}e^{-\beta(t)}V_{0,i}^{I}(T_v). \tag{19}$$

where

$$\beta(t) = R_0(T_v) - R(t) \tag{20}$$

denotes the discount factor used to discount from time t during the cycle to the beginning of the next cycle.[13] The first term in (19) represents the discounted profit stream that accrues to incumbent firms with certainty during the current cycle, and the second term is the expected discounted value of being an incumbent at the beginning of the next cycle.

Lemma 1 *: In a cyclical equilibrium, the identification of commercially viable ideas by a non-incumbent can be credibly signalled immediately, and all search in the sector stops until the next round of implementation.*

Unsuccessful entrepreneurs have no incentive to falsely announce search success. As a result, an entrepreneur's signal is credible, and other entrepreneurs will exert their efforts in sectors where they have a better chance of becoming the dominant entrepreneur.

In the cyclical equilibrium, entrepreneurs' conjectures ensure no more entrepreneurship in a sector once a signal of success has been received, until after the next implementation. The time $t \in (T_v^*, T_v)$ expected value of a viable idea whose implementation is delayed until time T_v is thus:

$$V_i^D(t) = e^{-\beta(t)}V_{0,i}^{I}(T_v). \tag{21}$$

In the cyclical equilibrium, such delay is optimal; i.e. $V_i^D(t) > V_i^I(t)$ throughout the contraction. Successful entrepreneurs are happier to forego immediate profits and delay implementation until the boom in order to ensure a longer reign of incumbency. Since no implementation occurs during the cycle, by delaying, firms are assured of incumbency until at least T_{v+1}. Incumbency beyond that time depends on the probability that another viable idea is identified.[14]

The symmetry of sectors implies that search effort is allocated evenly over all sectors that have not yet experienced a success within the cycle. In the posited cyclical equilibrium, the probability of not being displaced at the next implementation is

$$P(T_v) = \exp\left(-\mu\int_{T_v^*}^{T_v} h(\tau)d\tau\right). \tag{22}$$

[13]Throughout, we use the subscript 0 to denote the value of a variable immediately after the boom. Formally, for any variable $X(\cdot)$, we define $X(t) = \lim_{\tau\to t^-} X(\tau)$ and $X_0(t) = \lim_{\tau\to t^+} X(\tau)$.

[14]A signal of further entrepreneurial success submitted by an incumbent is not credible in equilibrium because incumbents have incentive to lie to protect their profit stream. No such incentive exists for entrants since, without a success, profits are zero. Note also that the reason for delay here differs from Shleifer (1986) where the length of incumbency is exogenously given.

4 Within–Cycle Dynamics

It can be seen from (11) and (13) that employment is identical for every sector i, $L_i(t) = x_i/A_i = L(t)$. It follows from (4) and (6) that $X(t) = \bar{A}_{v-1}L(t)$ where

$$\bar{A}_{v-1} = \exp\left(\int_0^1 \ln A_i(T_{v-1})di\right). \tag{23}$$

Consequently, in equilibrium, the aggregate production function can be expressed as

$$Y(t) = \bar{A}_{v-1}^{1-\alpha}K(t)^\alpha L(t)^{1-\alpha}, \tag{24}$$

Note that TFP is fixed through the cycle. In order to afford a stationary representation of the economy it is convenient to normalize aggregates by dividing by total factor productivity using lower–case letters to denote these deflated variables:

$$k(t) = \frac{K(t)}{\bar{A}_{v-1}}, \quad c(t) = \frac{C(t)}{\bar{A}_{v-1}}, \quad y(t) = \frac{Y(t)}{\bar{A}_{v-1}}. \tag{25}$$

Consequently, the intensive form production function is given by

$$y(t) = k(t)^\alpha L(t)^{1-\alpha}. \tag{26}$$

The household's Euler equation during the cycle can be expressed as

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t)-\rho}{\sigma}, \tag{27}$$

where $r(t) = \dot{R}(t)$. The economy's aggregate resource constraint is

$$\dot{k}(t) = y(t) - c(t) - \delta k(t) \tag{28}$$

Finally, factor prices can be expressed as

$$q(t) = \alpha k(t)^{\alpha-1}L(t)^{1-\alpha} \tag{29}$$

$$w(t) = e^{-\gamma}(1-\alpha)\bar{A}_{v-1}k(t)^\alpha L(t)^{-\alpha}. \tag{30}$$

Note that the wage rate is less than its marginal product by a factor $e^{-\gamma}$, reflecting the fact that a fraction $1-e^{-\gamma}$ goes in the form of profits to intermediate producers. Moreover,

Lemma 2 *: Free entry of replacement capital and reversible investment imply that*

$$r(t) = q(t) - \delta. \tag{31}$$

4.1 Phase 1: The Expansion $(T_{v-1} \rightarrow T_v^*)$

We now trace out the evolution of the economy implied by the behavior posited above. We start immediately following an implementation boom, when capital, consumption and output take on the initial values $k_0(T_{v-1})$, $c_0(T_{v-1})$ and $y_0(T_{v-1})$, respectively. During the expansion all labour is used in production so that

$$L(t) = 1. \tag{32}$$

Combining this condition with (26), (27), (28), (29) and (31) yields transitional dynamics that are identical to those of the Ramsey model:

$$\frac{\dot{c}(t)}{c(t)} = \frac{\alpha k(t)^{\alpha-1} - \delta - \rho}{\sigma} \tag{33}$$

$$\frac{\dot{k}(t)}{k(t)} = k(t)^{\alpha-1} - \frac{c(t)}{k(t)} - \delta. \tag{34}$$

These dynamics are illustrated using a phase diagram in Figure 2.

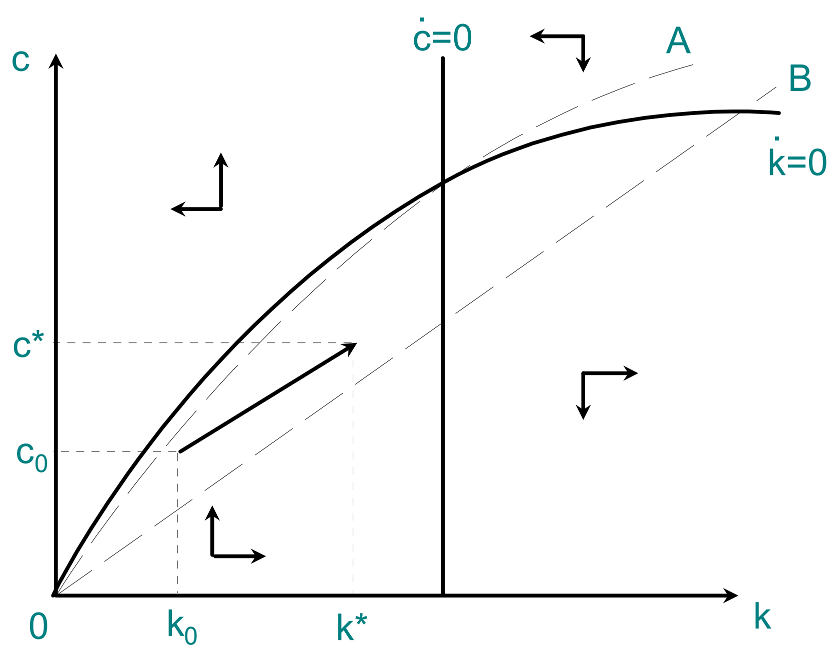

Figure 2: Dynamics in Phase 1

During this expansionary phase, both consumption and capital grow, so we restrict attention to the lower left quadrant of the phase diagram. As capital accumulates, the wage grows and the interest rate declines. However, for the dynamic path to be consistent with the cyclical equilibrium, more stringent conditions must be met.

Proposition 1 *: If, during an expansion, the dynamic paths of consumption and capital satisfy*

$$\left(1-\frac{\alpha}{\sigma}\right)k(t)^{\alpha}+\left(\frac{\rho+(1-\sigma)\delta}{\sigma}\right)k(t)>c(t)>\left(\frac{1-\alpha}{\alpha}\right)\delta k(t), \tag{35}$$

then there exists a T_v^* *such that for the first time*

$$\mu V^D(T_v^*)=w(T_v^*), \tag{36}$$

The left hand inequality in (35) is depicted in Figure 2 by points below the curve OA. This curve is a concave function passing through the origin and the intersection of the $\dot{k}=0$ and $\dot{c}=0$ loci. As long as the path of the economy lies below this curve during this phase, the consumption–capital ratio declines though time. This is consistent with the fact that the marginal product of capital is relatively high, inducing rapid investment and a capital stock that is growing relative to consumption.

The right hand inequality in (35) is depicted in Figure 2 by points above the line OB. This line is a ray from the origin that intersects the $\dot{k}=0$ locus at its peak. During the first phase of the cycle, entrepreneurial search with delayed implementation cannot be optimal. That is

$$\mu V^D(t)<w(t). \tag{37}$$

As the capital stock accumulates and TFP grows, $w(t)$ rises through time. Moreover, as the subsequent boom approaches $V^D(t)$ grows at the rate of interest. As long as the path of the economy lies between OA and OB it must be true that

$$r(t)>\frac{\dot{w}(t)}{w(t)}. \tag{38}$$

Consequently, the first phase of the cycle comes to an end in finite time.

After the date, T_v^*, if all labour were to remain in production, returns to search effort would strictly dominate those in production. As a result, labour effort is re–allocated from production and into search and this triggers the next phase of the cycle. The following Lemma demonstrates that during the transition from one phase to the next, all aggregate variables evolve smoothly.

Lemma 3 *: At time* T_v^**, when entrepreneurial search first commences in a cycle,* $L(T_v^*)=1$ *and output, investment and consumption evolve continuously. Growth rates of wages and employment change discretely.*

4.2 Phase 2: The Downturn ($T_v^* \to T_v$)

During this phase, capital continues to be accumulated so that (31) must still hold. However, now there is search, so that $L(t) < 1$. Free entry into entrepreneurship implies $\mu V^D(t) = w(t)$, so that it must be the case that

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{w}(t)}{w(t)}. \tag{39}$$

Combining these conditions with (26), (27), (28), (29) and (30) implies that during the slowdown, consumption, capital and the labour force in production evolve according to the following dynamical system:

$$\frac{\dot{c}(t)}{c(t)} = \frac{\alpha k(t)^{\alpha-1}L(t)^{1-\alpha} - \delta - \rho}{\sigma} \tag{40}$$

$$\frac{\dot{k}(t)}{k(t)} = k(t)^{\alpha-1}L(t)^{1-\alpha} - \frac{c(t)}{k(t)} - \delta \tag{41}$$

$$\frac{\dot{L}(t)}{L(t)} = -\frac{c(t)}{k(t)} + \left(\frac{1-\alpha}{\alpha}\right)\delta < 0 \tag{42}$$

The negative value of $\dot{L}(t)/L(t)$ at the beginning of this phase is ensured by condition (35). Initially, the consumption–capital ratio $c(t)/k(t)$ also continues to decline. However, as $L(t)$ declines, the marginal product of capital falls and investment starts to fall. Eventually, in the hypothesized cycle, $c(t)/k(t)$ starts to rise again.

Note that the implied path for $L(t)$ during this phase implies a path for the fraction of the labour effort engaged in search, $H(t) = 1 - L(t)$. This, in turn, determines the measure of sectors in which commercially viable ideas are identified at each date:

$$-\dot{P}(t) = \mu\left[1 - L(t)\right], \tag{43}$$

where $P(T_v^*) = 1$.[15] At the end of the cycle, the fraction of sectors that have experienced successful search is therefore

$$1 - P(T_v) = \int_{T_v^*}^{T_v} \mu\left[1 - L(\tau)\right] d\tau. \tag{44}$$

4.3 The Implementation Boom

We denote the improvement in total factor productivity during implementation, $e^{(1-\alpha)\Gamma_v}$, where $\Gamma_v = \ln\left[\bar{A}_v/\bar{A}_{v-1}\right]$. Productivity growth at the boom is given by

$$\Gamma_v = \gamma\left(1 - P(T_v)\right). \tag{45}$$

[15] The rate of change in P is given by $\frac{\dot{P}}{P} = -\mu h_i$. But since labor is allocated symmetrically to innovation only in the measure P of sectors where no innovation has occurred, $h_i = \frac{H}{P}$, so that $\dot{P} = -\mu H$.

A key implication of the assumption that investment is (at least partially) reversible is that household consumption must evolve smoothly over the period T_v — it cannot jump discontinuously. Intuitively, if households anticipated a sharp rise in consumption in the future they could raise their utility by converting some of the capital stock into consumption goods immediately. As a result, the household's Euler equation implies that the rate of return on any asset held over the boom must equal zero.[16] In particular, the return to storing intermediate goods until after the boom must be zero. A positive return would exist if the wage rose discontinuously upon implementation because it would be cheaper to produce extra intermediates at the low wage just before the boom and substitute them for production at the high wage afterwards. The fact that, in equilibrium, the wage must therefore evolve smoothly across the boom pins down a tight relationship between the growth in productivity and the labour effort allocated back into production:

Proposition 2 *Asset market clearing under reversible investment at the boom requires that*

$$(1-\alpha)\Gamma_v = -\alpha \ln L(T_v) \tag{46}$$

Note that the smooth evolution of wages through the boom is a consequence of there being diminishing returns to labour. During the boom, firm values and wages grow in proportion to labour productivity. Since, just before the boom $\mu V^I(T_v) = w(T_v)$, an immediate corollary is that

$$\mu V_0^I(T_v) = w_0(T_v) = (1-\alpha)e^{-\gamma}\bar{A}_v k_0(T_v)^{\alpha}. \tag{47}$$

Output growth through the boom is given by

$$\Delta \ln Y(T_v) = (1-\alpha)\Gamma_v - (1-\alpha)\ln L(T_v) = \left(\frac{1-\alpha}{\alpha}\right)\Gamma_v \tag{48}$$

It follows directly from Proposition 2 that growth in output exceeds the discount factor across the boom. Since profits are proportional to output, this explains why firms are willing to delay implementation during the downturn. Because investment is reversible, consumption cannot jump at the boom, and so all of the increase in output must be associated with a sharp rise in investment.

4.4 Optimal Entrepreneurial Behavior During the Cycle

Optimal entrepreneurial behavior imposes the following requirements on our hypothesized equilibrium cycle:

[16] This is in stark contrast to Shleifer (1986) and Francois and Lloyd-Ellis (2003), where consumption jumps at the boom.

• At time $t = T_v$, firms must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) \geq V_0^D(T_v). \tag{E1}$$

• In sectors where viable ideas are identified during the downturn, firms must prefer to wait until the beginning of the next cycle rather than implement earlier and sell at the limit price:

$$V^I(t) \leq V^D(t) \qquad \forall\, t \in (T_v^*, T_v) \tag{E2}$$

• Search is not optimal during the expansion of the cycle. Since in this phase of the cycle $\mu V^D(t) < w(t)$, this condition requires that

$$\mu V^I(t) \leq w(t) \qquad \forall\, t \in (T_{v-1}, T_v^*) \tag{E3}$$

• Finally, in constructing the equilibrium above, we have implicitly imposed the requirement that the downturn is not so long that viable ideas are identified in every sector:

$$P(T_v) \geq 0. \tag{E4}$$

Taken together conditions (E1) through (E4) are restrictions that must be satisfied for the cyclical growth path we have posited to be an equilibrium.

5 The Stationary Cyclical Growth Path

5.1 Characterization

We focus on a stationary cyclical equilibrium growth path in which the boom size is constant at Γ every cycle and the cycle length is given by

$$\Delta = T_v - T_{v-1} \quad \forall\, v. \tag{49}$$

In addition, we denote the length of the stationary expansion phase as

$$\Delta^* = T_v^* - T_{v-1} \quad \forall\, v. \tag{50}$$

Along this path, $\bar{A}$ rises by e^{Γ} at each implementation boom, but consumption and capital evolve continuously so that their normalized values at the beginning of each cycle are given by

$$c_0(T_v) = e^{-\Gamma} c(T_v) = \hat{c} \tag{51}$$

$$k_0(T_v) = e^{-\Gamma} k(T_v) = \hat{k}. \tag{52}$$

In Appendix B, we demonstrate that, for a given stationary cycle length and boom size, a stationary equilibrium is equivalent to that which would be chosen by a social planner who is constrained to follow the specific innovation and implementation path associated with (Γ, Δ). This implication holds despite the presence of imperfect competition in the intermediate goods market for two reasons: (1) all intermediate sectors consist of monopolists who behave symmetrically and (2) within the cycle, labour effort is supplied inelastically. Consequently, relative prices are the same as they would be under perfect competition. The only implication of monopoly *within the cycle* is for the distribution of household income between profits and wages. The existence and uniqueness of a stationary solution to the constrained planner's problem, $\hat{k}(\Gamma, \Delta)$, is demonstrated using a standard contraction mapping theorem, and requires only that utility is bounded:[17]

$$\rho + (\sigma - 1)\frac{\Gamma}{\Delta} > 0. \tag{53}$$

Note that if $\sigma \geq 1$, this condition is satisfied for all (Γ, Δ) pairs.

Thus, for each pair (Γ, Δ) satisfying (53), there is a unique stationary path for the endogenous variables $\{c(t), k(t), L(t)\}_{T_{v-1}}^{T_v}$ which repeats itself every cycle. We can summarize these within–cycle dynamics in terms of the consumption–capital ratio and the capital–labour ratio:

$$\frac{\dot{(c/k)}}{(c/k)} = \left(\frac{\alpha}{\sigma} - 1\right)\left(\frac{k(t)}{L(t)}\right)^{\alpha-1} + \frac{c(t)}{k(t)} + \frac{\delta(\sigma-1)-\rho}{\sigma} \tag{54}$$

$$\frac{\dot{(k/L)}}{(k/L)} = \left(\frac{k(t)}{L(t)}\right)^{\alpha-1} - \frac{\delta}{\alpha}, \tag{55}$$

where $L(t) = 1$ in phase 1 and $L(t)$ is determined by (42) in phase 2. The associated phase diagram is shown in Figure 3. In a stationary cycle, the consumption–capital ratio must be the same at the end as at the beginning. Also the capital–labour ratio must be higher at the end than at the beginning, which implies the economy must be to the left of the $\dot{(k/L)} = 0$ locus. In phase 1 ($T_{v-1} \rightarrow T_v^*$), condition (35) implies that the economy lies below the $\dot{(c/k)} = 0$ locus, so that c/k falls and (since $L(t) = 1$) $k(t)$ rises. During phase 2 ($T_v^* \rightarrow T_v$), c/k initially continues to fall, but eventually the economy crosses the $\dot{(c/k)} = 0$ locus, so that it starts to rise again.

For any arbitrary pair of values (Γ, Δ) such a stationary cyclical path need not, however, satisfy either the free entry into innovation condition (Proposition 1) or the asset market clearing condition at the boom (Proposition 2). In Appendix B we show that under fairly weak parametric restrictions, only one pair of values $\left(\hat{\Gamma}, \hat{\Delta}\right)$ can satisfy these conditions along a stationary cyclical path.[18]

[17] The fact that household utility is bounded in equilibrium is equivalent to (E1). To see this observe that (E1) can be expressed as $V_0^I(T_v) > e^{-\left[R_0(T_{v+1}) - R_0(T_v)\right]} e^{\Gamma + \Delta\phi} V_0^I(T_v)$, which holds only if $R_0(T_{v+1}) - R_0(T_v) = \sigma(\Gamma + \Delta\phi) + \rho\Delta > \Gamma + \Delta\phi$. Re-arranging yields (53).

[18] Obviously this does not imply that the equilibrium is globally unique, only that it is so within the class of

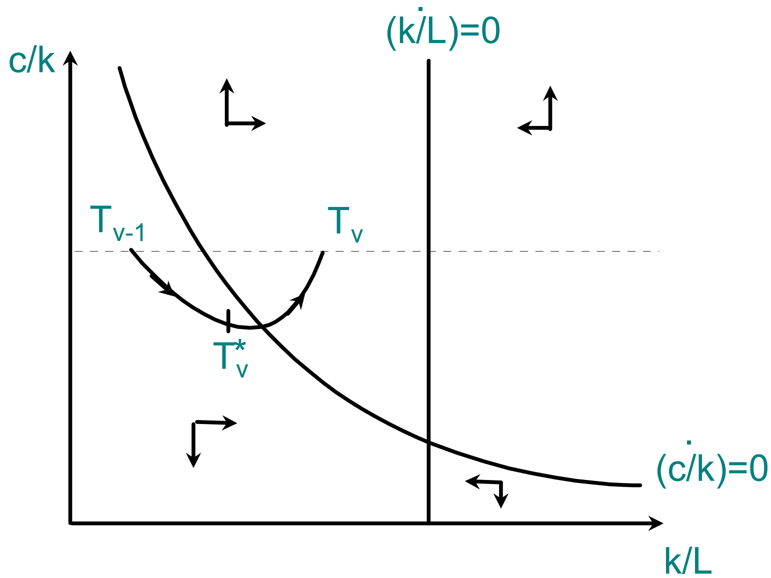

Figure 3: Within–Cycle dynamics in the stationary cyclical equilibrium

Proposition 3 *: There exists a $\sigma^* \in [0,1)$ and $\alpha^* \in (0,1)$ such that if $\sigma > \sigma^*$ and $\alpha < \alpha^*$, and if*

$$\mu(1-e^{-\gamma})(1-\omega) > \rho e^{-\gamma} \tag{56}$$

then a stationary cyclical equilibrium $(\hat{\Gamma}, \hat{\Delta}, \hat{k})$ satisfying all the conditions above is unique within this class of cycling equilibria.

5.2 Baseline Example

We numerically solve the model for various combinations of parameters that satisfy (56) and check the existence conditions (E1)–(E4). The parameters for our baseline example are given in Table 1.[19] The parameters α and γ imply a capital share of 0.3, a profit share of 0.13 and a markup rate of around 25%. The implied measured labour share averaged over the cycle is 62%: during an expansion it is 57%, but during contractions it rises because not all employed labour is used in production. The unit–valued intertemporal elasticity of substitution implies logarithmic preferences. Given these values, we chose μ, ρ and ω so as to match a long–run annual growth rate of about 2%, an average risk–free real interest rate of roughly 4%, and a cycle length of

stationary cyclical paths described above. In particular, we know that there exists at least one other equilibrium growth path — the standard acyclical one.

[19] The Gauss program used to generate the numerical simulations and the diagrams contained here is downloadable from the following URL: http://qed.econ.queensu.ca/pub/faculty/lloyd-ellis/research.html

approximately 8 years. These values roughly correspond to average data for the post–war US. The implied value of ω is admittedly rather high if we interpret it purely as a tax on profits.[20] However, as noted earlier, we view ω as also representing a number factors that affect the ratio of profits to wages (e.g. the fraction of rents accruing to upstream parties in the innovation process).

Table 1: Baseline Parameters

Parameter	Value
α	0.30
γ	0.20
σ	1.00
ρ	0.025
μ	1.80
δ	0.10
ω	0.70

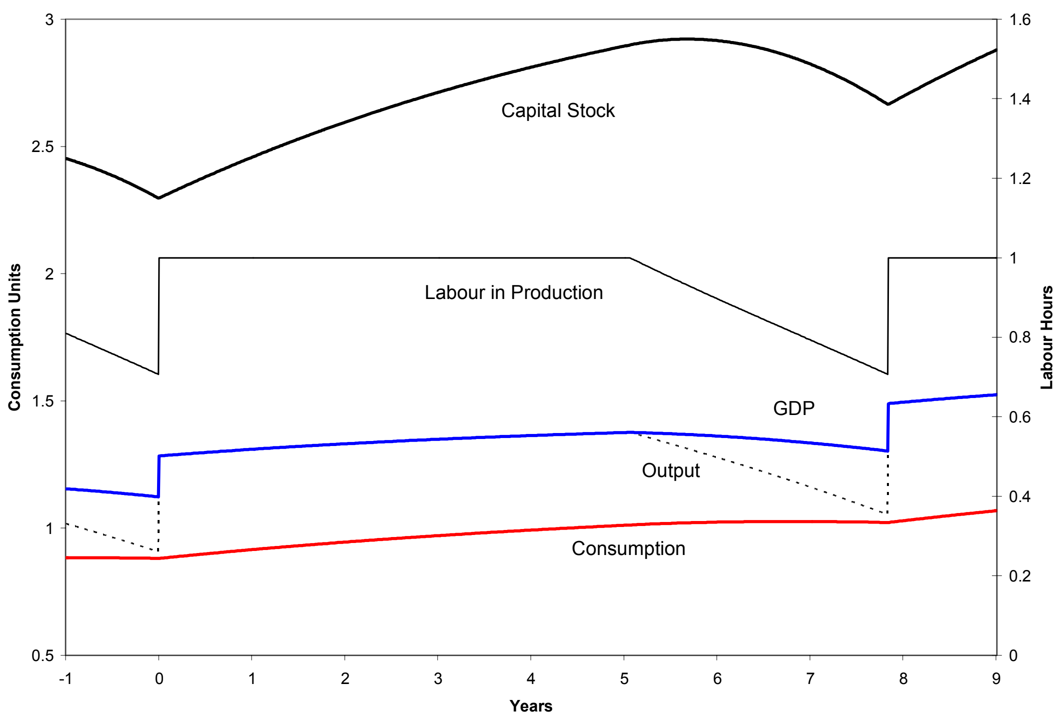

Figure 4: Evolution of Key Aggregates

[20]McGrattan (1994) estimates taxes on capital income in the US to be approximately 50%.

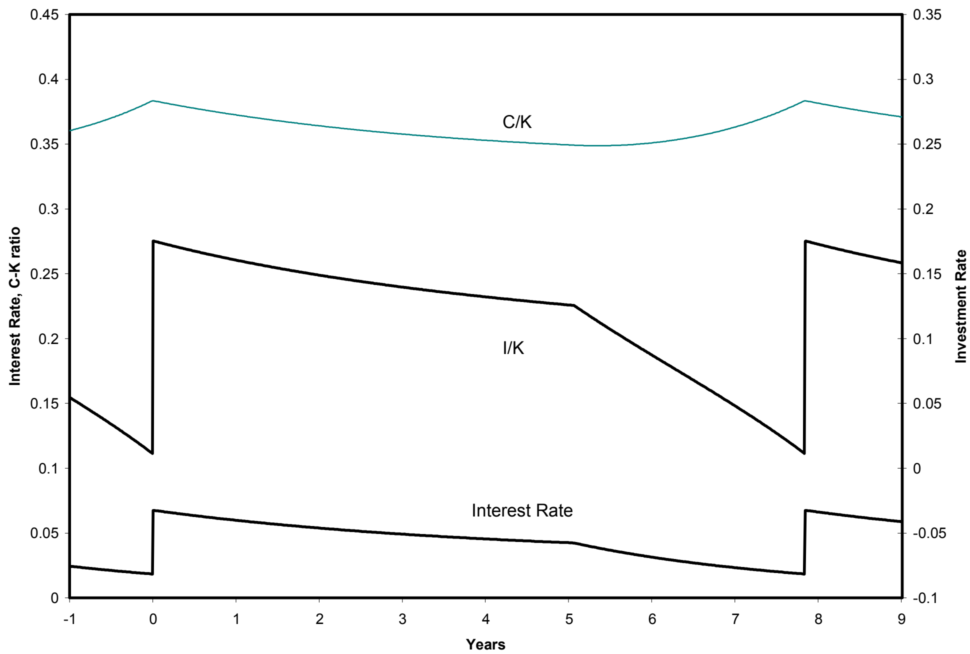

Figure 5: Rates and Ratios during the Cycle

Figure 4 depicts the evolution of key aggregates over the cycle for this baseline case. In this example, the capital stock grows monotonically through the first phase and into the second, before starting to decline towards the end of the cycle. Note that this decline is purely due to lack of maintenance, not because of negative investment. As shown in Figure 5, although the investment rate falls rapidly in phase 2 it never goes below zero. Consumption evolves much more smoothly than investment, rising through the first phase the slowing down and falling somewhat in the second, before accelerating at the subsequent implementation boom. Note that though not as volatile as investment, movements in aggregate consumption, relative to trend are clearly pro-cyclical.[21]

After rising gradually during the expansion, output — defined here as the sum of consumption goods and capital formation — falls dramatically in phase 2. However, correctly measured GDP should include the payments made to labour used in entrepreneurial search. As illustrated in Figure 4, GDP also falls during phase 2, but much less dramatically. The reason GDP falls is that labour used in production is being paid below its marginal product. As labour effort is transferred into innovative activities, the marginal cost in terms of lost output exceeds the marginal benefit of search. In effect, the transfer of labour imposes a negative externality on the

[21] This feature distinguishes our cycle from one generated by an anticipated TFP shock in a one sector RBC model. Under such a scenario consumption would accelerate during the recession that precedes the boom.

profits of incumbent producers. Because it is offset by the fall in this intangible investment, the rise in GDP at the boom is also less dramatic.

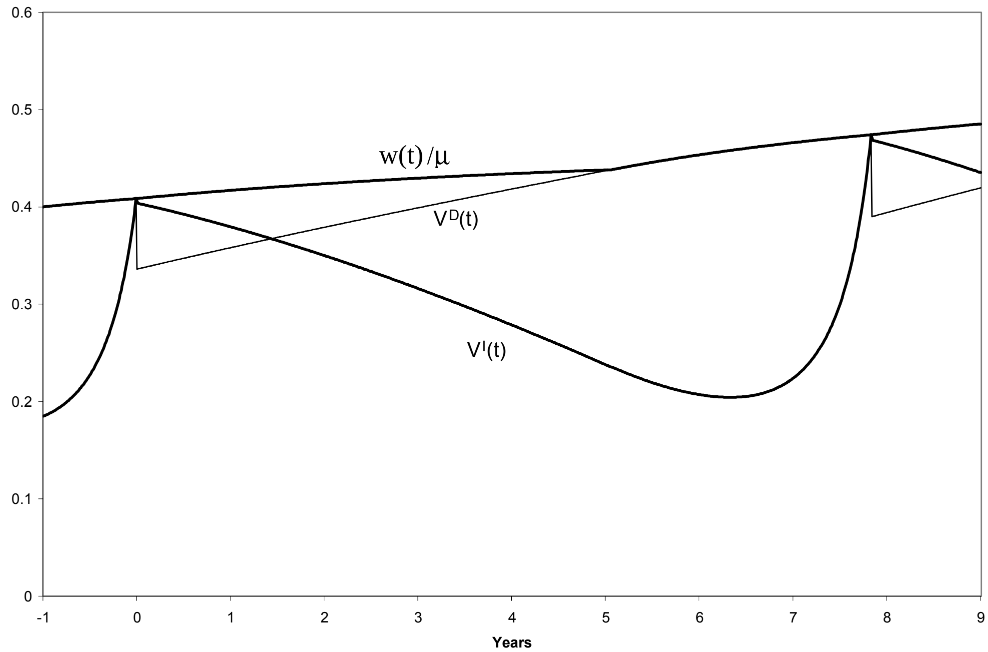

Figure 6: Evolution of Firm Values and the Wage

Figure 6 depicts the evolution of $w(t)/\mu$, the value of incumbent intermediate producers that have not been displaced, $V^I(t)$, and the value of viable ideas whose implementation is delayed until the subsequent boom. As can be seen, the value functions conform with conditions (E1)–(E3). $V^I(t)$ falls through the expansion as dividends are paid out. During the contraction the likelihood of being displaced at the boom declines and towards the end of the cycle this factor dominates, driving $V^I(t)$ sharply upward at the end. Interestingly, the wage does not vary much relative to trend over the cycle. In this baseline example it is mildly countercyclical, but in general it could rise more or less rapidly in the contraction. The growth of the wage over the cycle depends on the relative effects of falling capital versus falling labour in production on the rate at which the capital–labour ratio rises. The effect of the growth in TFP at the boom is exactly offset by the reallocation of labour effort back into production.

5.3 Comparative Stationary Cycles

Table 2 documents several statistics from the model for various deviations in parameter values from the baseline example. In most cases we raised or lowered the individual parameter by

10% from its baseline value. In all of the cases considered in the table, the investment is positive throughout the cycle. The implications for the average annualized rates of growth, $\bar{g}$, and interest, $\bar{r}$, are generally similar to what one obtains for the acyclical growth path with the exceptions discussed below. Changes in the share of income received by capital, α, or the depreciation rate, δ, have no impact on long–run growth. This is because capital plays a supportive role in this kind of endogenous growth model, accumulating to the extent necessary in order to complement the growth in TFP (see Grossman and Helpman, 1991).

Aside from their impact on long run growth and interest rates, parameter changes have a substantial impact on the nature of cycles in the short–run. In particular, the length of expansions Δ^* and hence overall cycle length. Increases in parameters that directly reward innovation: the step size of productivity increments, γ, the productivity of innovative efforts, μ, and the proportion of such efforts that can be kept by entrepreneurs, $1-\omega$, all increase the average growth rate. All of these factors also shorten cycle lengths. To see why, first note that all of these changes increase the average interest rate through the cycle, as they all raise the value of searching for viable ideas. But a higher interest rate implies that the value of innovating is rising more quickly through the cycle. This is because the withholding of implementation to the boom means that innovations yield returns only with a delay. A higher interest rate raises the costs of delay so that the value of delayed innovations rises more rapidly through time. Recall that the expansionary phase of each cycle ends when entrepreneurship first becomes profitable. As the value of innovations rises more rapidly with higher interest rates, this happens earlier, and thus shortens cycle lengths.

Table 2: Comparative Stationary Cycles

Parameters		Γ	Δ	Δ^*	k^*	$\bar{g}$ (%)	$\bar{r}$ (%)	$P(T)$
Baseline		0.15	7.83	5.06	2.30	1.90	4.40	0.26
$\alpha =$	0.27	0.12	6.53	4.15	1.94	1.90	4.40	0.38
	0.33	0.18	9.34	6.12	2.70	1.90	4.40	0.11
$\delta =$	0.08	0.15	7.86	5.12	2.89	1.90	4.40	0.26
	0.12	0.15	7.96	5.15	2.06	1.90	4.40	0.25
$\gamma =$	0.18	0.16	10.26	7.12	2.33	1.54	4.04	0.12
	0.22	0.14	6.19	3.70	2.22	2.33	4.83	0.34
$\mu =$	1.62	0.16	9.58	6.43	2.27	1.70	4.20	0.19
	1.98	0.14	6.51	4.03	2.29	2.14	4.64	0.30
$\rho =$	0.0225	0.14	7.06	4.32	2.37	1.97	4.23	0.30
	0.0275	0.16	8.40	5.59	2.19	1.93	4.68	0.19
$\sigma =$	0.9	0.14	7.39	4.65	2.37	1.89	4.20	0.30
	1.1	0.16	8.29	5.48	2.21	1.92	4.61	0.20
$\omega =$	0.6	0.17	6.56	3.75	2.08	2.57	5.07	0.16
	0.8	0.13	10.82	8.08	2.54	1.20	3.71	0.35

Changes in consumer preference parameters can alter cycle length in ways which countervail, and sometimes overshadow, the standard direct effects. For example, increasing σ, lowering intertemporal substitutability, generally induces lower growth in the acyclical steady state because consumers are less willing to delay consumption. A similar effect is present here. However, as the table shows, this increase also raises cycle length and amplitude, inducing more entrepreneurship and a larger boom. The net effect, as the table shows, is an increase in the average growth rate for the baseline parameter configuration. A similar sequence of effects is present for increases in ρ, but this is not large enough to offset the direct effect so that the qualitative effect on average growth is the same as in an acyclical steady state.

Changing these parameters too far in either direction results in one or more of the existence conditions (E1)–(E4) being violated. In particular, this places limits on the length of cycles that can arise in equilibrium. For example, holding other parameters constant and raising σ to a high enough value results in their being no downturn length for which the implied productivity gains are sufficient to induce households to delay consumption (given that $\dot{K} > -\delta K$). If one is willing to entertain average growth rates above 3%, it is possible to allow higher values of σ by increasing the value of μ, but this implies much shorter cycles. In general, cycle lengths, Δ, much longer than those documented below, do not appear to be feasible under the assumptions considered here.

6 Implications for Investment and Tobin's Q

Tobin's Q is measured as the ratio of the market value of firms liabilities to the replacement cost of their capital stock. According to neoclassical theory, with capital adjustment costs and constant returns to scale, Tobin's Q should summarize the incentives for investment. However, while there is some evidence of a long run relationship, neither micro nor macro level empirical work has generally found a significant short–run relationship between investment and Tobin's Q. As is well known, one cannot necessarily infer from this that investment is sub–optimal because Tobin's Q need not reflect the *marginal* incentives to invest (see Abel, 1979 and Hayashi, 1982). Moreover, as in the current model, equity values are likely to include the values of intangible, as well as tangible capital so that measured investment and capital stocks understate true levels. But then the question arises as to what kind of relationship we should expect to observe between investment and *measurable* proxies of financial incentives and financial values over the business cycle.

Figure 7 shows the investment rate and Tobin's Q for the US on a quarterly basis between 1948 and 2006. Our calculations of Tobin's Q extend the work of Hall (2001) beyond 1999.[22] Because

[22] Estimating Tobin's Q involves a number of strong assumptions, but in principle the number should exceed

the investment rate is most highly correlated with the value of Tobin's Q lagged about four quarters, one common interpretation of this data is that there is a lead–lag relationship between the two variables. In principle, such a relationship might occur for essentially mechanical reasons: a rise in Tobin's Q signals profitable investment opportunities, but actual investment can only respond with a considerable delay. Note that this is not the same as slow adjustment of the capital stock due, say, to a time–to–build constraint — RBC models require explicitly built–in "time to plan" constraints in order to explain the observed pattern in this way.[23]

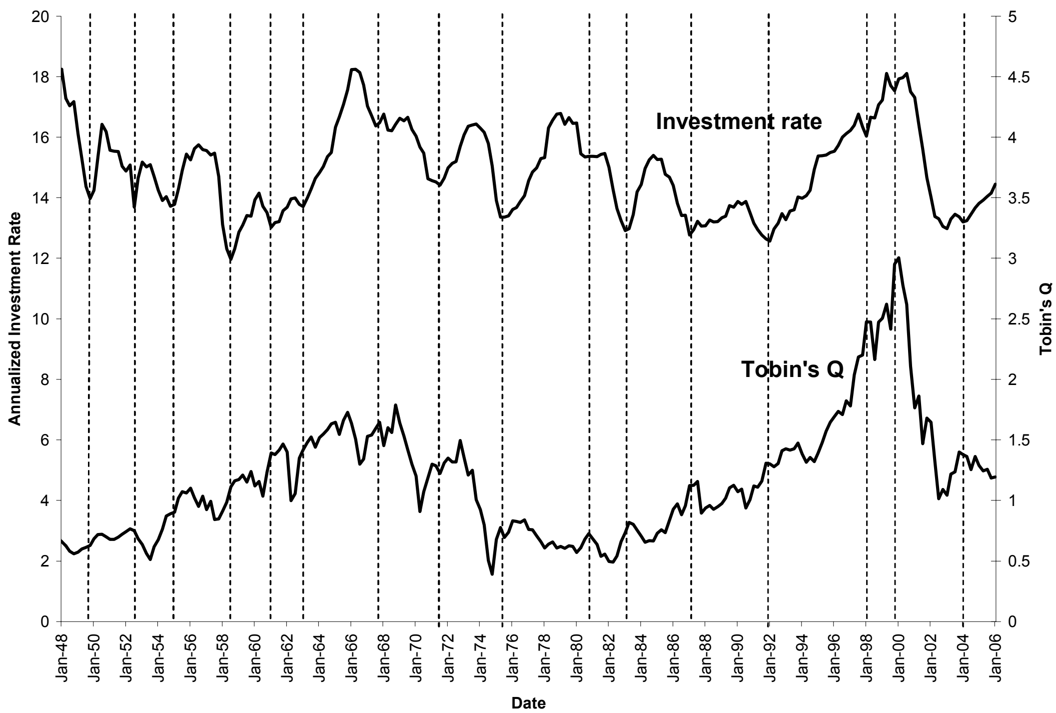

Figure 7: US Investment Rate and Tobin's Q (extension of Hall's (2001) calculations to 2006)

However, the relationship between the variables seems to be somewhat more complex than this. To illustrate this, the vertical lines in Figure 7 mark significant cyclical investment rate troughs during the post–war era.[24] There were 16 such troughs, 11 of which coincided with

unity. The fact that it often doesn't remains an unresolved puzzle. Possibilities include that the estimated value of reproducible capital is generally too large, or that asymmetric information plays a large role (see Robertson and Wright, 2005). Here we take the view that while these factors may affect the estimated level and short term volatility, the medium term cyclical properties would remain unchanged.

[23] Christiano and Todd (1996), Bernanke, Gertler and Gilchrist (1999) and Christiano and Vigfusson (2001) introduce "time to plan" as a fixed time period between the date when the decision to invest more (less) is made and the date when the actual funds are allocated.

[24] The criteria we used for selecting these troughs were that the investment rate declined for at least 2 quarters prior to the trough and increased for at least 2 quarters after it.

NBER–dated recessions. Following 10 of these troughs, during investment expansions, Tobin's Q rises initially reaching a cyclical peak prior to, or coincident with, the cyclical investment peak, and then declines. In the other 6 cases Tobin's Q declines immediately following the trough. On average investment and Tobin's Q are positively correlated during these investment expansions, although this is especially driven by the long expansions of the 1960s and 1990s.

A more robust observation, however, can be made regarding investment recessions. During 15 of these recessionary phases leading up to an investment trough, Q reaches a cyclical trough and then expands. The only exception occurs during the small investment recession at the end of 1997, when Q continued to expand throughout. The average behavior of the two series in the 2 year lead up to these troughs is depicted in Figure 8. As can be seen, the average investment recession lasted 6 quarters. On average Tobin's Q reaches a cyclical trough 4 quarters ahead of that reached by the investment rate and then rises rapidly as the trough approaches. On average, the two series are negatively correlated during phases where investment is declining.

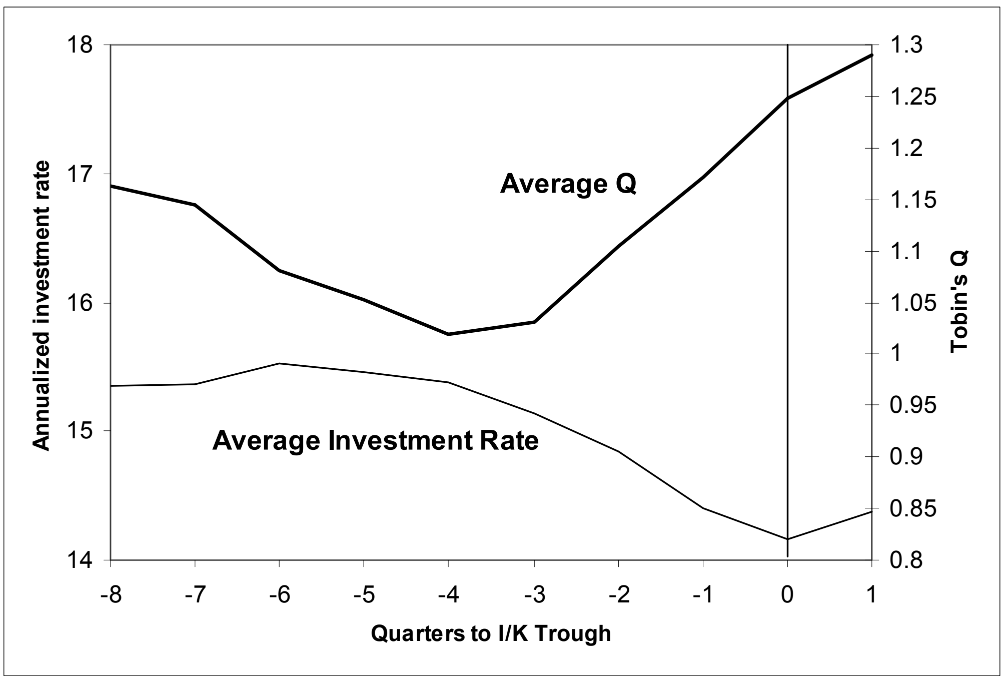

Figure 8: Average Behaviour during 16 investment recessions (1948-2006)

In our model, even though an entrepreneur or manager does not implement a discovery, he is able to credibly signal it. Consequently, Tobin's Q reflects the value of both implemented and unimplemented innovations. Tobin's Q is thus given by

$$Q(t) = 1 + \frac{\Pi(t)}{K(t)}, \tag{57}$$

where $\Pi(t)$ denotes the market value of the intangible capital tied up in firms. Figure 9 illustrates the evolution of Tobin's Q and the aggregate investment rate over the cycle in the baseline example. During an expansion the value of intangible capital is equal to the value of incumbent firms: $\Pi(t) = V^I(t)$. Since $K(t)$ rises and $V^I(t)$ declines, Tobin's Q falls monotonically throughout this phase. During a contraction, some sectors experience innovations, so there exist production methods that are certain to be made obsolete at the next round of innovation. At time t the measure of such sectors is $1 - P(t)$, and so

$$\Pi(t) = (1 - P(t))[V^T(t) + V^D(t)] + P(t)V^I(t), \tag{58}$$

where

$$V^T(t) = V^I(t) - \frac{P(T_v)}{P(t)} V^D(t) \tag{59}$$

is the value of these "terminal" production methods. During the contraction $Q(t)$ initially declines as $K(t)$ continues to grow. However, eventually the growth in the value of intangible capital starts to dominate as we approach the boom, so that $Q(t)$ rises in anticipation.

Comparing Figures 8 and 9, it can be seen that the predicted pattern of Q and the investment rate during contractions matches well (in a qualitative sense) the relationship between their counterparts in US data. In particular, the trough in Tobin's Q occurs part way through the recession after which Q rises rapidly, so that I and Q are negatively correlated. The model also predicts that during the booms and expansion, Tobin's Q and the investment rate are positively correlated, as in the data. However, because of the sharp increase in investment generated by the implementation boom, the similarities during the expansion phase are less compelling. A better match between the model's predictions and the data could be obtained if the boom were more drawn out, so that the investment rate grows throughout the first part of the expansion. We discuss possible ways of extending the model in this direction in the conclusion.

Note finally, that in our framework, the Tobin's Q is not an index of aggregated "news" about sectoral investment opportunities. Rather it is forecasting the boom in aggregate demand that will affect all sectors symmetrically, even though only some experience productivity increments. Although news about future investment opportunities in different sectors is also undoubtedly important, this is not the mechanism at work here. Interestingly, most studies of the relationship between investment and Tobin's Q at the firm or industry level have generally found at best weak evidence of a relationship (contemporaneous or lagged), and certainly much weaker than suggested by the correlation between the investment rate and lagged Q at the aggregate level.[25]

[25] There is a large literature here, most of which has found these effects to be weak – Blundell, Bond, Devereux, Schiantarelli (1992), Abel and Blanchard (1986), Hayashi and Innoue (1991) and Gilchrist and Himmelberg (1995) are a few. However recent research constructing an alternative measure of Q based on securities analysts' earnings forecasts: Cummins, Hassett and Oliner (1999) and Bond and Cummins (2001), do much better suggesting that perhaps basic Q theory, with the correct measure of firm value, can provide guidance to investment behavior at the micro level.

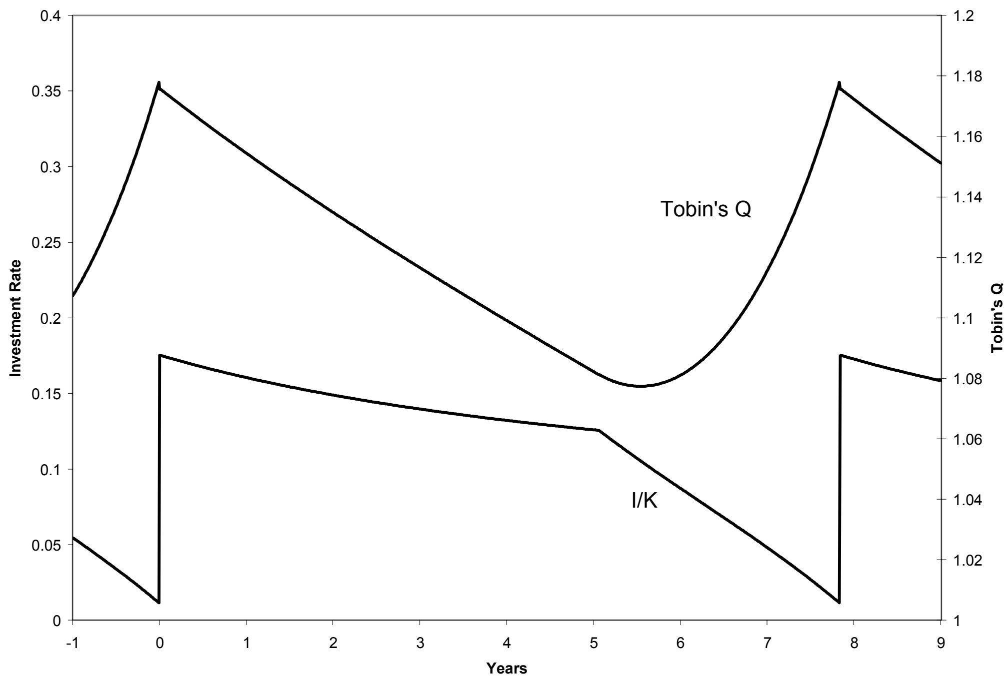

Figure 9: Tobin's Q and the Rate of Investment

Our model offers one interpretation of this difference — part of the aggregate relationship may be driven by general equilibrium effects reflecting endogenous delay.

7 Extension: Production Labour

One counter–factual implication of our model is the fact that wages rise more somewhat more rapidly during recessions than expansion. However, the only type of labour in our basic model is a high skilled type of manager that can be used in both production and entrepreneurship. It is straightforward to extend the model to include less skilled workers that are used only in production. To do this, we replace (3) with the following Cobb–Douglas production function

$$Y(t) = K(t)^{\alpha} X(t)^{\beta} L_P(t)^{1-\alpha-\beta}, \tag{60}$$

where $L_P(t)$ denotes production labour. Assume that production labour is in inelastic supply at $\bar{L}_P$ and earns an equilibrium wage $w_P(t)$. With this simple extension, the qualitative nature of the cyclical dynamics described above remains unchanged.[26] The only additional implications

[26] Calculations can be obtained from the authors upon request. Note that production labour, like capital, is allocated competitively and receives it marginal product. Consequently, the constrained social planner's problem still coincides with the decentralized outcome.

of the model are that the demand for production labour rises during expansions, as TFP grows and capital is accumulated, and falls during downturns as the effort of more skilled labour is reallocated away from production. Since full employment obtains, the wages of production labour therefore rise during the expansion, and fall during the contraction. The wages of the two types of labour are given by

$$w_P(t) = (1-\alpha-\beta)\bar{A}_{v-1}^{\beta}K(t)^{\alpha}L(t)^{\beta}\bar{L}_P^{-\alpha-\beta} \tag{61}$$

$$w(t) = e^{-\gamma}\beta\bar{A}_{v-1}^{\beta}K(t)^{\alpha}L(t)^{\beta-1}\bar{L}_P{}^{1-\alpha-\beta}. \tag{62}$$

We consider a representative numerical example in which the parameters are the same as in the baseline case, except that $\alpha = 0.26$ and $\mu = 2.6$, and we add two more parameters: $\bar{L}_P = 10$ and $\beta = 0.62$.[27] In this case, the measure of production workers is 10 times that of manager/entrepreneurs and the implied total labour share in production is 64%.[28] Figure 10 depicts the evolution of the wage of production workers, w_P, the average wage in production and the average wage of all workers. During the downturn, even though the skilled wage is growing, the average wage in production declines rapidly, both because w_P falls and because of a composition effect: the measure of skilled workers used in production is declining. However, the average wage across all workers grows at almost exactly the same rate as in the expansion. This does not imply that this average wage is acyclical, however, because it rises dramatically at the boom, reflecting the rise in the production wage.

The implied skill–premium in this model is given by the ratio of the two wages:

$$\text{Skill Premium} = \frac{w(t)}{w_P(t)} = \frac{e^{-\gamma}\beta\bar{L}_P}{(1-\alpha-\beta)L(t)}.$$

Thus, the model predicts a somewhat counter–cyclical skill premium: falling during booms, constant during expansion and rising during downturns. This cyclical pattern is consistent with the findings of Barlevy and Tsiddon (2006), once trend effects are removed. In Francois and Lloyd–Ellis (2006b), we consider a related model without capital, but in which involuntary unemployment arises as a result of incentive problems. There we show that various patterns in the cyclical nature of wages are possible, depending on parameters.

8 Concluding Remarks

This paper develops a general equilibrium theory of endogenous cycles, which emphasizes the asynchronous evolution of investment in tangible and intangible capital. Because of the dynamic

[27] Reducing β below 0.7, causes the growth rate to fall and the cycle to become longer because it reduces the profit share. The parameter μ was increased in order to maintain a similar growth rate and cycle length to our baseline case.

[28] This implies an average of 67% over the cycle if we also include the wages of skilled workers engaged in entrepreneurship.

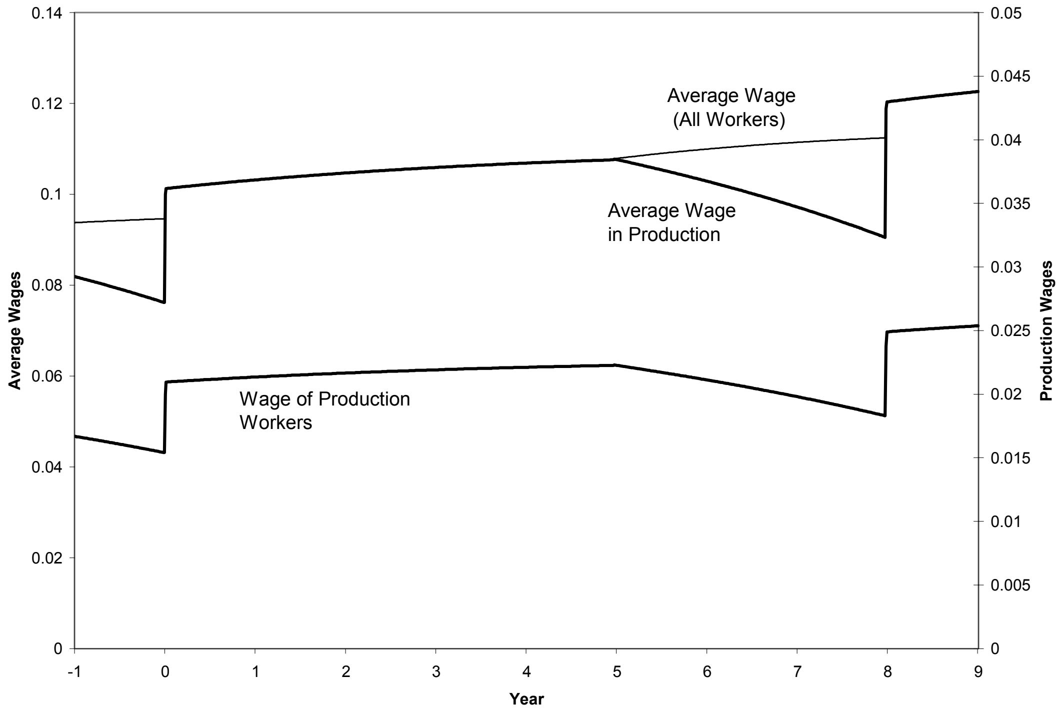

Figure 10: Wages over the Cycle

externalities inherent in the process of entrepreneurship, independently acting firms have incentives to undertake both types of investment in a clustered and seemingly coordinated fashion. Consequently, the aggregate economy oscillates between periods of intensive physical capital accumulation, rapid productivity growth and expanding output, and periods of slowing capital formation and declining output, but intangible capital accumulation. The model endogenously generates a volatile investment rate in the presence of relatively smooth consumption growth and the relationship between the investment rate and Tobin's Q conforms reasonably well with that observed during US recessions.

The relationship between I/K and Q in our model is the result of the endogenous delay at the centre of our cyclical equilibrium. In contractions, although potential productivity is higher (i.e. knowledge capital is being built) and the market value of firms' assets are rising, the anticipated boom induces innovators to hold off on implementation. Since investment will not pick up again until after productivity improvements are implemented, investment lags the movement of equity values (or Tobin's Q). In contrast, in the endogenous cycle models of Bental and Peled (1996), Freeman et al. (1999), Matsuyama (1999, 2001) and Walde (2005) the market value of intangible capital is perfectly correlated with productivity, so it rises contemporaneously with the increase in incentives for physical capital accumulation.

There are a number of possible extensions to the basic framework outlined here that we are currently working on:

• Fluctuations in employment — Our focus here has been on investment and stockmarkets, and we have deliberately abstracted from employment fluctuations. In Francois and Lloyd–Ellis (2006a) we develop a related model in which production workers are hired through relational contracts and incentive problems result in involuntary unemployment. In that paper we find that, although productivity improvements are clustered at the boom, a significant component of entry and exit can occur during recessions. Moreover, this framework provides a useful perspective on the behavior of job creation, job destruction, and worker flows over the cycle.

• Endogenous R&D — Here the search for commercially viable ideas and productivity improvements is counter–cyclical. There is some evidence that this kind of innovative activity is undertaken by managers during periods of slack demand (see Nickell, Nicolitsas and Patterson, 2001). However, recent evidence suggests that R&D is procyclical, even for firms that are not obviously cash–constrained during downturns (see Barlevy, 2005, and Walde and Woitek, 2004). In Francois and Lloyd-Ellis (2006b) we introduce endogenous R&D as a separate, knowledge-intensive activity that generates ideas whose commercial viability is unclear. The resulting stock of ideas can then be drawn upon by manager/entrepreneurs in their search for productivity improvements, and matched with specific markets. As in the current paper, entrepreneurial search is counter-cyclical, but R&D investment is pro-cyclical.

• Stochastic fluctuations — The cycle that we study here is deterministic and stationary. Variations in the aggregate growth rate and the length and amplitude of the cycle could be introduced by adding exogenous noise components to productivity growth. Such a framework can generate non–linear output dynamics akin to a duration-dependent, Markov switching process.

• Smoothing the boom — Although, due to capital accumulation, the economy experiences expansion throughout most of the cycle, the boom is unrealistically dramatic as it occurs only for the instant that delayed innovations are implemented.[29] This feature also creates obvious difficulties for the model in matching variables during the expansion. One way to remedy this would be to allow implementation to have a stochastic component. In this cas, though targeted at a common date, implementation would occur in a distribution around that date causing aggregate productivity to rise gradually, and for a sustained interval.

[29] Note, however, that Dahl and Gonzalez-Rivera (2003, p. 1) provide evidence supporting "three phases in the business cycle: rapid linear contractions, *aggressive short-lived convex early expansions,* and moderate/slow relatively long concave late expansions."

Appendix A – Proofs

Proof of Lemma 1 We show: (1) that if a signal of success from a non-incumbent entrepreneur is credible, search stops in that sector; (2) given (1), entrepreneurs have no incentive to falsely claim success.

Part (1): If entrepreneur i's signal of success is credible then all other entrepreneurs believe that i has a productivity advantage which is e^{γ} times better than the existing incumbent. If continuing to search in that sector, another entrepreneur will, with positive probability, also identify a productive advantage of e^{γ}. Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of others, so that both firms Bertrand compete with the same idea. Returns to searching in another sector where there has been no signal of success, or from simply allocating labour effort to production, $w(t) > 0$, are thus strictly higher.

Part (2): If success signals are credible, entrepreneurs know that upon success, further search in their sector will cease, from Part (1), by their sending of a costless signal. They are thus indifferent between falsely signalling success when it has not arrived, and sending no signal. Thus, there exists a signalling equilibrium in which only successful entrepreneurs send a signal of success.

An incumbent entrepreneur is also free to send a signal of success in his own sector. However, even without a success, an incumbent would have a strict incentive to send such a signal. Consequently, such signals are never credible.■

Proof of Lemma 2: The present value of the capital owners' net income in any final goods sector j is:

$$V_j^K(t) = \int_t^{\infty} e^{-[R(\tau)-R(t)]} \left[q(\tau) K_j(\tau) - \dot{K}_j(\tau) - \delta K_j(\tau) \right] d\tau. \tag{63}$$

Differentiating (63) with respect to time yields

$$\dot{V}_j^K(t) = r(t) V_j^K(t) - q(t) K_j(t) + \dot{K}_j(t) + \delta K_j(t). \tag{64}$$

Free entry of replacement capital and reversibility imply that $V_j^K(t) = K_j(t)$ and $\dot{V}_j^K(t) = \dot{K}_j(t)$ $\forall$ j. The result follows immediately.■

Proof of Proposition 1: Subtracting (34) from (33) we get

$$\frac{\dot{c}(t)}{c(t)} - \frac{\dot{k}(t)}{k(t)} = \frac{c(t)}{k(t)} - \left(1 - \frac{\alpha}{\sigma}\right) k(t)^{\alpha-1} - \frac{\rho + (1-\sigma)\delta}{\sigma} \tag{65}$$

It follows that in order for $c(t)/k(t)$ to be declining in the first phase

$$\frac{c(t)}{k(t)} < \left(1 - \frac{\alpha}{\sigma}\right) k(t)^{\alpha-1} + \frac{\rho + (1-\sigma)\delta}{\sigma}. \tag{66}$$

which is the left hand inequality in (35).

For $r > \dot{w}/w$, we require that

$$\alpha k(t)^{\alpha-1} - \delta > \alpha \frac{\dot{k}(t)}{k(t)} \tag{67}$$

Substitution using (34)yields

$$\alpha k(t)^{\alpha-1} > \delta + \alpha \left(k(t)^{\alpha-1} - \frac{c(t)}{k(t)} - \delta \right), \tag{68}$$

which rearranges to the right hand inequality in (35).■

Proof of Lemma 3: Under the posited cycle, since capital depreciation rates are independent of utilization, and the marginal product of capital is strictly positive, installed capital is fully utilized. At T_v^*, the first point in each cycle where $\mu V^D(t) = w(t)$, since the discount factor does not change discretely, and $V^D(t) = e^{-\beta(t)} V_0^I(T_v)$ neither does V^D. With full capital utilization, the wage must move upwards discretely if $L(t)$ moves down discretely. But since at $t \to T_v^*$, $\mu V^D(t) \to w(t)$ from below, this is not possible. It follows that $L(T_v^*) = 1$ and, if it is to change at that point, must change smoothly. Since $L(T_v^*)$ adjusts smoothly at this point, and capital utilization is non–variable, output cannot discretely fall at T_v^*. Since $r(t) = q(t) - \delta$ cannot discretely fall instantaneously and the discount factor does not discretely change, the Euler equation ensures that consumption cannot discretely change T_v^* either. Consequently, investment, $\dot{K}$, cannot jump at T_v^* so that wage growth $\frac{\dot{w}(t)}{w(t)}$ discretely rises to the new level at which r is growing, i.e. $\alpha k(t)^{\alpha-1} - \delta$. Consequently employment growth in production $\frac{\dot{L}(t)}{L(t)}$ must discretely fall to a negative level.■

Proof of Proposition 2: During the boom, for entrepreneurs to prefer to implement immediately, it must be the case that $V_0^I(T_v) > V_0^D(T_v)$. Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom: $\mu V^I(T_v) = \mu V^D(T_v) = w(T_v)$. Free entry into entrepreneurship at the boom requires that $\mu V_0^I(T_v) \leq w_0(T_v)$. The opportunity cost of financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where new ideas have been identified. Since this return across the boom must equal zero, it must be the case that $V_0^I(T_v) = V^I(T_v)$. It follows that asset market clearing at the boom requires

$$\log\left(\frac{w_0(T_v)}{w(T_v)}\right) = (1-\alpha)\Gamma_v - \ln L(T_v) \geq 0. \tag{69}$$

In sectors with no new ideas, incumbent firms could sell claims to stored output, use them to finance greater current production and then store the good to sell at the beginning of the next boom. Free entry into storage implies that the rate of return (the growth in the wage) to this activity must satisfy

$$\log\left(\frac{w_0\,(T_v)}{w(T_v)}\right) = (1-\alpha)\Gamma_v - \ln L(T_v) \leq 0. \tag{70}$$

Combining (69) and (70) yields (46).■

Appendix B – Uniqueness of the Stationary Equilibrium Cycle

We first show that the within–cycle decentralized equilibrium is equivalent in its aggregate implications to that which would be chosen by a social planner who is constrained to follow the innovation and implementation cycle assumed above.[29]

Lemma B1: *For a given cycle length, target value of $P(T_v)$, and boundary values for the capital stock, k_0 and k_T, the within–cycle dynamics are equivalent to that which would be chosen by a social planner that is constrained to attain $P(T_v)$ and k_T.*

Proof: Fix the value of Γ and let $P^* = 1-\Gamma/\gamma$. Consider first the within–cycle problem, taking the cycle length Δ, and the boundary values k_0 and k_T as given

$$V^P(k_0, k_T; \Gamma, \Delta) = \max_{P(.),c(.),L(.),k(.)}\left\{\int_0^{\Delta} e^{-\rho t}\left(\frac{c(t)^{1-\sigma}-1}{1-\sigma}\right)dt\right\} \tag{71}$$

subject to

$$\begin{aligned}
\dot{k}(t) &= k(t)^{\alpha}L(t)^{1-\alpha} - \delta k(t) - c(t) && (72)\\
-\dot{P}(t) &= \mu\left[1-L(t)\right] && (73)\\
k(0) &= k_0,\ k(\Delta) = k_T,\ P(0) = 1,\ P(\Delta) = P^* = 1-\Gamma/\gamma && (74)\\
L(t) &\leq 1 && (75)
\end{aligned}$$

where T_{v-1} and T_v are normalized to 0 and Δ respectively.

The Hamiltonian associated with the within–cycle planning problem is

$$\begin{aligned}
&H(c(t), k(t), L(t), P(t), \lambda_1(t), \lambda_2(t), \psi(t), t)\\
=\ & e^{-\rho t}\left(\frac{c(t)^{1-\sigma}-1}{1-\sigma}\right) + \lambda_1(t)\left[k(t)^{\alpha}L(t)^{1-\alpha} - \delta k(t) - c(t)\right]\\
&+\lambda_2(t)\mu\left[1-L(t)\right] + \psi(t)\left[1-L(t)\right]
\end{aligned}$$

[29] Lemmas B1 and B2 are inspired by Freeman, Hong and Peled (1996).

where $\lambda_1(t)$ and $\lambda_2(t)$ are the costate variables and $\psi(t)$ is the Lagrange multiplier on the labour constraint such that $\psi(t)\,[1 - L(t)] = 0$. The Hamiltonian conditions are (72), (73) and

$$\frac{\partial H}{\partial c} = e^{-\rho t}c(t)^{-\sigma} - \lambda_1 = 0 \tag{76}$$

$$\frac{\partial H}{\partial L} = \lambda_1(t)(1-\alpha)k(t)^{\alpha}L(t)^{-\alpha} - \lambda_2(t)\mu - \psi(t) = 0 \tag{77}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial k} = -\lambda_1(t)\left[\alpha k(t)^{\alpha-1}L(t)^{1-\alpha} - \delta\right] \tag{78}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial P} = 0 \tag{79}$$

Case 1: If $L(t) = 1$ then $\psi(t) > 0$. Then differentiating (76) w.r.t. time and using (78) to substitute out $\dot{\lambda}_1/\lambda_1$ we get

$$\rho + \sigma\frac{\dot{c}}{c} = \alpha k(t)^{\alpha-1} - \delta \tag{80}$$

This condition combined with (72) (with $L(t) = 1$) are, of course, the Ramsey conditions and are the same as those from the first phase of the cycle. Differentiating (77) w.r.t. time and using $\dot{\lambda}_2 = 0$, we can write

$$\frac{\dot{\psi}(t)}{\lambda_2\mu + \psi} = \frac{\dot{\lambda}_1}{\lambda_1} + \alpha\frac{\dot{k}(t)}{k(t)} = -\alpha k(t)^{\alpha-1} + \delta + \alpha\left(k(t)^{\alpha-1} - \delta - \frac{c(t)}{k(t)}\right) \tag{81}$$

$$= -\alpha\frac{c(t)}{k(t)} + (1-\alpha)\delta \tag{82}$$

This condition implies that in order for the constraint on labour used in production to eventually become non–binding, the initial consumption level for a given k_0 must be in a range so that during the first phase

$$\frac{c(t)}{k(t)} > \left(\frac{1-\alpha}{\alpha}\right)\delta \tag{83}$$

If this is the case, then along the optimal path, if $\psi > 0$ then $\dot{\psi} < 0$ and eventually hits zero. In this case there exists a T^* such that $L(t) = 1$ if $t < T^*$ and $L(t) < 1$ if $t > T^*$.

Case 2: If $L(t) < 1$ then $\psi(t) = 0$. In this case totally differentiating (76) and (77) w.r.t. time and noting that $\dot{\lambda}_2 = 0$ yields

$$-\frac{\dot{\lambda}_1}{\lambda_1} = \rho + \sigma\frac{\dot{c}}{c} \tag{84}$$

$$-\frac{\dot{\lambda}_1}{\lambda_1} = \alpha\left(\frac{\dot{k}}{k} - \frac{\dot{L}}{L}\right) \tag{85}$$

Substituting out $\dot{\lambda}_1/\lambda_1$ using (78) and $\dot{k}/k$ using (72) we get

$$\rho + \sigma\frac{\dot{c}(t)}{c(t)} = \alpha k(t)^{\alpha-1}L(t)^{1-\alpha} - \delta \tag{86}$$

$$\frac{\dot{L}(t)}{L(t)} = -\frac{c(t)}{k(t)} + \left(\frac{1-\alpha}{\alpha}\right)\delta \tag{87}$$

But these two conditions combined with (72) are identical to those from the decentralized equilibrium in phase 2 of the cycle.■

Lemma B2: *Given* (Γ, Δ) *satisfying*

$$\rho + (\sigma - 1)\frac{\Gamma}{\Delta} > 0$$

the stationary cyclical path implies that the normalized capital stock at the start of each cycle takes on a unique stationary value, $k_0(T_v) = \hat{k}\ \forall\ v$.

Proof: We can express the social planner's problem as a Bellman equation given by

$$W(k; (\Gamma, \Delta)) = \max_{k'} \left\{ V^P(k, e^{\Gamma} k'; (\Gamma, \Delta)) + e^{-\rho\Delta} e^{-(\sigma-1)\Gamma} W(k'; (\Gamma, \Delta)) \right\} \tag{88}$$

The optimal choice for k' must satisfy

$$\begin{aligned} \lambda_1(T) &= e^{-\rho\Delta} e^{(1-\sigma)\Gamma} \frac{\partial W(k(T))}{\partial k} \\ c(T)^{-\sigma} &= e^{-(\sigma-1)\Gamma} \frac{\partial W(k(T))}{\partial k} = e^{-\sigma\Gamma} c_0^{-\sigma}(T) \\ c(T) &= e^{\Gamma} c_0\,(T) \end{aligned}$$

Note that this implies $C(T) = C_0\,(T)$, (i.e. consumption cannot jump).

We show that the right–hand side of (88) is a contraction mapping in the space of relevant bounded functions so that it has a fixed point. Let $\Xi = [k_{\min}, k_{\max}]$. and let $f(\cdot)$ and $g(\cdot)$ be any two continuous functions from Ξ to Ξ. The maximized within cycle utility function can be expressed as

$$\begin{aligned} V^P(k_0, k_T; (\Gamma, \Delta)) \;=\; & \int_0^{\Delta} \left[H(\hat{c}(t), \hat{k}(t), \hat{L}(t), \hat{\lambda}_1(t), \hat{\lambda}_2(t), \psi(t), t) + \dot{\lambda}_1(t)\hat{k}(t) \right] dt \\ & + \hat{\lambda}_1(0)k_0 - \hat{\lambda}_1(T)k_T - \hat{\lambda}_2(T)(1 - P(\Gamma)) \end{aligned} \tag{89}$$

Observe that the value function is increasing and concave in k_0 and decreasing in k_T:

$$\begin{aligned} V_1^P &= \frac{dV^P}{dk_0} = \hat{\lambda}_1(0) = c(0)^{-\sigma} > 0 \\ V_{11}^P &= \frac{d^2V^P}{dk_0^2} = -\sigma c(0)^{-\sigma-1} \frac{dc(0)}{dk_0} < 0 \\ V_2^P &= \frac{dV^P}{dk_T} = -\hat{\lambda}_1(T) = -e^{-\rho\Delta} c(T)^{-\sigma} < 0 \end{aligned}$$

Define the operator Ψ by

$$\Psi \circ f(k) = \max_{k'} \left\{ V^P(k, e^{\Gamma} k') + e^{-\rho\Delta} e^{-(\sigma-1)\Gamma} f(k') \right\}$$

subject to $k' \in \Xi$. To show that Ψ is a contraction mapping we must show that it satisfies two sufficient conditions (Blackwell's conditions):

(a) Monotonicity: Suppose $f(k) \geq g(k)\ \forall k \in \Xi$. Let k_f and k_g attain $\Psi \circ f$ and $\Psi \circ g$, respectively for some arbitrary given $k \in \Xi$. Then

$$\begin{aligned}
\Psi \circ g(k) &= V^P(k, e^{\Gamma}k_g) + e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}g(k_g) \\
&\leq V^P(k, e^{\Gamma}k_g) + e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}f(k_g) \quad (\text{since } f(k_g) \geq g(k_g)) \\
&\leq V^P(k, e^{\Gamma}k_f) + e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}f(k_f) \quad (\text{by definition}) \\
&= \Psi \circ f(k)
\end{aligned}$$

(b) Discounting: We have to show that there exists a $\beta \in (0,1)$ such that for any constant $x \geq 0$,

$$\begin{aligned}
&\max_{k'}\left\{V^P(k, e^{\Gamma}k') + e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}\left[f(k') + x\right]\right\} \\
\leq\ &\max_{k'}\left\{V^P(k, e^{\Gamma}k') + e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}f(k')\right\} + \beta x
\end{aligned}$$

Clearly, setting $\beta = e^{-\rho\Delta}e^{(1-\sigma)\Gamma}$ will satisfy this condition as long as

$$\rho + (\sigma - 1)\frac{\Gamma}{\Delta} > 0$$

which must be true in the cyclical equilibrium.

Let $k' = F(k)$ be the optimal policy attaining $W(k)$. Since $F(k) : \Xi \to \Xi$, and is continuous, it has a fixed point k^* in Ξ, so that $k^* = F(k^*)$ and

$$W(k^*, \Gamma, \Delta) = V^P(k^*, e^{\Gamma}k^*, \Gamma, \Delta) + e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}W(k^*, \Gamma, \Delta).\blacksquare$$

Lemma B3: *There exists a $\sigma_0 \in [0,1)$ such that if $\sigma > \sigma_0$ then*
(a) an increase in Γ decreases the steady state capital stock, $\hat{k}$.
(b) an increase in Δ increases the steady state capital stock, $\hat{k}$.

Proof: The proof uses the following first and second derivatives of the maximized within-cycle value function, $V^P(\hat{k}, e^{\Gamma}\hat{k}, \Gamma, \Delta)$:

$$\begin{aligned}
V_1^P &= c_0^{-\sigma} > 0, \qquad V_2^P = -e^{-\rho\Delta}c_T^{-\sigma} < 0 \\
V_{11}^P &= -\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial k_0} < 0, \quad V_{21}^P = -\sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial k_0} > 0 \\
V_{12}^P &= -\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial k_T} > 0, \quad V_{22}^P = -\sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial k_T} > 0 \\
V_{1\Gamma}^P &= -\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial \Gamma} > 0, \quad V_{2\Gamma}^P = -\sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial \Gamma} < 0 \\
V_{1\Delta}^P &= -\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial \Delta} < 0, \quad V_{2\Delta}^P = -\rho V_2^P - \sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial \Delta} > 0
\end{aligned}$$

Along an optimal stationary cyclical path for a given Γ and Δ, the planner's problem implies the Euler condition that

$$Z(\hat{k}, e^{\Gamma}\hat{k}, \Gamma, \Delta) = V_2^P(\hat{k}, e^{\Gamma}\hat{k}, \Gamma, \Delta) + e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P(\hat{k}, e^{\Gamma}\hat{k}, \Gamma, \Delta) = 0.$$

Differentiating Z with respect to $\hat{k}$ we get

$$\begin{aligned}
\frac{\partial Z}{\partial \hat{k}} &= V_{21}^P + e^{\Gamma}V_{22}^P + e^{-\rho\Delta-(\sigma-1)\Gamma}\left(V_{11}^P + e^{\Gamma}V_{12}^P\right) \\
&= -\sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial k_0} - e^{\Gamma}\sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial k_T} - e^{-\rho\Delta-(\sigma-1)\Gamma}\left(\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial k_0} + e^{\Gamma}\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial k_T}\right)
\end{aligned}$$

Using the Euler condition, we can write this as

$$\begin{aligned}
\frac{\partial Z}{\partial \hat{k}} &= \sigma e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P\left(\frac{1}{c_T}\frac{\partial c_T}{\partial k_0} + \frac{e^{\Gamma}}{c_T}\frac{\partial c_T}{\partial k_T} - \frac{1}{c_0}\frac{\partial c_0}{\partial k_0} - \frac{e^{\Gamma}}{c_0}\frac{\partial c_0}{\partial k_T}\right) \\
\frac{\partial Z}{\partial \hat{k}} &= \sigma e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P\left(\frac{\partial \ln(c_T/c_0)}{\partial k_0} + e^{\Gamma}\frac{\partial \ln(c_T/c_0)}{\partial k_T}\right) < 0
\end{aligned}$$

The negative sign follows from the fact that both derivatives in brackets must be negative within a cycle. The first because, $\frac{\partial r(t)}{\partial k_0} < 0$ for all t in the cycle, and $\frac{\partial \ln(c_T/c_0)}{\partial R} > 0$, where R is the interest rate discounting from 0 to T. The second holds similarly because $\frac{\partial r(t)}{\partial k_T} < 0$ for all t in the cycle up to T, and again $\frac{\partial \ln(c_T/c_0)}{\partial R} > 0$, where R is the interest rate discounting from 0 to T.

(a) Differentiating the Euler condition with respect to Γ yields

$$\begin{aligned}
\frac{\partial Z}{\partial \Gamma} &= V_{2\Gamma}^P + e^{-\rho\Delta-(\sigma-1)\Gamma}V_{1\Gamma}^P - (\sigma-1)\, e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P \\
&= -\sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial \Gamma} - e^{-\rho\Delta-(\sigma-1)\Gamma}\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial \Gamma} - (\sigma-1)\, e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P \\
&= \sigma e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P\left(\frac{\partial \ln(c_T/c_0)}{\partial \Gamma}\right) - (\sigma-1)\, e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P
\end{aligned}$$

When Γ increases more labour must be allocated towards innovation in the second phase. Consequently, consumption must grow less during the cycle so that the first term above is negative. Clearly if $\sigma \geq 1$ the second term is negative. Define $\sigma_0 < 1$: the two terms are equal. For $\sigma > \sigma_0$ the first term dominates and $\frac{\partial Z}{\partial \Gamma} < 0$. It follows that at the stationary optimum:

$$\left.\frac{d\hat{k}}{d\Gamma}\right|_{Z=0} = -\frac{\left(\frac{\partial Z}{\partial \Gamma}\right)}{\left(\frac{\partial Z}{\partial \hat{k}}\right)} < 0. \tag{90}$$

(b) Differentiating the Euler condition with respect to Δ yields

$$\begin{aligned}
\frac{\partial Z}{\partial \Gamma} &= V_{2\Delta}^P + e^{-\rho\Delta-(\sigma-1)\Gamma}V_{1\Delta}^P - \rho e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P \\
&= -\rho V_2^P - \sigma\frac{V_2^P}{c_T}\frac{\partial c_T}{\partial \Delta} - e^{-\rho\Delta-(\sigma-1)\Gamma}\sigma\frac{V_1^P}{c_0}\frac{\partial c_0}{\partial \Delta} - \rho e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P \\
&= \sigma e^{-\rho\Delta-(\sigma-1)\Gamma}V_1^P\left(\frac{\partial \ln(c_T/c_0)}{\partial \Delta}\right) > 0
\end{aligned}$$

When Δ increases, relatively more output can be allocated towards consumption in the second phase so that the derivative in brackets must be positive. It follows that

$$\left.\frac{d\hat{k}}{d\Delta}\right|_{Z=0} = -\frac{\left(\frac{\partial Z}{\partial \Delta}\right)}{\left(\frac{\partial Z}{\partial \hat{k}}\right)} > 0. \tag{91}$$

Proof of Proposition 3: The proof shows that given the social planning solution for each pair (Γ, T) only one of these pairs $(\hat{\Gamma}, \hat{T})$ is consistent with both
(1) no–arbitrage in the presence of intermediate storage (with an ε storage cost)

$$e^{-[R(t)-R(s)]}\frac{w(t)}{w(s)} \leq 1 \;\forall t, s$$

(2) free entry into entrepreneurial search

$$\left[\mu V^D(t) - w(t)\right] L(t) = 0 \;\forall t.$$

Essentially, we show that (1) can be represented by a negative relationship between Γ and Δ like that labelled AMC in Figure 11 and (2) can be represented by a positive relationship like that labelled FEI.

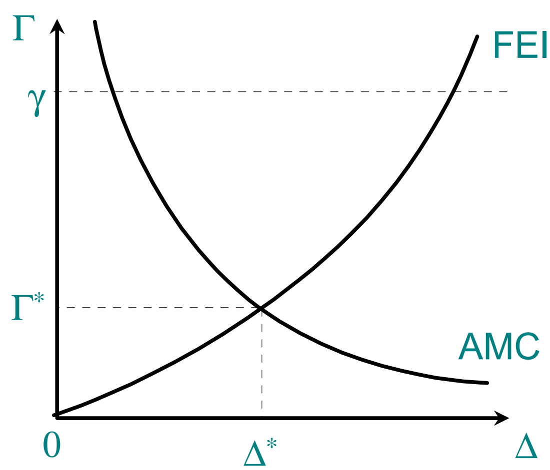

Figure 11: Uniqueness of the Stationary Equilibrium Cycle

(1) Since the social planner's solution implies that

$$r(t) \geq \frac{\dot{w}(t)}{w(t)} \;\forall t$$

at all points within the cycle, condition (1) boils down to the equilibrium requirement that the (Γ, Δ) must satisfy

$$w(T) = w_0(T)$$

which implies that the expected return from storage across the boom must be zero:

$$Z = \Gamma + \left(\frac{\alpha}{1-\alpha}\right) \ln L_T(\Gamma, \Delta, \hat{k}(\Gamma, \Delta)) = 0$$

where $L_T(\Gamma, \Delta, \hat{k}(\Gamma, \Delta)) = L(T; \Gamma, \Delta, \hat{k}(\Gamma, \Delta))$. Totally differentiating yields

$$\begin{aligned} \frac{\partial Z}{\partial \Delta} &= \left(\frac{\alpha}{1-\alpha}\right) \left[\frac{\partial \ln L_T}{\partial \Delta} + \frac{\partial \ln L_T}{\partial \hat{k}} . \frac{\partial \hat{k}}{\partial \Delta}\right] \\ \frac{\partial Z}{\partial \Gamma} &= 1 + \left(\frac{\alpha}{1-\alpha}\right) \left[\frac{\partial \ln L_T}{\partial \Gamma} + \frac{\partial \ln L_T}{\partial \hat{k}} . \frac{\partial \hat{k}}{\partial \Gamma}\right]. \end{aligned}$$

It follows that along the AMC curve where $Z = 0$:

$$\left.\frac{d\Gamma}{d\Delta}\right|_{AMC} = \frac{-\left(\frac{\alpha}{1-\alpha}\right)\left[\frac{\partial \ln L_T}{\partial \Delta} + \frac{\partial \ln L_T}{\partial \hat{k}} . \frac{\partial \hat{k}}{\partial \Delta}\right]}{1 + \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{\partial \ln L_T}{\partial \Gamma} + \frac{\partial \ln L_T}{\partial \hat{k}} . \frac{\partial \hat{k}}{\partial \Gamma}\right]} < 0$$

To see that this slope is indeed negative, note that from Lemma B3 we have that $\frac{\partial \hat{k}}{\partial \Gamma} < 0$ and $\frac{\partial \hat{k}}{\partial \Delta} > 0$. Also note that $\frac{\partial \ln L_T}{\partial \hat{k}} > 0$ since the higher is the capital stock the more costly it is to divert a marginal unit of labour from production at any date (since its marginal product is higher). Since innovative efforts are cumulative, increasing Δ makes it optimal to reduce innovation $(1 - L(t))$ for all t, ceteris parabus, consequently $\frac{\partial \ln L_T}{\partial \Delta} > 0$. It follows that the numerator is positive. Similarly increasing Γ, holding Δ fixed, requires more labour diverted from production at any t, so that $\frac{\partial \ln L_T}{\partial \Gamma} < 0$. Although the term in square brackets in the denominator is therefore negative, provided that α is sufficiently small the negative slope of AMC follows.■

(2) Since the social planners solution implies that whenever $L(t) < 1$,

$$r(t) = \frac{\dot{w}(t)}{w(t)},$$

condition (2) reduces to the equilibrium requirement that

$$\begin{aligned} \mu V^D(\Delta^*; \Gamma, \Delta) &= w(\Delta^*; \Gamma, \Delta) \\ \mu e^{-[R(T)-R(T^*)]} e^{\Gamma} V_0^I(0) &= w(\Delta^*; \Gamma, \Delta) \\ \frac{\mu e^{-[R(\Delta)-R(\Delta^*)]} e^{\Gamma}}{1 - P(\Gamma) e^{-\rho\Delta} e^{(1-\sigma)\Gamma}} \int_0^{\Delta} e^{-\int_0^{\tau} r(s) ds} \pi(\tau) d\tau &= w(\Delta^*; \Gamma, \Delta) \\ \frac{\mu(1-\omega)(\frac{1-e^{-\gamma}}{e^{-\gamma}}) e^{-[R(\Delta)-R(\Delta^*)]} e^{\Gamma}}{1 - P(\Gamma) e^{-\rho\Delta} e^{(1-\sigma)\Gamma}} \int_0^{\Delta} e^{-\int_0^{\tau} r(s) ds} w(\tau) L(\tau) d\tau &= w(\Delta^*; \Gamma, \Delta) \end{aligned}$$

Re-arranging we have

$$\begin{aligned}
\frac{e^{-[R(\Delta)-R(\Delta^*)]}e^{\Gamma}}{1-P(\Gamma)e^{-\rho\Delta}e^{(1-\sigma)\Gamma}}\int_0^{\Delta} e^{-\int_0^\tau r(s)ds}\frac{w(\tau)L(\tau)}{w(\Delta^*)}d\tau &= \frac{e^{-\gamma}}{\mu(1-e^{-\gamma})(1-\omega)} \\
\frac{e^{-[R(\Delta)-R(\Delta^*)]}e^{\Gamma}}{1-P(\Gamma)e^{-\rho\Delta}e^{(1-\sigma)\Gamma}}\left[\int_0^{\Delta^*} e^{-\int_0^\tau r(s)ds}\frac{w(\tau)}{w(\Delta^*)}d\tau + e^{-R(\Delta^*)}\int_{\Delta^*}^{\Delta} L(\tau)d\tau\right] &= \frac{e^{-\gamma}}{\mu(1-e^{-\gamma})(1-\omega)} \\
\frac{e^{-\rho\Delta}e^{(1-\sigma)\Gamma}}{1-P(\Gamma)e^{-\rho\Delta}e^{(1-\sigma)\Gamma}}\left[\int_0^{\Delta^*}\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}d\tau + (\Delta-\Delta^*) - \frac{\Gamma}{\mu\gamma}\right] &= \frac{e^{-\gamma}}{\mu(1-e^{-\gamma})(1-\omega)} = X
\end{aligned}$$

Now observe that for $\tau < \Delta^*$,

$$\begin{aligned}
\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)} &= \exp\left(\int_\tau^{\Delta^*}\left(r(s)-\frac{\dot{w}(s)}{w(s)}\right)ds\right) \\
&= \exp\left(\int_\tau^{\Delta^*}\left(\alpha\frac{c(s)}{k(s)} - (1-\alpha)(\phi+\delta)\right)ds\right)
\end{aligned}$$

For all $\tau < \Delta^*$, $\frac{c(\tau)}{k(\tau)}$ must be increasing in the steady–state capital stock at the start of each cycle — an increase in k^* reduces the marginal product of capital, so that the social planner allocates marginally less income to investment and more to consumption. It follows from Lemma B3 that at each date $\tau < \Delta^*$

$$\begin{aligned}
\frac{d}{d\Gamma}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right) &< 0 \\
\frac{d}{d\Delta}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right) &> 0.
\end{aligned}$$

Differentiating X with respect to Γ yields

$$\frac{\partial X}{\partial \Gamma} = -\frac{e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}}{1-P(\Gamma)e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}}\left[\frac{1}{\mu\gamma}+\frac{X}{\gamma}+\frac{(\sigma-1)X}{e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}} - \int_0^{\Delta^*}\frac{d}{d\Gamma}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right)d\tau.\right]$$

Since $\int_0^{\Delta^*}\frac{d}{d\Gamma}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right)d\tau \le 0$, a sufficient condition for $\frac{\partial X}{\partial \Gamma} < 0$ is that $\frac{1}{\mu\gamma}+\frac{X}{\gamma}+\frac{(\sigma-1)X}{e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}} > 0$. Clearly this must hold for $\sigma \ge 1$, and will hold for $\sigma < 1$ if sufficiently large.

Differentiating X w.r.t. Δ we get

$$\frac{\partial X}{\partial \Delta} = \frac{e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}}{1-P(\Gamma)e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}}\left[1-\rho X + \int_0^{\Delta^*}\frac{d}{d\Delta}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right)d\tau\right]$$

Since $\int_0^{\Delta^*}\frac{d}{d\Delta}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right)d\tau \ge 0$, a sufficient condition for $\frac{\partial X}{\partial \Delta} > 0$ is

$$\frac{\rho e^{-\gamma}}{\mu(1-e^{-\gamma})(1-\omega)} < 1$$

Thus, the slope of the FEI curve is given by

$$\left.\frac{d\Gamma}{d\Delta}\right|_{FEI} = -\frac{\left(\frac{\partial X}{\partial T}\right)}{\left(\frac{\partial X}{\partial \Gamma}\right)} = \frac{1 - \rho X + \int_0^{\Delta^*} \frac{d}{d\Delta}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right) d\tau}{\frac{1}{\mu\gamma} + \frac{X}{\gamma} + \frac{(\sigma-1)X}{e^{-\rho\Delta}e^{-(\sigma-1)\Gamma}} - \int_0^{\Delta^*} \frac{d}{d\Gamma}\left(\frac{e^{-R(\tau)}w(\tau)}{e^{-R(\Delta^*)}w(\Delta^*)}\right) d\tau} > 0$$

If $\sigma \geq 1$ and $\rho e^{-\gamma} < \mu(1-e^{-\gamma})(1-\omega)$ (so that $\rho X < 1$), this is clearly positive. For lower values of σ it may still be positive.■

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