Chapter 4

EMPIRICAL EVIDENCE ON THE TREATMENT OF SEASONAL PRODUCTS: THE ISRAELI CPI EXPERIENCE

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1. Introduction

The treatment of seasonal products in a Consumer Price Index (CPI) or Producer Price Index (PPI) is dealt with in chapter 22 of the new international manuals on the CPI and PPI.[2] Several methods are explored. In chapter 22 of the manuals (referred to hereafter simply as the Manual chapter), numerical demonstrations of performance for the alternative methods were carried out using a single artificial dataset: the modified Turvey data.[3] In this article we report on the results of implementing many of the methods proposed in the Manual chapter using real data from the Israeli CPI program.[4]

This new study has three main objectives. As background for our empirical research, we selectively summarize the material in the Manual chapter. Second, we briefly describe some of the special methods used in the Israeli CPI program for dealing with seasonal fluctuations in a month-to-month index. Third, we summarize our empirical results. Israeli CPI data are used to implement the methods considered in the Manual chapter. Results are obtained first using real Israeli CPI data and then using the Israeli CPI data with artificial modifications to check the sensitivity of key findings to data characteristics. We comment on how these results compare with the Manual chapter results. Formulas for the various methods used for dealing with seasonal products are given in appendix A.[5]

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[2] Web addresses for the *CPI Manual* (Hill 2004) and the *PPI Manual* (Armknecht 2004) are given in the references. The original chapter 22 was authored by Erwin Diewert, the first author of this paper, together with Paul Armknecht (2004) of the International Monetary Fund who is also the editor for the new 2004 *PPI Manual*.

[3] Diewert modified Turvey's (1979) artificial dataset to facilitate the complex computations in chapter 22.

[4] For more on the Israeli CPI program, see Karshai (1992), Sabag and Finkel (1994), Burck and Salama (2003), and Artsev, Roshal and Finkel (2006).

[5] In appendix A we indicate where full explanations of the formulas and methods used can be found in the Manual chapter and in the extended overview of the Manual chapter provided by Diewert, Armknecht and Nakamura (2011).

Citation for this chapter:
W. Erwin Diewert, Yoel Finkel and Yevgeny Artsev (2011),
"Empirical Evidence on the Treatment of Seasonal Products: The Israeli CPI Experience," chapter 4, pp. 53-78 in
W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (eds.),
PRICE AND PRODUCTIVITY MEASUREMENT: Volume 2 -- Seasonality. Trafford Press.

2. The Treatment of Seasonal Products in the Manual Chapter

The Manual chapter 22 is organized into 12 sections. Here we indicate where the results can be found that relate to the findings of this study.

The first section in the Manual chapter (par. 22.1-22.13)[6] explains the problem of seasonal products. Products are defined as being (a) *strongly seasonal* when they are not available at all in the marketplace during certain seasons of the year and (b) *weakly seasonal* when they are available throughout the year but have regular fluctuations in prices or quantities that are synchronized with the season or the time of the year.[7] Strongly seasonal products are most challenging for compilers of price indexes since having different bundles of products in the index basket in the time periods being compared leads to a breakdown of traditional bilateral index number methodology. Two categories of approaches are presented in the Manual chapter for dealing with strong seasonality: month-to-month and year-over-year index approaches.

In the second section (par. 22.14-22.15), the modified Turvey data is introduced.

The third section (par. 22.16-22.34) presents the concept of year-over-year monthly comparisons. Even strongly seasonal products typically reappear in the same months each year. The overlap of products -- that is, the extent to which the same products appear in both the periods for which prices are being compared -- is maximized in the year-over-year monthly indexes. The remainder of the third section introduces and compares different types of year-over-year monthly indexes, including true versus approximate, and chained versus fixed base. Both true and approximate Laspeyres, Paasche and Fisher formulas are presented, and both the fixed base and the chained variants are defined.[8] It was found in the Manual chapter that use of chained indexes tends to reduce the spread between the Paasche and Laspeyres indexes compared to their fixed base counterparts. Since an approximate Fisher is just as easy to compute as an approximate Paasche or Laspeyres, and is more appropriate on other grounds, it is recommended that statistical agencies make the approximate Fisher indexes available to the public.[9]

The fourth section in the Manual chapter (par. 22.35-22.44) moves on from year-over-year monthly indexes to year-over-year annual ones.[10] The approach to computing annual indexes, which essentially involves taking monthly expenditure share-weighted averages of the 12 year-over-year monthly indexes, should be distinguished from taking a simple arithmetic mean of the 12 monthly indexes. The key problem with the latter approach is that months when

[6] The paragraph numbers for the sections are the same for chapter 22 in both the CPI Manual and the PPI Manual except for a one-paragraph difference in the last two sections. The paragraph numbers shown in parentheses in this paper thus apply to both manuals, except for the final two sections where these are the numbers for the CPI Manual.

[7] This classification of seasonal products corresponds to Balk's narrow and wide sense seasonal products; see Balk (1980a, p. 7; 1980b, p. 110; 1980c, p. 68). See also Balk (1981). Diewert (1998, p. 457) used the terms type 1 and type 2 seasonality. For other related references, see also Baldwin (1990) and Diewert (2002).

[8] See appendix section A.1 in this paper for the relevant formulas.

[9] Both the year-over-year and month-to-month Laspeyres and Paasche indexes fail the time reversal test: the Laspeyres with an upward bias and the Paasche with a downward bias.

[10] See appendix section A.2 in this paper for the formulas.

expenditures are below average (for example, February) are given the same weight as months when expenditures are above the average (for example, December).

Rolling year-over-year indexes are the subject of the fifth section of the Manual chapter (par. 22.45-22.54).[11] In the previous section, the indexes are defined over a calendar year. However, there is no need to confine comparisons to calendar year time periods. Prices can be compared for any specified 12 consecutive month period in different years, provided that January data is compared to January data, February data is compared to February data, and so on.[12] Rolling year indexes are computed using Laspeyres, Paasche and Fisher formulas, and the approximate Laspeyres, Paasche and Fisher formulas. Alterman, Diewert, and Feenstra (1999, p. 70) termed the resulting indexes rolling year indexes.[13] The specifics of constructing rolling year indexes for the modified Turvey data set are spelled out in the Manual chapter for both fixed base and chained rolling year indexes.

The sixth Manual chapter section (par. 22.55-22.62) shows how a year-over-year monthly index may be used to predict a rolling year index that is centered at the current month. It is shown that under a regime where the long run trend in prices is smooth, the current month year-over-year monthly index along with last month's year-over-year monthly index can be used to forecast a rolling year index that is centered on the last two months.

The seventh through tenth sections examine the treatment of seasonality using the traditional month-to-month approach. Section seven (par. 22.63-22.77) considers the maximum overlap month-to-month price index method which uses the set of products that are present in the marketplace in both months for which the price comparison is being made.[14] When strongly seasonal products are included, the maximum overlap index suffers from a serious downward bias for the artificial modified Turvey dataset. In the eighth section (par. 22.78-22.84), annual basket indexes with carry forward of unavailable prices are evaluated using the Lowe (1823), Young (1812) and geometric Laspeyres formulas,[15] and the movements of these are compared to the fixed base Laspeyres centered rolling year index. The ninth section (par. 22.85-22.86) replicates the section eight exercise, except that now imputed prices are computed rather than using the "carry forward" method. The indexes are still found to suffer from seasonality and do not closely approximate their rolling year counterparts.

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[11] For more on rolling year indexes, see appendix section A.2, and also Diewert (1983), and Diewert, Armknecht and Nakamura (2007).

[12] Diewert (1983) suggested this type of comparison and termed the resulting index a split year comparison.

[13] Crump (1924, p. 185) and Mendershausen (1937, p. 245), respectively, used these terms in the context of various seasonal adjustment procedures. The term "rolling year" seems to be well established in the business literature in the United Kingdom. In order to theoretically justify the rolling year indexes from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1996, pp. 32-34; 1999, pp. 56-61).

[14] See appendix section A.3 for a brief introduction to maximum overlap indexes.

[15] The fixed base geometric Laspeyres annual index, P_{GL}, is the weighted geometric mean counterpart to the fixed base Laspeyres index, which is equal to a base period weighted arithmetic average of the long-term price relative. It can be shown that P_{GL} approximates the approximate fixed base Fisher index P_{AF} to the second order around a point where all of the long-term price relatives are equal to unity.

In the tenth section (par. 22.87-22.90), a final month-to-month index is considered: the Bean and Stine (1924) Type C, also known as the Rothwell (1958) type index.[16]

The eleventh section (par. 22.91-22.96) presents an attempt to forecast current month rolling year indexes using month-to-month annual basket indexes.[17] Seasonal adjustment factors are computed for the Lowe, Young, geometric Laspeyres and Rothwell indexes based on a centered rolling year index that is used as the target index.

The final section restates some of the main conclusions of the Manual chapter

(1) Year-over-year monthly indexes and rolling year indexes should be computed by the statistical agencies, at least as analytical series, along side the month-to-month ones;

(2) Annual basket indexes can be successfully used in the context of seasonal products;

(3) The spread between Laspeyres and Paasche indexes will usually be reduced by chaining, but the results should be checked against the year-over-year counterparts periodically to guard against chain drift that can be expected when a Laspeyres or Paasche index is used;

(4) Laspeyres and Paasche indexes can typically be viewed as of equal importance, so a Fisher (1922) type index is preferable when the data are available to compute this;

(5) The approximate year-over-year Fisher index accurately tracks the true year-over-year Fisher index for the modified Turvey data, and this result should hold for other datasets so long as the current period expenditure shares are similar to the past period shares used in the approximate indexes;

(6) The approximate Fisher and also the geometric Laspeyres indexes can be computed with the data that statistical agencies normally have at hand on a current basis; and

(7) The maximum overlap month-to-month indexes suffer from substantial bias when they include seasonal products.

3. The Main Treatment of Seasonal Products in the Israeli Index

Taken together, seasonal products make up nearly one fifth of the total expenditures covered by the Israeli CPI. We describe the methods used in the CPI program for dealing with seasonal products, focusing on clothing and boots, and on fresh fruits and vegetables.

3.1 Clothing and footwear

Clothing and footwear represent 2.9 percent of total expenditures covered by the Israeli CPI. The weights for this consumption group and specific products in it reflect the average monthly expenditures on the specified products over the base year as a percentage of the average monthly expenditure on the total basket covered by the CPI basket. The data source is the

[16] See appendix section A.5 for the formulas.

[17] See appendix section A.6.

Household Expenditure Survey (HES) conducted annually by the Israeli Central Bureau of Statistics. The price indexes are computed using the Lowe (1923) formula, defined in appendix equation (A-17).[18] The price changes for products that have disappeared due to seasonality are imputed using the average weighted percentage change in the price indexes of all clothing (or footwear) products present in the market. When the seasonal products reappear, "true" price indexes are computed for them.

There are many end-of-season sales that take place twice yearly: at the end of the summer season (usually in August-September) and at the end of the winter season (usually in February-March). Products are then sold at large discounts. These sales will tend to cause a downward bias problem. In order to overcome this problem, the price indexes of affected products are corrected in June and December.[19] These months were chosen because they are mid-season. The correction process is done using a three-stage computation method introduced into the CPI in 1989.

In stage one, the price indexes of seasonal products are computed using biannual year-over-year comparisons (December-over-December and June-over-June) and those for the other products (those found throughout the year like underwear) are computed using a 6-months prior comparison month (December versus June in December, and June versus December in June).

For clothing and footwear, the frequent changes in fashion cause many changes in the products, whether they are seasonal or not. Thus, in stage two, the prices for the potentially large number of products that have been replaced during the 12-month or 6-month period are imputed; more specifically, their last observed prices are increased by the average percentage change of the other products that could be compared over the annual or 6 month period.

Stage three involves averaging indexes for all clothing or footwear products.

The price indexes for clothing and footwear products are computed like the price indexes of other products, by the method of chaining: comparing current period prices with the previous period rather than base period prices.

3.2 Fruits and vegetables

Fruits and vegetables represent 3.5 percent of total expenditure in the Israeli CPI. The weights of the consumption groups and specific products in the categories are determined using the percentages of the average monthly expenditure in the base year versus the average monthly expenditure on the total basket. The data source is the Household Expenditure Survey (HES).

Until 1987, the fruit and vegetable price indexes were computed using the Rothwell formula, defined in appendix equations (A-20) and (A-21). From January 1988 on, these indexes have been computed according to the annual basket month-to-month Lowe formula (appendix equation (A-17)). Missing prices were imputed (in contrast to being filled in using the carry forward method). Research, conducted by the Israeli Central Bureau of Statistics over the past decade has not been conclusive about whether this transition led to an "improved" index. This

[18] For this formula, the base period for the weights is price updated to the base period of the CPI. See Balk and Diewert (2011) and T.P. Hill (2011) for more on the properties of the Lowe index.

[19] Hedonic methods may be used to solve this problem.

motivated our experimentation with the year-over-year methods described in the Manual chapter. Our findings for these methods are noted in section 4.

3.3 Seasonal adjustment methods in the Israeli CPI

Various approaches to seasonal adjustment were used in the Israeli CPI program in the mid-eighties and early nineties. Seasonal adjustment was conducted, at first, by Arima X-11, which was then replaced by Arima X-12.[20] The Israeli Bureau of Statistics publishes every month seasonally adjusted indexes for the nine following series: the total CPI, the CPI excluding housing, the CPI excluding housing and fruits and vegetables, fresh fruit, fresh vegetables, clothing, footwear, recreation and vacation, and travel abroad.[21]

4. Implementation of Methods from the Manual Chapter

4.1 Israeli CPI program data

The Central Bureau of Statistics of Israel (CBS) conducts a monthly CPI and an annual HES. The sample for the Israeli HES consists of 6,200 households (with 500 households or more interviewed each month).[22]

For this study, we used actual CPI and HES data for the five-year period of 1997-2002 (72 monthly observations) for (1) fresh fruits[23] (lemons, apricots, avocado, watermelon, persimmon, grapefruits and bananas), and for (2) fresh vegetables[24] (cabbage, cauliflower, cucumbers, potatoes, carrots, lettuce and eggplants). For each of the selected fresh fruits, there are some months with no price observations, so these are strongly seasonal products. The fresh vegetables exhibit weak(er) seasonality; i.e., each of the selected vegetables is present throughout the year but there are large price fluctuations in the vegetable prices and quantities. We applied all the methods in the Manual chapter for both the fresh fruits and fresh vegetables.

Here we present only the findings that are connected to tentative conclusions reached in the Manual chapter where our results seem to differ in important ways. To focus the discussion, we examine only our results for fresh fruits.[25]

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[20] See Diewert and Armknecht (2006, section) for a brief introduction to these methods and references. See also Burck and Salama (2003) and Burck and Gabman (2003) for specifics of the seasonal adjustment problems in Israel and the efforts that have been made over time to deal with these.

[21] In addition, trend indexes are published monthly for three series: Total CPI, CPI excluding housing, and the CPI excluding housing and fruits and vegetables. The estimation of the trend is conducted by Symmetric Henderson Moving Averages. See Diewert, Armknecht and Nakamura (2011) for more on the Arima X-11 and Arima X-12 software packages and Henderson filters.

[22] The Israeli HES has been annual since 1997. Over 500 households are sampled each month.

[23] See appendix B for the price and expenditure weight data.

[24] See appendix C for the price and expenditure weight data.

[25] Due to weaker seasonality for vegetables, the seasonal adjustment methods were always more successful for the vegetables than the fruits.

4.2 Year-over-year monthly indexes

Can the approximate Laspeyres, Paasche and Fisher indexes introduced in section three of the Manual chapter -- formulas (A-4)-(A-6) in appendix A of this paper -- be used in place of the true Laspeyres, Paasche and Fisher -- formulas (A-1)-(A-3)? The key advantage of the approximate over the true current indexes is that they use only the data that statistical agencies normally have at hand.[26] Thus, in table 1 below we compare the year-over-year current month fixed base Fisher index with the approximate monthly fixed base Fisher index.[27] In 1997, by construction, the same data are used for both methods. In table 1 and the following five tables, entries where the relevant difference is 5 percentage points or more are shown in bold and larger sized type.

The relevant comparison in table 1 is for 1998-2002. In 11 out of the 60 months being compared, the differential is 5 percent or more. However, only six of these months have differentials of 10 percent or more (including four of the five Septembers).

Table 1. Ratio between Year-over-Year True and Approximate Fixed Base Monthly Fisher Indexes for Israeli CPI Data

Month/Year	1997	1998	1999	2000	2001	2002
1	1.00	1.01	0.99	0.99	1.02	0.98
2	1.00	1.00	1.01	1.00	1.00	1.00
3	1.00	1.00	1.00	1.01	1.01	1.01
4	1.00	1.00	0.98	1.01	1.00	1.02
5	1.00	0.99	1.04	**1.05**	**1.27**	0.97
6	1.00	1.01	1.00	1.01	1.01	1.03
7	1.00	1.01	**1.16**	1.01	**1.06**	1.02
8	1.00	0.99	1.02	1.00	1.04	**1.05**
9	1.00	**1.77**	**1.83**	1.04	**1.82**	**1.58**
10	1.00	1.01	1.00	1.01	1.01	0.96
11	1.00	0.99	0.98	0.99	0.98	**0.94**
12	1.00	1.01	0.99	1.00	1.00	**0.93**

In table 2 below, the same results are shown for year-over-year monthly chained Fisher indexes. The chained indexes are not approximated as well as the fixed base ones. The differences between the table 1 and 2 results show up beginning in 1999. In table 2, 16 observations (those shown in bold in table 2), the differences are 5 percent or more, and for 9 of those 16 the differences are 10 percent or more.

In the Manual chapter it is consistently found that the spread between the Laspeyres and Paasche type indexes is less for the chained than for the fixed base variants. In table 3 below we compare the spread between the Laspeyres and Paasche indexes for the fixed base and then for

[26] It is generally agreed that the Fisher index is to be preferred to the Laspeyres and Paasche indexes when the data are available that are needed to compute that Fisher index.

[27] The formulas for the true and approximate Fisher indexes are given in appendix equations (A-3) and (A-6).

the chained variants. The total period average Laspeyres/Paasche spread for the chained indexes seems to be slightly more, rather than less, on average, compared with the results for the fixed base indexes. These findings do not support the conclusion in the Manual chapter.

Table 2. Ratio between Year-over-Year True and Approximate Monthly Chained Fisher Indexes for Israeli CPI Data

MONTH/YEAR	1997	1998	1999	2000	2001	2002
1	1.00	1.01	0.98	0.99	**1.05**	1.01
2	1.00	1.00	1.03	0.99	1.00	1.00
3	1.00	1.00	1.00	1.02	1.03	1.04
4	1.00	1.00	0.98	1.03	1.01	**1.07**
5	1.00	0.99	**1.09**	**1.12**	**1.58**	**1.32**
6	1.00	1.01	1.00	1.00	1.00	**1.06**
7	1.00	1.01	**1.14**	1.01	**1.06**	1.02
8	1.00	0.99	1.01	1.00	**1.05**	1.04
9	1.00	**1.77**	**1.82**	**1.23**	**1.75**	**1.77**
10	1.00	1.01	1.02	1.01	1.02	**1.07**
11	1.00	0.99	0.99	1.02	1.01	0.99
12	1.00	1.01	1.01	1.01	1.01	0.99

Table 3: Mean Annual Ratio between Year-over-Year True Monthly Laspeyres and Paasche Indexes for Israeli CPI Data

	1997	1998	1999	2000	2001	2002
Fixed Base	1.00	0.97	0.974	1.03	0.94	0.93
Chained	1.00	0.97	0.972	0.98	0.93	0.94

4.3 Month-to-month indexes

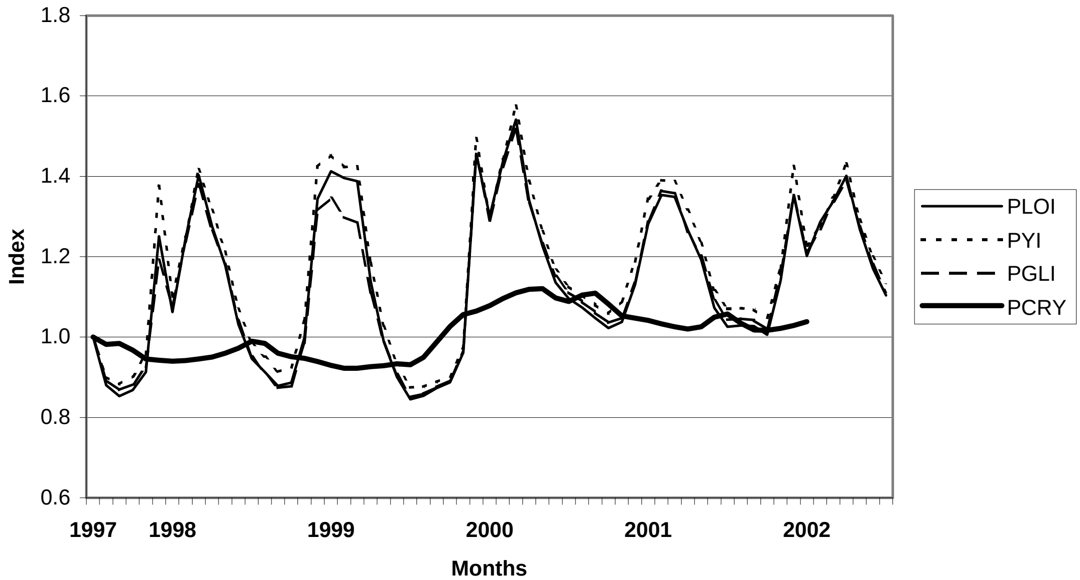
The Manual chapter reports that the traditional month-to-month indexes all have large seasonal fluctuations and the maximum overlap method[28] is downward biased. Our findings are consistent with these Manual chapter conclusions. In figure 1 below, extreme (though regular) seasonal fluctuations are found for the annual basket Lowe (denoted by PLOI in figure 1; equation (A-27) in the appendix of this paper), Young (denoted by PYI; equation (A-30)) and geometric Laspeyres (PGLI; equation (A-19)) annual basket month-to-month indexes[29] with the target centered year rolling index[30] using imputed prices[31] (denoted in figure 1 by PCRY; see section A.6 in the appendix).

[28] See appendix section A.3 in this paper.

[29] See appendix section A.3 in this paper for the formulas for the maximum overlap method.

[30] This series was normalized to equal 1 in December 1997 so that it would be comparable to the other month-to-month indexes. Also, the centered rolling year indexes cannot be calculated for the last 6 months, since the data set does not extend 6 months into 2003.

Figure 1. Lowe, Young, Geometric Laspeyres, and Centered Rolling Year Indexes with Imputed Prices



The year-over-year indexes defined in section 3, and their annual averages studied in sections 4 and 5, offer a theoretically satisfactory method for dealing with strongly seasonal products. However, because these methods rely on year-over-year comparisons of prices, they cannot be directly used to produce month-to-month or quarter-to-quarter indexes: typically the main focus of a CPI or PPI price index program. Thus, in sections 7-11, more traditional month-to-month price index methods are explored. If strongly seasonal products are to be included in the basket for traditional month-to-month price indexes, ways must be found to fill in the missing price information for products not available in one or the other of each pair of months compared.

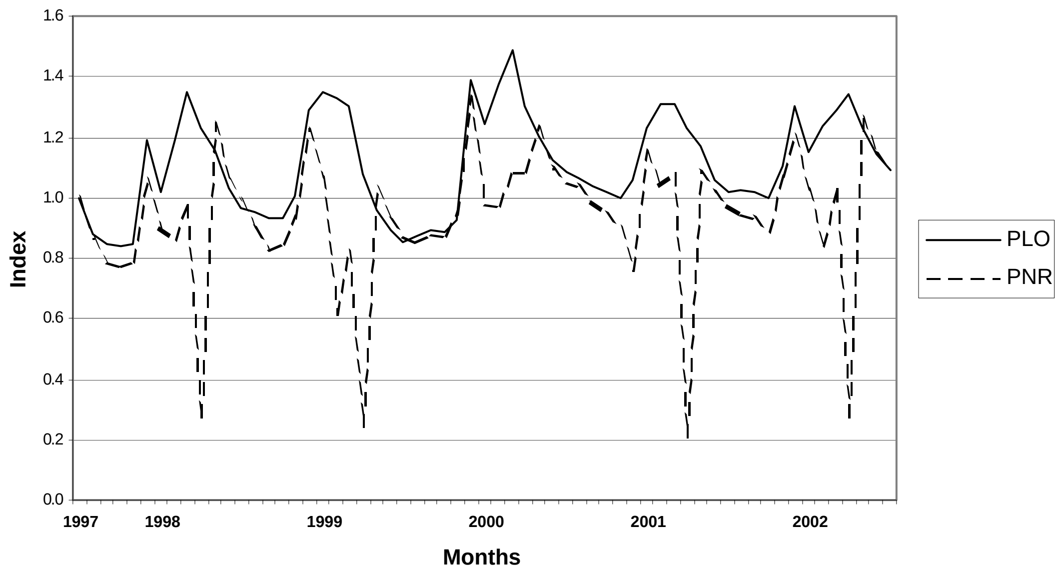
It is of interest to compare the above three indexes that use annual baskets to the fixed base Laspeyres rolling year indexes. However, the rolling year index that ends in the current month is centered five and a half months backwards. Hence the above annual basket type indexes may be compared with an arithmetic average of two rolling year indexes that have their last month 5 and 6 months forward. This latter *centered rolling year index* is labeled P_{CRY}. It can be seen from figure 1 that the Lowe, Young and geometric Laspeyres indexes do not closely track their rolling year counterpart (PCRY).[32]

[31] The idea of the imputation method is to take the last available price and impute prices for the missing periods that trend with the category of fresh fruits. For each month m, the imputed price equals the price of the previous month (m-1) multiplied by the average of the fresh fruit category index for three adjacent months (m-1, m and m+1).

[32] The sample means of the indexes are 1.1220 (Lowe), 1.1586 (Young), 1.1190 (geometric Laspeyres) and 1.0072 (rolling year). Of course, the geometric Laspeyres index will always be equal to or less than the Young counterpart since a weighted geometric mean is always equal to or less than the corresponding weighted arithmetic mean.

Another finding from the Manual chapter was that the Rothwell index (PNR in figure 2)[33] has smaller seasonal movements than the Lowe index (PLO in figure 2) and is less volatile in general. The corresponding findings using the Israeli CPI program data are quite different, as can be seen in figure 2. The normalized Rothwell index is seen to be more volatile than the annual basket month-to-month Lowe index[34]. This finding supports the decision made in the Israeli CPI program to move to the Lowe index. However, both indexes exhibit large seasonal fluctuations.

Figure 2. Lowe and Normalized Rothwell Indexes for Israeli CPI Data[35]



Section 11 of the Manual chapter documents an attempt to forecast rolling year indexes using month-to-month annual basket indexes. A seasonal adjustment factor was computed for the Lowe, Young and geometric Laspeyres indexes with imputed prices and the Rothwell index based on the centered rolling year index. Here, we introduce the actual Israeli CPI "fresh fruits" price index that has been seasonally adjusted using an Arima X-11 multiplicative model. This series enables us to compare whether the pattern of price change for the chosen set repeats the trend of the general category of the product or higher-level component of the CPI. The series has been normalized by dividing the original indexes by the first observation to make the series more comparable with the others. Our finding is that the predicted values of these "seasonally adjusted" indexes using the Israeli CPI data are fairly close to the corresponding target index

[33] See appendix section A.5.

[34] The Lowe index has a mean of 1.1087 and a standard deviation of .1677 while the normalized Rothwell has a mean of .9368 and a standard deviation of .2282.

[35] The Rothwell index is compared to the Lowe index with carry forward of missing prices (i.e., for the prices that are not available in the current month, the last available price is carried forward).

values.[36] The centered rolling year index for the chosen data does not seem to reliably represent the overall price index of fresh fruits with X-11 seasonal adjustment.[37] However, only the Seasonally-Adjusted Rothwell had outliers compared to the target indexes. This result perhaps weakens the argument for using this index in the Israeli CPI.

5. Sensitivity Results Using on a Modified Israeli CPI Program Dataset

Comparison of the real Israeli CPI dataset used in this study with the artificial Turvey one on which the examples in the Manual chapter are based reveals structural differences that may account for some of the differences in findings. These data differences are summarized in table 4.

Table 4. Artificial Modified Turvey Versus Israeli CPI Program Data Differences

Presumption Built into the Artificial Turvey Data	Observed Reality
Annual price change of a product is always in one direction. An increase (decrease) in price in one year, compared to the previous one, cannot be followed by a decrease (increase) in price in the next year.	The trend of price change can change direction from year to year.
There is one seasonal cycle a year for each product.	Two seasonal cycles are possible.
A product is available (unavailable) in the same months each year.	Seasonal fluctuations are not completely synchronized with the calendar months for the products with strong seasonality. Thus, a product may appear/disappear a month before/later than in the previous year.
There is constant consumption behavior, in comparison with the same month in the previous year, independent of price behavior.	Consumption patterns are often erratic.

In the actual Israeli CPI program data, there is some variation over years regarding the months when strongly seasonal products appear/disappear. This seasonal irregularity might explain some of the difference between our findings and some of those in the Manual chapter, since strongly seasonal products are assumed to always be available only in the same months year after year in the artificial Turvey data used for used to explore the performance of the different methods in the Manual chapter. We explored this possibility by producing a modified Israeli data set. Starting with the Israeli CPI program dataset for fresh fruit used in the previous section,[38] some observations were altered so as to align the availability of products with strong

[36] For observations 13 through 66, we regressed the seasonally adjusted series on the centered rolling year series. For the seasonally adjusted Lowe index, an R^2 of .1916 is obtained; for the seasonally adjusted Young index, an R^2 of .1707 is obtained. For the seasonally adjusted geometric Laspeyres index, an R^2 of .3050 is obtained. And for the seasonally adjusted Rothwell index, an R^2 of .1298 is obtained, which is lower than for the other three indexes.

[37] For the X-11 seasonally adjusted series regressed on the centered rolling year series, the R^2 value is .0410.

[38] We modified the dataset of fruits only. There are no products with strong seasonality in the dataset of vegetables.

seasonality over the years. More specifically, 16 observations out of 504 were modified (3.2 percent). Eight of these observations were omitted for products that appeared in months when they were not usually available, and eight other observations were imputed by carrying forward/back[39] the price of the adjacent month for products that were unavailable in months when they usually were available.

The data modification led to substantial changes in some of the findings reported in section 4 above. We report these results to draw attention to the fact that some of the Manual conclusions seem quite sensitive to the properties of the data at hand.

In table 5, we again compare the year-over-year "current month" fixed base Fisher index with the approximate Fisher and find that the approximate Fisher index provides adequate approximations. The number of cases in which the difference between the indexes is 5 percentage points or more (shown in bold and larger sized type) is now just 5 (in contrast to 11 in table 1). Also, now there are no longer any differences of 10 percentage points or more.

Table 5. Ratio Between the Approximate and True Year-over-Year Monthly Fixed Base Fisher Indexes for the Modified Israeli CPI Data

Month/Year	1997	1998	1999	2000	2001	2002
1	1.00	1.01	0.99	0.99	1.02	0.98
2	1.00	1.00	0.99	1.00	1.00	1.00
3	1.00	1.00	0.99	1.00	1.00	1.01
4	1.00	1.00	0.98	1.01	1.00	1.02
5	1.00	0.99	1.04	**1.05**	**1.05**	1.02
6	1.00	1.01	1.00	1.01	1.01	1.03
7	1.00	1.01	1.02	1.01	1.04	1.00
8	1.00	1.01	1.02	1.01	1.04	1.03
9	1.00	1.00	1.03	1.01	1.00	**0.89**
10	1.00	1.01	1.00	1.01	0.99	0.96
11	1.00	0.99	0.98	0.99	0.98	**0.94**
12	1.00	1.01	0.99	1.00	1.00	**0.93**

In table 6 we return to the comparison of year-over-year monthly chained Fisher indexes. There is a decrease of about 50 percent in the number of cases in which the difference between indexes is 5 percent or more (seven in contrast to 16 cases), and only two observations are 10 percent or over.

[39] Imputation by carrying back the price of the following month was preferred only in cases when there was no price in the previous months of the same year.

Table 6. Ratio Between the Approximate and True Year-over-Year Monthly Chained Fisher Indexes for the Modified Israeli CPI Data

Month/Year	1997	1998	1999	2000	2001	2002
1	1.00	1.01	0.98	0.99	**1.05**	1.01
2	1.00	1.00	0.98	1.00	1.01	1.01
3	1.00	1.00	1.00	1.02	1.02	1.03
4	1.00	1.00	0.98	1.03	1.01	**1.07**
5	1.00	0.99	1.09	**1.12**	**1.09**	**1.15**
6	1.00	1.01	1.00	1.00	1.00	**1.06**
7	1.00	1.01	1.01	1.01	1.04	1.00
8	1.00	1.01	1.01	1.01	1.04	1.02
9	1.00	1.00	1.02	1.00	0.98	0.98
10	1.00	1.01	1.02	1.01	1.00	**1.05**
11	1.00	0.99	0.99	1.02	1.01	0.99
12	1.00	1.01	1.01	1.01	1.01	0.99

Table 7. Mean Annual Ratio Between Year-over-Year Monthly Laspeyres and Paasche Indexes for the Modified Israeli CPI Data

Year	1997	1998	1999	2000	2001	2002
Fixed Base	1.00	1.03	1.06	1.03	1.05	.99
Chained	1.00	1.03	1.06	1.12	1.15	1.21

And in table 7 below, we compare the spread between the Laspeyres and Paasche indexes without and with chaining. That is, we compare the fixed base with the chained indexes. Looking at the mean annual ratios in table 7, we see that from year 2000 on, the Laspeyres/Paasche spread is systematically greater for the chained variants. These findings, as in section 4 above for the unmodified Israeli CPI program data, *contradict* the conclusion from the Manual chapter.

A major change in findings occurs with the Lowe (denoted by PLO in figure 3) and Rothwell (PNR) indexes. Now the Rothwell exhibit smaller seasonal movements than the Lowe indexes (means of 1.01 and 1.12, respectively) and are less volatile (see figure 3).

Finally, the seasonally adjusted series of the Lowe, Young and geometric Laspeyres indexes with imputed prices and the Rothwell index seem to perform less well in tracking the trend of the target index values (the centered rolling year index).[40] The centered rolling year index for the chosen data set still fails to reliably represent the overall price index of fresh fruits with X-11 seasonal adjustment.[41] However, the extreme outliers found in the Seasonally-Adjusted Rothwell compared to the target indexes are absent with the modified Israeli data.

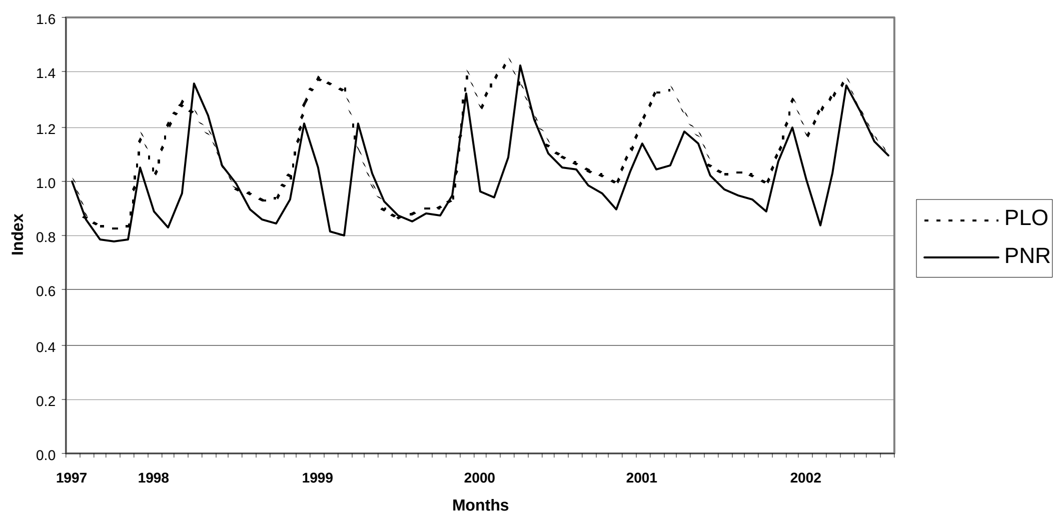
[40] For the seasonally adjusted Lowe index, an R^2 of .1094 is obtained for regression on the centered rolling year series (the lowest fit); for the seasonally adjusted Young index, the R^2 is .1274 and for the seasonally adjusted geometric Laspeyres index, the R^2 is .2197. For the seasonally adjusted Rothwell index, the R^2 is .1477.

[41] The R^2 value is .0026.

7. Concluding Remarks

This paper has three objectives: (1) to summarize the methods and findings of the chapter on the treatment of seasonal products from the new international PPI and CPI Manuals, (2) to describe some of the methods used in the Israeli CPI to overcome seasonal fluctuations (and bias) in a month-to-month index, and (3) to examine some of the conclusions from the Manual chapter by simulating the methods with real Israeli CPI data.

Figure 3. Lowe and Normalized Rothwell Indexes for the Modified Israeli CPI Data



Using real data from the Israeli CPI led to findings that mostly support those reported in the Manual chapter, but there are interesting exceptions. The exceptions are the focus of our report. One conclusion is that the methods presented in the Manual chapter should be compared using fuller data sets and in many countries around the world. This kind of empirical research can improve the methods used for seasonal products. In this regard, analytical series (i.e., unofficial alternative series produced and made available to researchers and other interested users by official statistics agencies) have an important role to play. Computing analytical series on a current basis will enable comparisons that can lead to enhanced procedures in the production of price indexes. Setting rules for modifications in a way that will approach the results obtained by the theoretical dataset may help to treat these problems in a more effective manner. In addition, the investment in analytical series is once only.

Appendix A. Formulas for Methods of Treatment of Seasonal Products

In this appendix, two sets of the equation number are provided for the convenience of readers interested in studying the fuller presentations of these methods in Diewert and Armknecht (2004) and in Diewert, Armknecht and Nakamura (2007). The same numbering is used as in the latter reference except that an A has been added as a prefix, and the equation numbers from the first of two related reference works are shown in square brackets.

A.1 Year-over-year monthly indexes

For each month $m = 1,2,\ldots,12$, let $S(m)$ denote the set of products that are available for purchase in each year $t = 1,2,\ldots,T$. Let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is available in month m of year t for n belongs to $S(m)$. Then the year-over-year monthly Laspeyres, Paasche and Fisher indexes going from month m of year t to month m of year t+1, in price relative and monthly revenue share form, can be defined as follows:

(A-1)[22.4] $$P_L = \sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right), \qquad m=1,2,\ldots,12,$$

(A-2)[22.5] $$P_P = \left[\sum_{n\in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}, \qquad m=1,2,\ldots,12,$$

(A-3)[22.6] $$P_F = \sqrt{P_L P_P}\,.$$

where the monthly revenue share for product $n \in S(m)$ for month m in year t is defined as:

$$s_n^{t,m} = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i\in S(m)} p_i^{t,m} q_i^{t,m}}, \qquad m=1,2,\ldots,12,\ n \in S(m);\ t = 0,1,\ldots,T$$

Approximate year-over-year monthly Laspeyres and Paasche indexes are defined as:

(A-4)[22.8] $$P_{AL} = \sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right), \qquad m=1,2,\ldots,12;$$

(A-5)[22.9] $$P_{AP} = \left[\sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}, \qquad m=1,2,\ldots,12;$$

Where $s_n^{0,m}$ is the base period monthly revenue share.

Using the approximate Laspeyres and Paasche indexes, the approximate Fisher year-over-year monthly indexes are defined by[42]

[42] If the monthly expenditure shares for the base year, $s_n^{0,m}$, are all equal, then the approximate Fisher index defined by equation (A-6) reduces to Fisher's (1922, p. 472) formula 101. Fisher (1922, p. 211) observed that this index was empirically very close to the unweighted geometric mean of the price relatives, while Dalén (1992, p. 143) and Diewert (1995, p. 29) showed analytically that these two indexes approximate each other to the second order.

(A-6)[22.10] $P_{AF} = \sqrt{P_{AL}P_{AP}}$.

A.2 Year-over-year annual indexes

Using the notation introduced above, the Laspeyres and Paasche annual (chain link) indexes comparing the prices of year t with those of year t+1 can be defined as follows:

(A-9)[22.16] $$P_L = \sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m} \left(p_n^{t+1,m} / p_n^{t,m} \right);$$

(A-10)[22.17] $$P_P = \left[\sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^{t+1} s_n^{t+1,m} \left(p_n^{t+1,m} / p_n^{t,m} \right)^{-1} \right]^{-1},$$

where the revenue share for month m in year t is defined as:

$$\sigma_n^t = \frac{\sum_{n \in S(m)} p_n^{t,m} q_n^{t,m}}{\sum_{i=1}^{12} \sum_{j \in S(i)} p_j^{t,i} q_j^{t,i}}, \qquad m = 1,2,\ldots,12;\ t = 0,1,\ldots,T.$$

The current year weights, $s_n^{t,m}$ and σ_m^t and $s_n^{t+1,m}$ and σ_m^{t+1} can be approximated by the corresponding base year weights, $s_n^{0,m}$ and σ_m^0.

There is no need to restrict attention to calendar year comparisons: any 12 consecutive months of data could be compared to the price and quantity data of the base year, provided that the January data in the non-calendar year is compared to the January data of the base year, the February data of the non-calendar year is compared to the February data of the base year, …, and the December data of the non-calendar year is compared to the December data of the base year. Alterman, Diewert and Feenstra (1999, p. 70) call these indexes *rolling year indexes*.

A.3 Maximum overlap month-to-month price indexes

Let there be N products that are available in some month of some year and let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is in the marketplace in month *m* of year *t* (if the product is unavailable, let $p_n^{t,m}$ and $q_n^{t,m}$ be 0). Let $p^{t,m} \equiv [p_1^{t,m}, p_2^{t,m}, \ldots, p_N^{t,m}]$ and $q^{t,m} \equiv [q_1^{t,m}, q_2^{t,m}, \ldots, q_N^{t,m}]$ be the month m and year *t* price and quantity vectors, respectively. Let $S(t,m)$ be the set of products that is present in month *m* of year *t and the following month*.

Define the revenue shares of product n in month m and m+1 of year t, using the set of products that are present in month m of year *t* and the subsequent month, as follows:

(A-12)[22.23] $$s_n^{t,m}(t,m) = \frac{p_n^{t,m}q_n^{t,m}}{\sum_{i \in S(t,m)} p_i^{t,m}q_i^{t,m}}, \qquad m=1,2,\ldots,11;\ n \in S(t,m),$$

(A-13)[22.24] $$s_n^{t,m+1}(t,m) = \frac{p_n^{t,m+1}q_n^{t,m+1}}{\sum_{i \in S(t,m)} p_i^{t,m+1}q_i^{t,m+1}}, \qquad m=1,2,\ldots,11;\ n \in S(t,m),$$

where $s_n^{t,m+1}(t,m)$ must be distinguished from $s_n^{t,m+1}(t,m+1)$. The revenue share $s_n^{t,m+1}(t,m)$ is the share of product n in month m+1 of year t with n restricted to the set of products that are present in month m of year t and the subsequent month, whereas $s_n^{t,m+1}(t,m+1)$ is the share of product n in month m+1 of year t with n restricted to the set of products that are present in month m+1 of year t and the subsequent month.

If product n is present in month m of year t and the following month, define $s_n^{t,m}(t,m)$ using (a); if this is not the case, define $s_n^{t,m}(t,m) = 0$. Similarly, if product n is present in month m of year t and the next one, define $s_n^{t,m+1}(t,m)$ using (b); if not, define $s_n^{t,m+1}(t,m) = 0$.

Using these share definitions, Laspeyres and Paasche formulas can be written in revenue share and price form as follows[43]:

(A-14)[22.25] $$P_L = \sum_{n \in S(t,m)} s_n^{t,m}(t,m)\left(p_n^{t+1,m} / p_n^{t,m}\right), \qquad m=1,2,\ldots,11,$$

(A-15)[22.26] $$P_P = \left[\sum_{n \in S(t,m)} s_n^{t,m+1}(t,m)\left(p_n^{t+1,m} / p_n^{t,m}\right)^{-1}\right]^{-1}, \qquad m=1,2,\ldots,11.$$

A.4 Annual basket indexes

The Lowe index for month m is defined by the following formula:

(A-17)[22.28] $$P_{LO} = \frac{\sum_{n=1}^{N} p_n^m q_n}{\sum_{n=1}^{N} p_n^0 q_n}$$

[43] It is important that the revenue shares that are used in an index number formula add up to unity. The use of unadjusted expenditure shares would lead to a systematic bias in the index number formula.

where $p^0 \equiv [p_1^0, p_2^0, \ldots, p_N^0]$ is the price vector for the price reference period, $p^m \equiv [p_1^m, p_2^m, \ldots, p_N^m]$ is the current month m price vector, and $q \equiv [q_1, \ldots, q_N]$ is the weight reference year quantity vector.

The Young (1812) index is defined as follows:

(A-18)[22.30] $$P_Y = \sum_{n=1}^{N} s_n (p_n^m / p_n^0)$$

where $s \equiv [s_1, \ldots, s_N]$ is the weight reference year vector of revenue shares.

The geometric Laspeyres index is defined as follows:

(A-19)[22.32] $$P_{GL} = \prod_{n=1}^{N} (p_n^m / p_n^0)^{s_n}$$

Thus the geometric Laspeyres index makes use of the same information as the Young index except that a geometric average of the price relatives is taken instead of an arithmetic one.

It is of interest to compare the above three indexes that use annual baskets to the fixed base Laspeyres rolling year indexes. However, the rolling year index that ends in the current month is centered five and a half months backwards. Hence the above annual basket type indexes may be compared with an arithmetic average of two rolling year indexes that have their last month 5 and 6 months forward. This latter *centered rolling year index* is labeled P_{CRY}.

A.5 Bean and Stine Type C or Rothwell indexes

The Bean and Stine Type C (1924, p. 31) or Rothwell (1958, p. 72) index makes use of seasonal baskets in the base year, denoted as the vectors $q^{0,m}$ for the months m = 1,2,…,12. The index also makes use of a vector of base year unit value prices, $p^0 \equiv [p_1^0, \ldots, p_5^0]$ where the nth price in this vector is defined as:

(A-20)[22.33] $$p_n^0 \equiv \frac{\sum_{m=1}^{12} p_n^{0,m} q_n^{0,m}}{\sum_{m=1}^{12} q_n^{0,m}};$$

The *Rothwell price index* for month m in year t can now be defined as follows:

(A-21)[22.34] $$P_R = \frac{\sum_{n=1}^{N} p_n^{t,m} q_n^{0,m}}{\sum_{n=1}^{N} p_n^{0} q_n^{0,m}}, \qquad m = 1,\ldots,12.$$

To make the different series more comparable, the normalized Rothwell index P_{NR} is introduced; this index is equal to the original Rothwell index divided by its first observation.

A.6 Forecasting rolling year indexes using month-to-month annual basket indexes

For each of the series -- Lowe, Young and geometric Laspeyres -- a seasonal adjustment factor (SAF) is defined, as the centered rolling year index P_{CRY} divided by P_{LO}, P_Y and P_{GL}, respectively for the first 12 observations. Now for each of the three series, repeat these 12 seasonal adjustment factors for the remaining observations. These operations will create 3 SAF series for all the observations (label them SAF_{LO}, SAF_Y and SAF_{GL}, respectively).

Finally, define seasonally adjusted Lowe, Young and geometric Laspeyres indexes by multiplying each unadjusted index by the appropriate seasonal adjustment factor.

(A-22)[22.35] $$P_{LOSA} \equiv P_{LO} SAF_{LO}; P_{YSA} \equiv P_Y SAF_Y; P_{GLSA} \equiv P_{GL} SAF_{GL}.$$

A seasonally adjusted version of the Rothwell index presented in the paper may also be defined in the same way.

Appendix B. Seasonal Data Set for Fresh Fruit

Table B-1. Prices for Fresh Fruits (New Israeli shekel per kilo)

Year	Month	Lemons	Apricots	Avocado	Watermelon	Persimmon	Grapefruit	Bananas
1997	1	3.42	0	3.42	0	5.81	2.81	3.79
	2	3.34	0	3.71	0	5.81	2.74	3.88
	3	3.43	0	3.78	0	6.67	2.78	3.76
	4	3.89	0	4.03	0	0	2.9	4.24
	5	4.35	0	5.07	3.65	0	2.81	5.39
	6	6.76	8.81	6.44	2.03	0	3.01	6.77
	7	7.7	8.01	7.25	1.56	0	3.41	9.73
	8	9.15	0	0	1.46	0	3.63	9.43
	9	8.36	0	7.65	1.56	0	4.48	7.57
	10	6.47	0	5.65	0	6.7	4.31	7
	11	4.79	0	4.35	0	5.34	3.61	6.74
	12	3.9	0	3.95	0	5.44	2.9	5.86
1998	1	3.51	0	3.82	0	5.75	2.69	4.49
	2	3.45	0	3.72	0	5.88	2.42	4.09
	3	3.42	0	3.78	0	0	2.46	4
	4	3.68	0	3.98	0	0	2.57	3.98
	5	4.19	0	4.6	3.34	0	2.95	3.97
	6	5.9	6.11	5.18	1.67	0	3.39	5.01

Year	Month	Lemons	Apricots	Avocado	Watermelon	Persimmon	Grapefruit	Bananas
	7	6.38	6.64	5.81	1.57	0	3.77	7.12
	8	7.39	0	9.16	1.74	0	3.7	7.52
	9	7.58	0	8.79	0	0	4.74	5.88
	10	7.06	0	6.34	0	7.85	4.36	5.71
	11	5.88	0	5.44	0	6.71	4.16	4.76
	12	4.94	0	5.71	0	7.1	3.37	4.19
1999	1	4.55	0	6.28	0	7.61	3.2	3.89
	2	4.22	0	6.14	0	0	3	3.75
	3	4.17	0	6.5	0	0	3.05	3.67
	4	4.62	0	7.56	0	0	3.22	4.16
	5	5.47	0	10.58	2.47	0	3.45	5.07
	6	7	8.82	13.66	1.7	0	3.88	6.14
	7	7.88	0	0	1.4	0	0	6.36
	8	7.96	0	0	1.42	0	4.13	5.94
	9	7.19	0	8.42	0	0	4.4	4.69
	10	5.68	0	5.84	0	7.5	4.33	4.3
	11	4.85	0	4.95	0	6.23	3.73	4.06
	12	4.32	0	4.64	0	6.41	3.4	3.81
2000	1	4.06	0	4.56	0	7.14	3.29	4.07
	2	3.83	0	4.35	0	7.66	3.19	4.4
	3	3.69	0	3.85	0	0	3.1	4.58
	4	3.49	0	3.67	0	0	3.27	5.13
	5	4.24	0	4.48	2.74	0	3.44	7.58
	6	5.7	7.32	5.56	1.65	0	3.87	7.58
	7	8.15	7.61	6.55	1.76	0	4.24	8.12
	8	10.92	0	9.04	1.93	0	0	7.85
	9	7.84	0	9.26	1.93	0	0	6.12
	10	6.18	0	6.55	0	7.41	5.65	5.83
	11	5.3	0	5.09	0	6.11	4.26	5.71
	12	4.65	0	4.93	0	6.02	3.73	5.49
2001	1	4.15	0	5.03	0	6.35	3.41	5.33
	2	3.86	0	4.86	0	7.01	3.19	5.11
	3	3.70	0	5.04	0	0	3.17	4.84
	4	3.91	0	5.14	0	0	3.32	4.45
	5	4.4	0	6.73	0	0	3.59	4.66
	6	5.78	8.45	8.33	2.21	0	3.75	5.31
	7	6.46	8.86	0	1.97	0	4.66	6.56
	8	6.69	0	0	1.96	0	5.69	6.42
	9	5.62	0	8.88	0	0	0	5.42
	10	5.21	0	6.69	0	7.77	0	5.4
	11	4.57	0	4.97	0	6.75	4.12	4.91
	12	4.31	0	4.75	0	6.82	3.9	4.56
2002	1	4.1	0	4.97	0	7.15	3.56	4.65
	2	3.91	0	4.62	0	7.76	3.48	4.67
	3	3.67	0	4.32	0	0	3.44	4.54
	4	3.94	0	4.7	0	0	3.64	5.72
	5	4.05	10.6	4.74	2.89	0	3.75	5.94
	6	4.21	6.46	5.07	1.99	0	4	6.2
	7	5.84	6.51	0	1.6	0	3.83	7.81
	8	6.58	0	0	1.91	0	0	7.64
	9	6.19	0	9.61	0	0	5.69	6.8
	10	5.48	0	6.32	0	7.93	5.11	6.52
	11	4.8	0	6.22	0	6.28	4.23	5.84
	12	4.22	0	6.33	0	5.91	3.76	5.36

Table B-2. Expenditures for Fresh Fruits[44]

Year	Month	Lemons	Apricots	Avocado	Watermelon	Persimmon	Grapefruit	Bananas
1997	1	1.7	0.1	4.3	0	1.1	0.3	17.7
	2	2.5	0	3.6	0.3	0.9	1.7	14.8
	3	1.9	0	3.7	0.7	0.2	1.4	15.3
	4	2.4	0.1	3	3.4	0	1.4	17.5
	5	2.1	0.2	2.7	11	0	1.5	11.8
	6	3.2	6.4	2.3	28.9	0	1.6	6.2
	7	2.2	7.4	1.6	27.8	0.1	0.8	1
	8	2.9	0.8	0.5	22.2	0	0.7	1.5
	9	2.8	0	0.5	13.3	0	0.5	2.4
	10	2.8	0	1.7	2.7	1.6	0.6	6.7
	11	2.4	0	3.5	0.4	3.6	1.2	13.3
	12	2.2	0.1	5.1	0.2	3.6	0.9	15
1998	1	1.7	0	3.8	0	3	0.7	14.4
	2	2.2	0	4.8	0	1.9	0.8	16.9
	3	2.6	0.1	3.8	0.6	0.7	1	17.4
	4	2.8	0.2	3.2	2.6	0.1	1.7	17.5
	5	2.6	1.1	2.8	22.2	0.1	0.7	12.5
	6	2.4	10.4	1.6	26	0	0.4	7.2
	7	3.7	6.9	1.4	23.6	0	0.7	3.2
	8	2.6	0.3	0.8	24.6	0.2	0.7	3.2
	9	2.9	0.1	1.1	11.7	0.2	1.1	4.5
	10	3.6	0.1	2.1	1.8	0.8	0.4	8.9
	11	3.2	0	4.3	0.3	3.4	1.2	13.8
	12	2.8	0	4	0.2	2.7	0.9	14.7
1999	1	2.1	0.1	4.3	0	1.5	1	16.1
	2	2.4	0.1	4.3	0.1	1.7	1.1	14.2
	3	2.1	0	4.4	0.4	0.3	1.1	15
	4	3	0.1	4	4.3	0.2	1.2	13.6
	5	3.2	2.2	2.2	21	0.3	1.7	11.5
	6	2.8	11	1.9	26.7	0	0.8	6.7
	7	3.1	6	0.4	25.7	0	0.8	4
	8	2.6	0.5	0.2	19.4	0.1	0.4	3.7
	9	2.8	0.2	1.1	9.4	0.4	0.6	6.1
	10	2.8	0	2.6	1.4	1.6	0.9	8.3
	11	2.5	0.3	5.2	0.2	3.7	1.4	12.7
	12	2.6	0	4.4	0	3.4	0.7	12.3
2000	1	2.2	0.2	3.7	0	2.9	1	11
	2	2.7	0	4.2	0	2.3	1	13.6
	3	3.1	0	3.6	0.1	0.6	1.4	12.7
	4	2.6	0	3.2	3.6	0.1	1	14.2
	5	3.1	1.2	3	18	0	1.1	8.5
	6	2.4	8.9	1.6	25.4	0.1	0.6	4.7
	7	3.2	7.1	1.6	25.7	0	0.2	2
	8	3.8	0.4	1.1	21.3	0	0.5	2.5
	9	2.6	0.1	1.1	9.9	0.1	0.3	4.6
	10	2.9	0.1	2.5	1.8	1.5	0.6	9.4
	11	3.1	0	5	0.7	3.9	1	11.3

[44] The given data set of expenditures has been further adjusted for the purpose of calculations: in the months where the price for the good is equal to 0, we assume that the expenditure also equals 0.

Year	Month	Lemons	Apricots	Avocado	Watermelon	Persimmon	Grapefruit	Bananas
	12	2.6	0.3	4	0.2	3.5	0.9	13.7
2001	1	2.3	0	4.2	0	4.1	1.4	13.5
	2	2.9	0.2	3.7	0.2	2	0.9	14
	3	2.6	0.2	3.6	0.8	1.7	1.2	13.8
	4	2.9	0.1	3	5.9	0.1	0.8	13.7
	5	2.4	2.5	2.5	21.1	0	0.7	10.1
	6	2.8	10.1	2.3	23.3	0	0.9	5.8
	7	3	3.6	2.1	23.6	0	0.4	4
	8	3.3	0.1	1.1	17	0	0.3	3
	9	3.4	0.1	1.4	5.2	0.2	0.1	4.8
	10	3.7	0.2	4.1	1.8	2.4	0.6	9.4
	11	3.1	0	6.3	0.5	4.4	1	13.2
	12	2.5	0.1	5	0.5	3.4	0.8	13.5
2002	1	1.8	0.2	16	0	3.2	1.2	12.7
	2	3	0.1	15	0.5	1.9	1.3	16.4
	3	3.5	0.1	14.2	0.7	1.1	1.1	14.4
	4	3.7	0.1	14.2	9.9	0.4	1.1	14.5
	5	2.8	4	13.6	17.3	0.1	1.2	11
	6	2.8	10.6	13.1	21.6	0	0.7	6.9
	7	3.3	3.8	16	25.1	0	0.7	3.4
	8	3.9	0.3	20.5	18.4	0	0.1	3.2
	9	3.4	0.1	16.8	10.6	0.2	0.4	3.8
	10	2.8	0	17.3	1	2.1	0.4	7.3
	11	2.8	0	16.7	0.3	4.5	0.7	11.5
	12	2.8	0	21.4	0.4	5	0.9	14.7

Appendix C. Seasonal Data Set for Fresh Vegetables

Table C-1. Prices for Fresh Vegetables (New Israeli shekel per kilo)

Year	Month	Cabbage	Cauliflower	Cucumbers	Potatoes	Carrots	Lettuce	Eggplants
1997	1	2.09	3.1	3.21	2.37	3.16	3.01	3.28
	2	2.5	3.77	5	2.54	3.16	3.05	6.31
	3	2.67	3.92	5.49	3.23	3.26	3.14	6.49
	4	2.34	4.04	4.46	3.17	3.26	3.12	5.55
	5	2.4	3.63	2.98	2.91	3.13	3.18	4.07
	6	2.24	4.1	2.56	2.64	3.02	3.26	3.33
	7	2.12	4.5	2.96	2.56	3.07	3.25	2.63
	8	2.61	4.54	2.96	2.93	3.33	3.46	2.82
	9	2.83	4.51	2.73	2.93	3.55	3.45	2.74
	10	2.71	4.19	3.35	3.05	3.86	3.53	2.99
	11	2.55	4	3.44	3.04	3.52	3.61	3.12
	12	2.45	3.8	3.27	2.86	3.11	3.44	3.03
1998	1	2.36	3.4	3.11	2.71	2.81	3.29	3.21
	2	2.28	3.13	2.99	2.58	2.76	3.1	3.61
	3	2.18	3.54	3.47	2.42	2.67	3.17	4.12
	4	2.18	3.51	4.14	2.46	2.77	3.18	4.64
	5	2.12	4.24	3.26	2.44	2.84	3.28	5.03
	6	2.27	4.8	2.67	2.34	3.12	3.32	3.14
	7	2.33	4.88	2.69	2.36	3.39	3.39	2.94
	8	3.76	5.65	3.35	2.65	3.88	3.99	2.95

Year	Month	Cabbage	Cauliflower	Cucumbers	Potatoes	Carrots	Lettuce	Eggplants
	9	7.4	7.24	3.75	2.94	4.27	5.1	3.33
	10	6.38	6.18	3.53	3.25	4.54	5.04	3.43
	11	3.84	5.56	3.09	3.32	4.14	4.23	3.32
	12	3.05	4.89	4.43	3.26	3.72	3.7	3.16
1999	1	3.21	3.99	3.25	3.18	3.59	3.26	4.39
	2	2.72	3.4	3.19	3.09	3.47	3.26	4.8
	3	2.27	3.98	3.05	2.81	3.21	2.89	4.15
	4	2.34	3.46	3.15	2.76	3.18	2.91	3.8
	5	2.2	3.54	2.78	2.45	3.12	3.07	3.38
	6	2.24	4.15	2.86	2.4	3.28	3.16	3.2
	7	2.33	5.61	3.05	2.39	3.38	3.28	3.07
	8	2.67	6.02	2.99	2.71	3.29	3.5	3.17
	9	2.93	5.33	3.51	2.77	3.31	3.68	3.13
	10	2.86	4.95	4.34	2.86	3.58	3.72	3.22
	11	2.65	5	3.67	2.97	3.59	3.56	3.03
	12	2.76	5.24	5.02	3.57	3.74	3.52	3.88
2000	1	2.6	3.69	4.33	3.18	3.86	3.5	4.45
	2	2.56	3.82	4.45	2.98	3.88	3.6	6.38
	3	2.44	4.44	4.18	2.9	3.7	3.29	5.67
	4	2.24	3.89	3.16	2.53	3.25	3	4.85
	5	2.28	3.79	2.68	2.52	3.38	3.27	4.51
	6	2.29	4.25	2.95	2.44	3.24	3.43	3.28
	7	2.86	5.09	3.33	2.61	3.28	3.45	2.85
	8	3.71	5.42	2.96	3.09	3.49	3.68	2.94
	9	3.65	5.1	3.21	3.14	3.69	3.84	2.96
	10	3.25	5.09	4.02	3.3	3.99	3.79	3.31
	11	3.03	5.32	4.26	3.21	3.89	3.77	3.41
	12	3.02	4.54	3.87	2.99	3.85	3.73	3.31
2001	1	2.96	4.16	3.06	2.67	3.63	3.63	3.31
	2	2.81	4.1	3.26	2.45	3.41	3.53	3.52
	3	2.65	4.14	3.13	2.34	3.21	3.3	3.62
	4	2.57	4.49	3.47	2.58	3.36	3.39	4.57
	5	2.5	4.46	3.5	2.88	3.55	3.49	4.14
	6	2.52	4.8	3.24	3.1	3.73	3.62	3.72
	7	2.55	5.12	3.35	3.43	3.9	3.56	3.4
	8	2.71	5.25	4.64	3.76	3.99	3.61	3.54
	9	2.87	6.21	5.18	3.77	4.26	3.93	4.11
	10	3.01	5.51	4.03	4.08	4.38	3.88	3.75
	11	2.95	5.1	3.7	4.29	4.23	3.89	3.65
	12	3.46	4.66	4.29	3.94	4.12	3.91	3.72
2002	1	3.38	4.64	5.96	3.51	3.97	3.95	5.19
	2	3.3	4.45	4.86	3.6	4.03	3.83	6.34
	3	2.97	4.17	3.75	3.44	3.93	3.53	4.74
	4	2.91	4.17	3.87	3.42	3.94	3.57	4.95
	5	2.6	4.24	3.09	3.27	3.83	3.57	4.4
	6	2.56	4.68	3.41	3.17	3.75	3.62	3.55
	7	2.44	5.51	3.41	3.07	3.63	3.52	3.22
	8	3.49	6	3.99	3.16	3.82	3.98	3.63
	9	4.72	6.38	4.11	3.33	4.06	4.31	3.79
	10	4.54	5.15	4.66	3.28	4.3	4.08	3.64
	11	3.36	5.5	4.53	3.03	4.18	3.93	3.24
	12	3.07	5.04	4.25	3.03	4.08	3.69	4.01

Table C-2. Expenditures for Fresh Vegetables

Year	Month	Cabbage	Cauliflower	Cucumbers	Potatoes	Carrots	Lettuce	Eggplants
1997	1	5.1	3	15.6	21.5	5.2	2.9	3.9
	2	4.2	3.1	13.3	16.4	4.4	2.7	3.1
	3	3.8	2.6	15.1	19.1	4.4	2.4	4.2
	4	4.1	2.2	17	21.1	4.8	3.4	2.8
	5	3.1	1.4	14.3	22.1	4.2	3.8	2.2
	6	3.1	1.2	12.2	18.6	4	2.2	3.9
	7	3.3	1.6	12	16.7	3.3	2.2	4
	8	2.5	1.1	11.8	16.9	3	1.8	4.7
	9	2.1	1.2	9.9	18.5	3.8	2.3	3.1
	10	3.4	2.2	12.2	21.2	4.5	2.6	3.6
	11	4.1	2	13.3	20	4.6	2.5	3.9
	12	3.9	2.4	11.2	17.5	4.1	2.4	3.3
1998	1	3.3	2.6	10.2	19.9	4.4	3.1	1.6
	2	4.3	2.7	12.9	19.9	4.1	2.9	3.7
	3	3.9	2.3	13.4	18	4.1	2.7	2.9
	4	3.2	2	15.2	18.7	3.7	4.1	3.9
	5	3.4	1.9	15.2	18.3	3.9	3.2	4.1
	6	2.9	1.3	13.2	16.6	3.6	1.8	4.4
	7	2.7	1.6	13.1	14.2	3.6	2.2	4.1
	8	2.8	1.4	12.6	17	3.3	2.1	3.4
	9	3.7	1.7	16.1	19.5	4	2.9	3.8
	10	6.4	2.1	15.5	19.2	4.2	3.5	4.9
	11	5.2	2.3	12.8	19.5	5.4	3.3	4.2
	12	4	2.8	11.5	19	4.7	3.4	3
1999	1	5.2	4	10.3	19.8	5.8	3.3	3.1
	2	5	3.1	11.6	19	4.6	2.9	3.9
	3	4.2	1.9	12.4	18.7	4.4	3.4	2.4
	4	4.2	2.9	13.3	20	4.3	3.7	2.5
	5	3.6	1.9	12.8	19.6	4.3	3.2	4.2
	6	3.7	1.2	15.2	18.8	4.1	3.3	4.5
	7	3.1	1.1	13.9	16.3	4.3	2.7	4.5
	8	2.8	1.2	11.6	17	4	1.8	4.7
	9	3.7	1.8	14.8	19.6	4	3.1	4.8
	10	3.3	2	13.1	16.1	3.9	3	4.3
	11	3.7	2.1	13.2	17.9	4.1	3.7	3.2
	12	4.5	2.8	16.3	22.3	4.7	2.9	3.8
2000	1	3.1	1.4	10	20.2	5.2	2.7	1.9
	2	4.4	2.6	13.8	21.6	7.2	3.3	3.1
	3	3.8	2.4	12.8	20	4.6	3.3	4
	4	3.7	2.7	13.4	19.2	4.5	4.1	3
	5	4.2	2.1	12.5	19.1	4.2	4.4	3.5
	6	3.5	1.3	12.5	14.4	4.2	3.4	3.8
	7	3.4	0.9	12.5	17.2	4.3	2.8	3.3
	8	4.7	1.3	12.6	17.6	3.5	3.1	3.7
	9	3.9	1.2	13.8	19.6	3.9	3.3	3.6
	10	5.5	2.9	14.4	22.7	5.6	4.5	4.9
	11	4.3	1.7	13.5	19.5	5.3	3.9	3.6
	12	4.1	2.7	13.3	20.5	4.6	3.6	3.4
2001	1	4.6	2.5	10.5	19.9	5	4.8	2.2
	2	4.1	2.2	10.7	15.8	4.9	3.9	3
	3	4.8	2.5	11.4	18	4.9	3.8	3.4
	4	3.6	2.8	13.1	19.2	4.2	3.8	3.4
	5	3.2	1.1	13.1	15.7	3.9	3.6	3.6

Year	Month	Cabbage	Cauliflower	Cucumbers	Potatoes	Carrots	Lettuce	Eggplants
	6	3.4	1.2	12	16.6	4.6	3.3	3.3
	7	4.1	1.5	12.5	17.6	5.3	3.7	3.8
	8	3.9	1.3	14.7	20.7	5	3.1	4.2
	9	4.4	1.9	16.2	22.3	4.8	3.6	4.7
	10	5.3	2.4	15.4	21	5.6	4.4	4.3
	11	4.5	2.7	14.5	25.5	5.9	4.8	3.5
	12	5	2.9	13.1	23.1	6.1	4.3	2.7
2002	1	4.8	2.9	13.6	22.7	5.5	3.9	2.8
	2	6.3	3.4	19.1	23.2	4.7	5.6	3.4
	3	5.3	2.9	14.5	21.5	5	5	3.2
	4	5.3	2.7	14.2	22.7	5.2	5.6	5.2
	5	4.4	1.8	13.6	19.4	4.7	4.7	3.6
	6	4.4	1.8	13.5	18.2	4.3	4.3	4.7
	7	4	0.9	13.9	16.8	4.8	4.4	4.3
	8	4.8	2.5	14.7	19.3	3.6	4.1	3.9
	9	4.5	1.3	13.3	17.9	4.9	4.3	3.2
	10	5.3	1.9	15.2	20.1	4.4	5.2	3.1
	11	5.3	2.4	16.3	18.4	6.7	4.5	3.9
	12	4.7	3.1	14.9	20.8	5.8	4.2	3.8

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