Preventing Self-fulfilling Debt Crises

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Abstract

This paper asks whether a government can implement policies that help to avert a crisis driven by self-fulfilling expectations. I consider two policies that are often at the center of political discussions: an austerity and a fiscal stimulus. I find that under plausible conditions austerity decreases the probability of a debt crisis, while a stimulus increases it. I show that endogenous expectations amplify the effects of government policies so that even a small policy adjustment can have significant effects. Finally, I find that policy uncertainty increases the attractiveness of austerity versus stimulus, but decreases the overall impact of both policies.

Key words: sovereign debt crises, expectations, policy uncertainty, taxes, fiscal stimulus

JEL codes: D82, D84, F34

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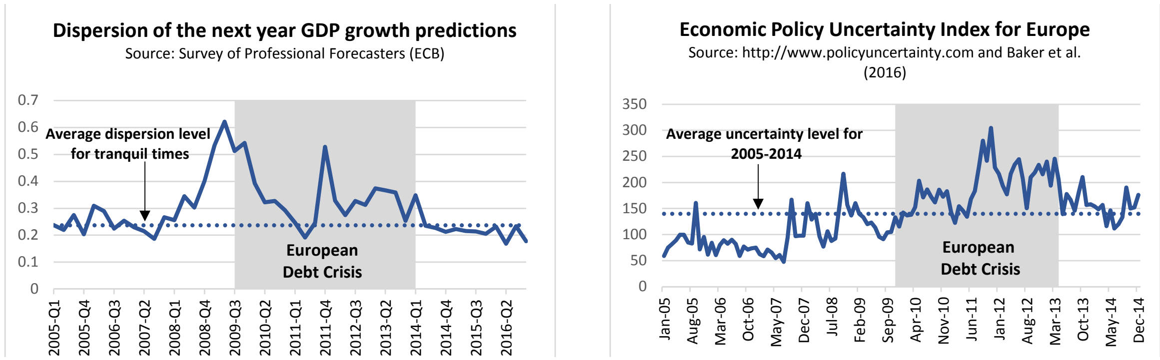
"[...] the assessment of the Governing Council is that we are in a situation now where you have large parts of the Euro Area in what we call a bad equilibrium, namely an equilibrium where you have self-fulfilling expectations. [...] So, there is a case for intervening, in a sense, to "break" these expectations."

Mario Draghi, Press Conference, Frankfurt am Main, September 6, 2012

1 Introduction

Sovereign debt crises are a recurring phenomena. After the turbulent 1980s and a series of defaults in the late 1990s and early 2000s, sovereign defaults once again became a hotly debated topic. One of the leading views on sovereign defaults, as exemplified by the above quote, is that they are the result of an interplay between poor economic fundamentals and self-fulfilling expectations.[1]



(a) Economic uncertainty (b) Politicy uncertainty

Figure 1: Economic and policy uncertainty in Europe 2005-2014.

It is important to note that confidence crises do not appear out of nowhere, but, rather, are preceded by a deterioration of a debtor country's economic situation and an increase in economic and political uncertainty. Since investors often have access to different sources of private information (or vary in their interpretation of common information), this increase in uncertainty translates into an increased dispersion of beliefs among investors. As a consequence, individual investors, afraid that other investors hold more pessimistic beliefs about the debtor country, may choose not extend new loans even if they believe that debtor country is solvent, thus triggering a default. Indeed, as shown in Figure 1, the recent European debt crisis was accompanied by both an increase in dispersion of beliefs about the future economic prospects of EU countries (Panel A) and an increase in economic policy uncertainty (Panel B).

Motivated by these observations, in this paper, I ask (1) whether a government can implement policies that help to avert a crisis driven by self-fulfilling expectations; and (2)

[1] See, also, Bocola and Dovis (2016), Conesa and Kehoe (2017), or De Grauwe and Ji (2013).

how the desirability of such policies depends on market participants' expectations and on the presence of economic policy uncertainty. I focus on two policies that have been at the center of political discussion in Europe during the recent debt crisis: austerity and fiscal stimulus (see Brunnermeier et al., 2016, Corsetti et al., 2013, and Reinhart and Rogoff, 2010). My findings suggests that under plausible conditions, austerity tends to decrease the probability of an imminent crisis, while stimulus tends to increase it.[2] I also show that endogenous expectations amplify the effects of government policies so that even a small policy adjustment can have significant effects. Finally, I find that the presence of policy uncertainty further increases the attractiveness of austerity versus stimulus, but tends to decreases the overall impact of government policies.

The paper consists of two parts. In the first part, I develop a simple model of self-fulfilling debt crises in which crises arise as a result of an interplay between poor fundamentals, foreign lenders' expectations, and domestic households' expectations. To model dispersed beliefs and to endogenize expectations about sovereign default, I assume that lenders and households do not observe the relevant fundamentals of the economy, but, instead, only receive noisy private signals. This realistic assumption not only captures the uncertainty surrounding the state of the economy during crises episodes, but also transforms lenders' and households' expectations into endogenous equilibrium objects and restores the uniqueness of equilibrium within the class of monotone equilibria.[3] The resulting environment is rich enough to capture the main trade-offs faced by governments during debt crises; however, in contrast to standard models of self-fulfilling sovereign debt crises, it also links beliefs and expectations to economic fundamentals.

In the second part of the paper, I use the model to analyze which policies available to the government can decrease the ex-ante likelihood of a debt crisis (i.e., prevent a debt crisis). I show first that a change in the probability of default implied by *any* policy adjustment can be decomposed into the product of the "direct effect" (the initial effect of the policy change on the government's incentive to default, holding households' and lenders' beliefs constant) and the "beliefs effect" (the change in the government's

[2] To be precise, I provide conditions under which austerity and stimulus decrease the probability of default and conditions under which they increase it. However, I argue that the conditions under which stimulus works are unlikely to hold in practice, while those under which austerity works are likely to be satisfied.

[3] Even though the model has a unique equilibrium outcome, a debt crisis is still driven by expectations in the following sense: There is a region of the fundamentals where both crisis and no-crisis outcomes are consistent with fundamentals, and whether a crisis occurs depends only on agents' endogenous expectations. In that sense, a crisis is self-fulfilling (see Morris and Shin, 1998).

default decision implied by the adjustment in households' and lenders' expectations). I show that the direct effect determines whether a given policy decreases or increases the likelihood of a crisis, while the beliefs effect, which captures the role played by expectations, acts as an amplification mechanism that always magnifies the initial response of the economy. These novel results indicate that if the government wants to avoid default, it can use expectations to its own advantage, as even a small policy change, when amplified by adjustments in expectations, can significantly decrease the likelihood of default.

I use the above observations to analyze the impact of an adjustment in a tax rate and the impact of a fiscal stimulus on the probability of default. In the model, increasing taxes decreases the government's incentives to default by filling the government's financing gap when lenders are unwilling to provide the funding. On the other hand, higher taxes distort investment and decrease future output, making it more difficult for the government to repay the debt later on. I find that an increase in a tax rate tends to decrease the probability of default as long as the initial level of taxes is not "very high," and I argue that this condition is typically satisfied in practice. I model a fiscal stimulus as an increase in government investment financed with debt. A fiscal stimulus, by increasing the output of the economy and, hence, government tax revenues, tends to decrease the government's incentives to default. On the other hand, the associated increase in government debt makes defaulting more attractive. I show that the positive effect dominates if the ratio of the government debt to the initial stock of capital in the economy is sufficiently high. However, I argue that the conditions under which stimulus works are unlikely to hold in practice. It follows that austerity is typically a preferred option.

The above analysis was conducted under the assumption that the government always implements its announced policies. However, debt crises are often accompanied by a substantial uncertainty as to whether the government will go through with its plans (e.g., see Panizza et al., 2009). Indeed, according to the recent index of economic political uncertainty constructed by Baker et al. (2016), this uncertainty reached historic heights in Europe during the recent debt crises (Panel B of Figure 1).

Motivated by these observations, I analyze how the presence of such uncertainty affects the above results and find that the presence of such an uncertainty tends to decrease the negative effect of austerity: Uncertain as to whether higher taxes will be implemented, households do not decrease their investment as much as they would otherwise. On the other hand, economic policy uncertainty decreases the benefits of fiscal stimulus: Unsure of whether the stimulus will be implemented, households do not expand their investment as much as they would otherwise. Thus, the presence of economic policy uncertainty further strengthens the case for austerity relative to fiscal stimulus.

However, I also find that economic policy uncertainty decreases the overall effect of both policies on the probability of default. This is because agents, uncertain about the final government decisions, do not adjust their expectations about the likelihood of default as much as they do in the absence of economic policy uncertainty, which implies that the amplifying effect of endogenous adjustments in expectations is weak. In the extreme case, when a policy change is unexpected and agents' information is very precise, the beliefs effect is completely missing, and government policies cease to have any impact on the probability of default. This last result provides a strong warning against unexpected policy U-turns.

In the final part of the paper, I investigate numerically how the effectiveness of the policies described above depends on the values of the model's main parameters. In addition, I look into the importance of the endogenous expectations (as captured by the beliefs effect) in driving these adjustments and link their importance to the characteristics of the economy. Finally, I investigate how my model's predictions of my model differ from those of a model in which crises are driven purely by fundamentals.

Related Literature — The framework developed in the paper unifies two popular approaches to modeling self-fulfilling debt crises: the micro-funded general equilibrium approach of Cole and Kehoe (2000) and the game-theoretic approach of global games as in Corsetti et al. (2006) and Morris and Shin (2006). The key difference between my model and that of Cole and Kehoe (2000) lies in the information structure, which captures the uncertainty surrounding debt crises and which leads to a unique equilibrium in my model. The equilibrium uniqueness follows from global games literature started by Carlsson and Damme (1993) and Morris and Shin (1998). Corsetti et al. (2006) and Morris and Shin (2006) use reduced-form global game models to study the effectiveness of IMF assistance in preventing a self-fulfilling debt crisis and the moral hazard that such assistance creates. In a parallel work, Zabai (2014), uses global games to study how the government can use tax and borrowing policies to manage probability of default in a model in the spirit of Calvo (1988). In contrast to the above work, the focus of this paper is on understanding the impact that endogenous expectations and policy uncertainty have on the effectiveness of fiscal policies.

Models of self-fulfilling crises have a long tradition in the literature on sovereign default, beginning with Sachs (1984) and Calvo (1988). Following the debt crisis in Europe, this literature has experienced a revival. Corsetti and Dedola (2011, 2016) and Aguiar et al. (2013) investigate how monetary policy can help to avoid a crisis. Lorenzoni and Werning (2013) focus on the role of the interest rate as the main driver of sovereign default. Finally, Cooper (2013) studies the role of debt guarantees as a way to avert a crisis within a federation of countries.

This paper is also related to the literature on sovereign debt in the spirit of Eaton and Gersovitz (1981), which is summarized well in Aguiar and Amador (2014) and Panizza et al. (2009). More recently, this line of research has focused on developing quantitative models of sovereign default that can account for the observed dynamics surrounding the default episodes (see Aguiara and Gopinath, 2006; Arellano, 2008; Hatchondo and Martinez, 2009; or Mendoza and Yue, 2012; and references therein). Cuadra and Sapriza (2008) study the role of political uncertainty quantitatively. In a recent paper, Bianchi et al. (2017) study optimal fiscal policy in the presence of default risk. Typically, this strand of literature assumes away the possibility of a belief-driven crisis.

A large body of work, motivated by the recent events in Europe, studies policy responses to the recession that accompanied the European debt crisis. For example, using a DSGE framework, Eggertsson et al. (2014) study the effects of structural reforms, while Corsetti et al. (2013) investigate the effects of fiscal and monetary policy adjustments. My work complements these papers by providing an analysis of austerity and fiscal stimulus in an environment with a self-fulfilling debt crisis and dispersed beliefs.

2 Model

There are two periods, $t = 1, 2$, and three types of agents: a continuum of identical households, a continuum of identical lenders, and the government. The economy is characterized by the average productivity level A, which is distributed according to a normal distribution with mean A_{-1} and standard deviation σ_A — that is, $A \sim N\left(A_{-1}, \sigma_A^2\right)$. Here, A_{-1} denotes the past average productivity level in the economy, which all agents know. The current average level of productivity, A, is realized at the beginning of period 1 and is constant across the two periods, but it is initially unobserved by the agents. Instead, households and lenders receive private noisy signals about A; its value is revealed to everyone at the end of period 1.

2.1 Households

There is a continuum of identical households, indexed by $i \in [0, 1]$. Households are risk-averse and have preferences given by

$$\sum_{t=1,2} \left[\log\left(c_t\right) + \log\left(g_t\right)\right],$$

where c_t is private consumption and g_t is government spending. Each household initially is endowed with the same amount of capital k_1, and has access to a production function:

$$y_t^i = \widetilde{Z} e^{A_i} f\left(k_t^i\right),$$

where $f(k) = k^\alpha$, $0 < \alpha < 1$. Here, A_i is a household-specific productivity level; $\widetilde{Z}$ is the aggregate productivity level, which depends on the government's default decision; and f is a production function that takes as inputs capital and, implicitly, inelastically supplied labor. The proceeds from production are the only source of income for the household and are taxed at a rate $\tau > 0$. Finally, capital fully depreciates each period.[4]

Households receive their idiosyncratic productivity shocks A_i at the beginning of period $t = 1$. The idiosyncratic productivity is constant across time and given by

$$A_i = A + \varepsilon_i,$$

where ε_i is *i.i.d.* across households and is uniformly distributed on $[-\varepsilon, \varepsilon]$, $\varepsilon > 0$. This implies that A is the average level of productivity in the economy, and that knowing A is equivalent to knowing the aggregate output. After the households observe their respective productivity realizations, household i makes its investment decision; that is, it chooses its capital stock, k_2^i, for period 2. Households make these choices before $\widetilde{Z}$ is determined (and before the actual production takes place). Thus, when making their investment decisions, households face uncertainty regarding their future income.[5] Households are committed to their investment decisions and they cannot adjust them later. The production takes place at the end of period 1, after $\widetilde{Z}$ is determined, at which point the households invest the amount chosen earlier and consume the rest of their income.

Households make no decisions in period 2. They simply use their capital to produce, and they consume all of their after-tax income.

2.2 The Government

The government is benevolent and maximizes households' utility. In each period t, it provides households with public consumption goods, g_t, and finances its expenditures

[4] The assumption that capital fully depreciates implies that the households' optimal investment choice is linear in e^{A_i}, which simplifies the subsequent analysis.

[5] This assumption captures two realistic features of an investment process. First, investment takes time and often requires prior planning. Second, investment decisions are made under uncertainty regarding future economic conditions (in this case, uncertainty about $\widetilde{Z}$).

by taxing households' income and (in period 1) by borrowing in the bond market. The government enters period 1 with a legacy debt, B_1, which is due later in this period, and it initially does not observe the average level of productivity in the economy, A.

At the beginning of period 1, the government announces an interest rate $r > 0$ at which it is willing to borrow in the bond market. Once the households and lenders make their choices, the government observes A and decides how much to borrow, B_2; whether or not to default, d_1; and how much of public goods to provide to households, g_1. In period 2, the government repays its debt, B_2, if it did not default on it earlier, and provides g_2 to households. The government can default only in period 1, in which case it defaults on all of its debt.[6]

Following the large literature on sovereign default, I assume that default is costly and associated with a drop in aggregate productivity (and, hence, in output) by a factor $\widetilde{Z}$. In particular, when the government defaults, $\widetilde{Z}$ takes a value $Z < 1$, while $\widetilde{Z} = 1$ otherwise. There is also an additional cost of default: If the government issues a positive amount of debt at $t = 1$ (i.e., $B_2 > 0$) and then decides to default, it faces a further cost of default equal to ξB_2, $0 < \xi \leq 1$. I interpret ξB_2 as a "litigation cost" associated with the legal battles between bondholders and the government following a default.[7]

2.3 Lenders and the Bond Market

There is a continuum of identical, risk-neutral lenders, indexed by $j \in [0, 1]$, each with finite wealth $b > 0$. Lenders choose at $t = 1$ whether to participate in the bond market or invest in a risk-free asset. The net return on the risk-free asset is normalized to 0, while the return from participating in the bond market is endogenous and determined in equilibrium. Lenders do not observe the realization of the average productivity; instead, each lender j observes a private signal x_j about A where

$$x_j = A + v_j, \;\; v_j \sim N\left(0, \sigma_x^2\right),$$

[6] I allow for default only in period 1, because of an inherent asymmetry between the two periods in the model. Since period 2 is the last period of the model, it is hard to support repayment as an equilibrium outcome in that period — compared to period 1— because in period 2, the government faces much smaller costs of default and lacks the ability to roll over part of its debt.

[7] Following a default, creditors tend to file a substantial number of lawsuits against a defaulting government. For example, in the case of default by Argentina in 2001, there were over 140 lawsuits filed abroad, including 15 class action lawsuits, in addition to a large number of lawsuits filed in Argentine courts (Panizza et al., 2009). I interpret ξB_2 as the costs to the government associated with these legal battles. For more discussion of this assumption, see Section 3.1 below.

with v_j being *i.i.d.* across lenders and independent of A and ε_i.

Only the government and lenders have access to the bond market. I assume that the government has all the market power in the bond market, and, therefore, the government sets an interest rate r at which it is willing to borrow new funds. Taking r as given, lenders decide whether to supply their funds to the bond market, determining the total funds available in the bond market, S. The government then chooses its new borrowing, B_2, where $B_2 \in [0, S]$. After the government raises new funds, the bond market shuts down and lenders invest the funds not borrowed by the government in storage. For each unit of funds lent to the government, lender j receives a gross return of $1 + r$ in period $t = 2$ if the government repays its debt, and nothing otherwise.

The above bond market structure differs substantially from a Walrasian market typically considered in the sovereign debt literature. However, the assumption that the government has all the market power in the bond market and the resulting lack of learning from prices is not unrealistic. Most governments issue debt using sealed-bid auctions and have considerable leeway in choosing the amount of borrowing based on the bids effectively controlling the volume and, to a lesser extent, the price.[8] This auctioning mechanism also means that the price in the primary bond market cannot be used directly to infer any information.[9]

2.4 Timing

The timing of period 1 is summarized in Figure 2. At the beginning of period 1, nature draws the productivity level A, which is initially unobserved by the government, as well as by the households and the lenders. Then, based only on the information

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[8]For example, the Spanish government provides only a lower and upper bound on the amount of funds, accompanied by a note says that says: "The announced issuance target is indicative and it may be modified according to market conditions" (for more information see http://www.tesoro.es/en). What this means is that, typically, if the demand is strong and bids are high, the government will decide to issue more debt and at a lower interest rate than if the demand is weak and bids are low. Thus, effectively, the government controls both the volume of and, to some extent, the interest rate on its debt.

[9]If in the model, r were determined in a Walrasian market, then, in the absence of additional frictions or additional sources of uncertainty, the interest rate would perfectly reveal the underlying productivity. In order to prevent this, one would need to inject additional sources of uncertainty, in which case the interest rate would act as a partially endogenous public signal. This would, however, complicate the model without altering the main conclusions of the paper (see Angeletos and Werning, 2006, Hellwig et al., 2006, or Tarashev, 2007)

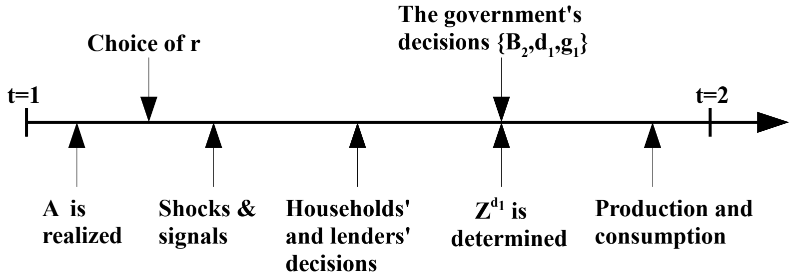

Figure 2: Timeline

contained in the prior belief, the government sets an interest rate r, at which it is willing to borrow from the lenders. Once r is announced, households receive their idiosyncratic productivity shocks and lenders observe their private noisy signals about A. Given their productivity shocks, households choose how much they want to invest, while lenders, using their private signals, decide whether to supply their funds in the market. At this point, the government learns the true A, and based on lenders' and households' decisions and the realization of A, it decides how much it will borrow today, B_2; whether or not to default, d_1; and how much of public goods to provide to households, g_1. Once the government borrows its desired amount, the bond market shuts down, and the lenders' remaining funds are invested in the risk-free asset. Finally, at the end of the period, production, actual investment, and consumption take place, and the average productivity level is revealed to all the agents.

Period 2 is much simpler. At the beginning of the period, production takes place. Then, the government collects the taxes, provides public goods, and, if it did not default earlier, repays its remaining debt. Finally, households consume their after-tax output.

3 Equilibrium Analysis

An equilibrium in the model is defined as follows:

Definition 1 *An equilibrium is a set of government policy functions* $\{r,\ d_1,\ g_1,\ g_2,\ B_2\}$*, a profile of households' consumption and investment choices* $\{c_1, c_2, k_2\}_{i\in[0,1]}$*, and a profile of lenders' supply decisions* $\{\beta\}_{j\in[0,1]}$*, such that:*

1. $\{r, d_1, g_1, g_2, B_2\}$ *solves the government's problems at* $t = 1,2$*, taking households' and lenders' decisions as given.*

2. *For every* i*,* $\{c_1^i, c_2^i, k_2^i\}$ *solves household* i*'s problems at* $t = 1,2$*, taking as given the other agents' decisions.*

3. *For every* j*,* β^j *solves lender* j*'s problem, taking as given the other agents' decisions.*

The above definition of an equilibrium is standard, and it requires that all the agents behave optimally in each subgame, taking the actions of the others as given.

The equilibrium can be computed by backward induction, starting with period 2 and then moving to period 1. The key step is to solve simultaneously for the households' investment choices, the lenders' supply decisions, and the government's default decision. In what follows, I will focus on equilibria in monotone strategies. This greatly simplifies the task of solving the model and renders the analysis more tractable.

3.1 Additional Assumptions

To simplify the analysis and ensure that the government problem is well-posed, I make the following assumptions (listed below from the least to the most restrictive).

Assumption 1 *The legacy debt is large enough, $B_1 > \overline{B}_1$ for some threshold $\overline{B}_1$.*

Assumption 1 ensures that if the government decides to repay its legacy debt, it will find it optimal to borrow a positive amount. Otherwise, lenders stop playing any role in the model.

Assumption 2 *The wealth of each lender j is bounded by $\overline{b}$ (i.e., $b < \overline{b}$).*

Assumption 2 simply implies that the total liquidity in the bond market is finite. This is a typical assumption in the models with risk-neutral traders and incomplete information (see, e.g., Albagli et al., 2015).[10]

Assumption 3 *$Z > \underline{Z}$; that is, output cost of default is not too large.*

Assumption 3 implies that the output cost of default at time t is bounded from below by $(1 - \underline{Z})\, Y_t$. This implies that the government's optimal unconstrained borrowing, the amount it would like to borrow if it repays the debt, is monotone in A.

Assumption 4 *The "litigation costs" are large (i.e., $\xi \to 1$).*

[10] For some parameters, this assumption is also needed to ensure that the difference in the value of repaying and defaulting is sufficiently monotone. See Section $A.1.3$ of the Appendix.

Assumption 4 implies that the main benefit to the government from defaulting comes from repudiation of the legacy debt, B_1, rather than from defaulting on the new debt, B_2, which seem to be the relevant case empirically. This assumption also ensures that the government's incentive to default decreases as the supply of funds in the market increases, and is essential for establishing the existence of equilibrium.[11]

Given the above assumptions, I now analyze the equilibrium of the model. I compute the equilibrium using backward induction. Note that once the government makes its choices of B_2, d_1, g_1, no agent makes any decision and the equilibrium outcomes are determined. Therefore, I begin the analysis by describing the government's new borrowing, default, and spending decisions in period 1.

3.2 Period $t = 1$: The Government's Decisions

The government decides how much to borrow, whether or not to default, and how much to spend to maximize the households' utility, internalizing how each of these decisions affects consumption, aggregate productivity, and future tax revenues. The government makes these decisions after observing households' investment decisions, the supply of funds in the market, and the average level of productivity in the economy.

Let $\mathbf{k}_2 = \{k_2^i\}_{i\in[0,1]}$, and let $V_1^R(A, \mathbf{k}_2, S)$ be the value to the government of repaying its debt when the average productivity is equal to A, the households' investment profile is $\mathbf{k}_2$, and the supply of funds in the bond market is S. Then, $V_1^R(A, \mathbf{k}_2, S)$ is given by

$$V_1^R(A, \mathbf{k}_2, S) = \max_{B_2\in[0,S]} \sum_{t=1,2} \left\{ \int_0^1 \left[\log\left(c_t^{i,R}\right) + \log\left(g_t^R\right)\right] di \right\}$$
$$s.t.\ g_1^R = \tau Y_1^R - B_1 + B_2$$
$$g_2^R = \tau Y_2^R - (1+r) B_2,$$

where g_t^R is government spending in period t and Y_t^R is the aggregate output at time t if the government repays the debt. When the government decides to repay its debt, it chooses its new borrowing, B_2, to maximize households' utility, subject to the available funds in the market, S, and its budget constraints.

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[11] Note that a high ξ is needed to ensure that there is a region where the government is exposed to self-fulfilling beliefs. For example, in Cole and Kehoe (2000) $\xi = 0$, and, as a consequence, they can ensure the existence of such a region at extreme parameter values only. A separate issue arises from the fact that, in my model, lenders and households have incomplete information. As Kletzer (1984) notes, in debt crises models with asymmetric information, an equilibrium may not exist. Assumption 4 ensures that this is not an issue.

Let $V_1^D(A, \mathbf{k}_2, S)$ be the value associated with defaulting; that is,

$$V_1^D(A, \mathbf{k}_2, S) = \max_{B_2 \in [0,S]} \sum_{t=1,2} \left\{ \int_0^1 \left[\log\left(c_t^{i,D}\right) + \log\left(g_t^D\right) \right] di \right\}$$
$$s.t.\ g_1^D = \tau\left(ZY_1^R\right) + (1-\xi) B_2$$
$$g_2^D = \tau\left(ZY_2^R\right)$$

If the governments defaults, it borrows the maximum possible amount in the market (i.e., $B_2 = S$) and then repudiates all of its debt, and both of these actions tend to increase government spending in period 1. When $\xi \to 1$, this effect of borrowing as much as possible vanishes, and the main benefit of default is an increase in the g_1 due to repudiation of the "legacy debt" B_1. The negative effect of defaulting is a drop in aggregate productivity by factor Z.

When deciding whether or not to default, the government compares $V_1^R(A, \mathbf{k}_2, S)$ with $V_1^D(A, \mathbf{k}_2, S)$ and chooses to repay its debt if and only if the value associated with repaying is larger than the value associated with defaulting; that is, if and only if

$$\Delta V(A, \mathbf{k}_2, S) \equiv V_1^R(A, \mathbf{k}_2, S) - V_1^D(A, \mathbf{k}_2, S) \geq 0 \tag{1}$$

3.3 Default Decisions and the Fragility Region

For sufficiently low productivity levels, the government finds it optimal to default, regardless of the households' and lenders' actions — when A is low, defaulting leads to an increase in government spending. On the other hand, when the average level of productivity is high, the government always finds it optimal to repay the debt. Intuitively, for high A, defaulting not only leads to a drop in private consumption, but also results in less government spending. Accordingly, for each interest rate r, there exist two thresholds, $\underline{A}(r)$ and $\overline{A}(r)$, such that the government always defaults if $A < \underline{A}(r)$ and never defaults if $A > \overline{A}(r)$.[12]

For all $A \in \left[\underline{A}(r), \overline{A}(r)\right)$, the government's default decision depends on the households' and lenders' choices. If the lenders expect default, they invest all of their funds in the risk-free asset. In this case, the government cannot roll over its debt, and, hence, repaying B_1 becomes very costly in terms of the forgone utility from government spending. If, on the other hand, the households expect default, they decrease their investment, leading to a drop in the government's revenues (taxes) in the future. This translates into a drop in government expenditures in both periods (since the government smooths

[12] While thresholds $\underline{A}(r)$ and $\overline{A}(r)$ also depend on all parameters of the model ,for notational convenience, I suppress this dependence.

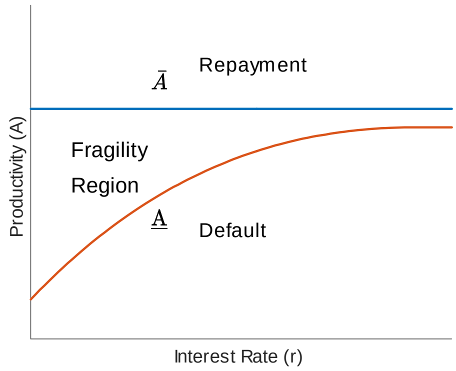


Figure 3: Fragility Region

out the drop in its revenue across time) and leads to a higher cost of repaying the legacy debt. If $A \in \left[\underline{A}(r), \overline{A}(r)\right)$, these costs are large enough that, in response to a shift in households' or lenders' expectations, the government finds it optimal to default. Figure 3 depicts the fragility region $\left[\underline{A}(r), \overline{A}(r)\right)$.

3.4 Households' Problem

Consider household i with an idiosyncratic productivity shock A_i that must choose how much to invest. This household's problem can be written as

$$\max_{k_2} \; E\left[\sum_{t=1,2}\left[\log\left(c_t\right)+\log\left(g_t\right)\right]\middle| A_i, \boldsymbol{\sigma}\right]$$
$$s.t. \; c_1 = (1-\tau)\, Z^{d_1(\boldsymbol{\sigma})} e^{A_i} f\left(k_1\right) - k_2$$
$$c_2 = (1-\tau)\, Z^{d_1(\boldsymbol{\sigma})} e^{A_i} f\left(k_2\right),$$

where $\boldsymbol{\sigma} = \{\mathbf{k}_2, \boldsymbol{\beta}, r, d_1, g_1, g_2, B_2\}$ is the strategy profile of all players, and the expectations are taken over the government default decisions, $d_1(\boldsymbol{\sigma})$, as well as over the average level of productivity, A. Household i chooses k_2 to maximize its utility, subject to the budget constraint, taking $\boldsymbol{\sigma}$ as given. Lemma 1 characterizes households' optimal investment when they believe that the government will always default if the average productivity is less than A^*.

Lemma 1 *Suppose that the government defaults if and only if $A < A^*$. Then, household i's optimal investment is given by*

$$k_2 = (1-\tau)\, e^{A_i} f\left(k_1\right) \Lambda\left(A_i; \varepsilon, A^*\right),$$

where $\Lambda\left(A_i; \varepsilon, A^*\right)$ is increasing in the idiosyncratic productivity, A_i, and decreasing in the default threshold, A^*.[13]

[13] See Section A of the Appendix for the exact definition of $\Lambda\left(A_i; \varepsilon, A^*\right)$.

3.5 Lenders' Problem

Simultaneously with the households' investment choices, the lenders must decide whether to supply their funds to the bond market or to invest their funds in storage. Lenders base their decisions on the prior belief about A and their private signals, x_j. Let $\mathcal{R}(\boldsymbol{\sigma})$ be the government repayment set for a fixed strategy profile $\boldsymbol{\sigma}$. Then the expected payoff to lender j from supplying the funds to the bond market is given by

$$\int\limits_{A\in\mathcal{R}(\boldsymbol{\sigma})} \left(1 + r \min\left\{1, \frac{B_2^{R,u}(A;\boldsymbol{\sigma})}{S(A;\boldsymbol{\beta})}\right\}\right) f(A|x_j)\, dA,$$

where $f(A|x_j)$ is lender j's posterior belief about A, $B_2^{R,u}(A;\boldsymbol{\sigma})$ is the unconstrained desired borrowing by the government in repayment, and $S(A;\boldsymbol{\beta})$ is the supply function implied by the lenders' supply strategy profile $\boldsymbol{\beta}$. Finally, $\min\left\{1, B_2^{R,u}(A;\boldsymbol{\sigma})/S(A;\boldsymbol{\beta})\right\}$ is the amount that lender j expects to lend to the government, given that the average productivity level is A.[14] Lender j supplies his funds to the bond market if and only if the expected return from supplying the funds is higher than 1, the return from investing in storage. The next lemma characterizes lenders' behavior.

Lemma 2 *Suppose that the government defaults if and only if $A < A^*$. Then, an optimal strategy for each lender j is to supply the funds to the bond market if and only if he receives a signal $x_j \geq x^*$. Moreover, x^* is the unique solution to the equation*

$$\int\limits_{A^*}^{\infty} \left(1 + r \min\left\{1, \frac{B_2^{R,u}(A;\boldsymbol{\sigma})}{S(A;\mathbf{x}^*)}\right\}\right) f(A|x^*)\, dA = 1,$$

where $S(A;\mathbf{x}^*)$ is the supply function when all lenders follow this strategy.

3.6 Equilibrium Default Threshold

Above, I characterized the optimal behavior of each type of agent. This, in turn, allows me to prove the following proposition, which states that for any interest rate r there exists a unique equilibrium in monotone strategies.

Proposition 1 *There exist $\overline{\varepsilon} > 0$ and $\overline{\sigma}_x > 0$ such that for any interest rate r, any $\varepsilon \in (0, \overline{\varepsilon}]$, and any $\sigma_x \in (0, \overline{\sigma}_x]$, the model has a unique equilibrium in monotone strategies where the following hold:*

[14] For all $A \notin \mathcal{R}(\boldsymbol{\sigma})$, the government borrows all available funds in the market and then defaults, implying that, in this case, lender j earns nothing. If $A \in \mathcal{R}(\boldsymbol{\sigma})$, the government would like to borrow $B_2^{R,u}$.

1. *The government defaults if and only if* $A < A^*(r)$.[15]

2. *Each lender provides the funds if and only if* $x_j \geq x^*(r)$.

3. *Households' investment rules,* k_2*, are increasing in* A_i.

The proof of Proposition 1 builds on the insights and results of Athey (1996) and Morris and Shin (2003). The above result is non-trivial for several reasons. First difficulty comes results the fact that in the model, the global game is played by three different types of agents, each with its own preferences and choice sets. Second, the lenders' payoff function satisfies only a weak single-crossing condition, rather than global strategic complementarities, as in typical global games.[16] Finally, the regime-change condition (i.e., the condition that determines whether default will occur) arises endogenously from the government's optimal behavior — unlike in the typical global games literature, where it is exogenously imposed.

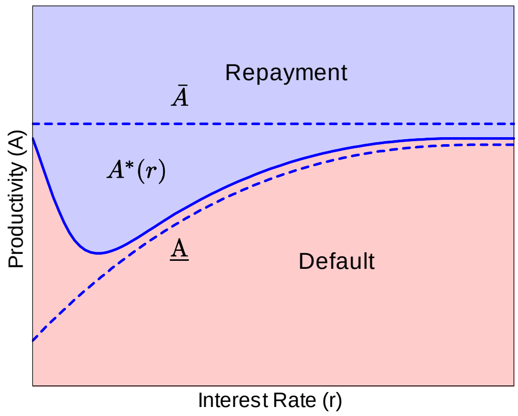


Figure 4: Default Threshold

Figure 4 depicts the equilibrium default threshold A^* as a function of the interest rate r. We see that $A^*(r)$ is a non-monotone function of r. To understand this, note that when the interest rate is low, few lenders supply their funds to the bond market. As a result, the government finds it optimal to default for most productivity values in

[15] The default threshold $A^*(r)$ depends also on all the parameters of the model such as the tax rate τ, the capital stock k_1, the legacy debt B_1, etc. For notational convenience, I suppress this dependence whenever this does not lead to a confusion.

[16] Applying global games results in a complex environment in which payoff functions satisfy only the weak single-crossing condition, rather than global strategic complementarities, is not without cost. In particular, I need to restrict my attention to monotone strategies. Morris and Shin (2003) discuss why, in general, the single-crossing condition is not enough to prove uniqueness without such a restriction.

the "fragility region." As r increases, the supply of funds increases since higher r compensates lenders for exposing themselves to default risk. At the same time, households' investment rules shift upwards since they anticipate that the government will choose to repay the debt for a larger set of productivity levels. This decreases the government's incentives to default and leads to a lower $A^*(r)$. A higher interest rate, however, increases the costs of rolling over the debt, discouraging the government from smoothing debt repayment over time. This tends to decrease the value of repaying debt to the government. For sufficiently high r, this negative effect dominates, implying that $A^*(r)$ becomes an increasing function of r.

It is important to stress that, while the default threshold is unique, the outcome of the model in the fragility region is driven fully by households' and lenders' expectations. For all productivity levels in the fragility region, both repayment and default could be supported as equilibrium outcomes if we had the freedom to choose the lenders' and households' expectations. However, the households' and lenders' expectations are not free objects. An incomplete-information structure transforms beliefs into equilibrium objects and requires them to be sequentially rational and consistent with agents' strategy profiles. This imposes requirements on the beliefs that are not present in the complete-information game.

3.7 Optimal Choice of r

It remains to characterize the government's optimal choice of interest rate, r. The government chooses the interest rate based on the current and past fundamentals of the economy, $\{B_1, k_1, A_{-1}\}$. The government also knows its future policy functions $\{d_1, g_1, g_2, B_2\}$ and realizes that it can affect consumption, investment, and the supply of funds through its choice of interest rate. To choose the optimal interest rate, the government solves the following problem:

$$W(A_{-1}, B_1, \mathbf{k}_1; \boldsymbol{\sigma}) = \max_r E\left[\sum_{t=1,2} \int_{i=0}^{1} \left[\log\left(c_t^i\right) + \log\left(g_t\right)\right] di \,\middle|\, A_{-1}\right]$$

$$\begin{aligned} s.t.\ & \text{optimal policy functions } \{c_1, c_2, d_1, B_2, g_1, g_2\} \\ & \text{optimal lenders' and households' strategies } \{\boldsymbol{\beta}, \mathbf{k}_2\}. \end{aligned}$$

When choosing the interest rate, the government faces the following trade-off: On the one hand, at least initially, a higher r tends to decrease the default threshold. On the other hand, a higher r increases the cost of borrowing at $t = 1$, making it more costly to roll over the maturing debt. Thus, the government weighs the positive effect of a lower default threshold against the increase in the borrowing costs. The above trade-off implies that the government will always set an interest rate on the decreasing portion of the $A^* (r)$-curve.

4 Preventing Self-fulfilling Debt Crises

Having characterized the equilibrium of the model, I now focus on the main questions that motivated this paper: (1) how the government can decrease the ex-ante probability of default (i.e., prevent a debt crisis), and (2) what role endogenous expectations play in determining the effect of government policies on the probability of crises.

I start by considering a case in which each policy change is announced in period 1, before the households and lenders make their decisions but after r is set, and in which the government is committed to implementing the announced policies. The policy itself is, however, is not implemented until the end of that period. These assumptions are made for simplicity and allow me to focus on the fundamental forces at play in the model, while abstracting from the effects of other factors. I relax these assumptions in the following sections. In Section 5, I analyze what happens if either the policy adjustment is unexpected or if there is uncertainty as to whether the government will implement the announced policy. In Section E of the Appendix, I analyze the case in which the policy announcement is made before the interest rate is set. Figure 5 depicts the timing for the policy adjustment considered in this section.

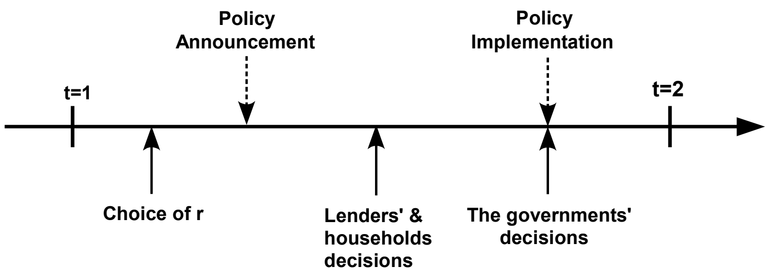


Figure 5: Timing of Policy Adjustments

In order to simplify the analysis and make the problem more tractable, I make the following assumption:

Assumption 5 *B_1 is large enough so that for all $A > \underline{A}(0)$, the government's desired borrowing in repayment exceeds the supply of funds in the market.*[17]

[17]Recall from Section 3.3 that $\underline{A}(0)$ is the lower bound for the fragility region when $r = 0$. Thus, it is the productivity level below which the government will always default, regardless of the interest rate and regardless of the households' and lenders' decisions.

Assumption 5 simplifies the problem by eliminating the issue of competition between lenders in the bond market, in which case the lender's problem can be solved in closed form.[18]

4.1 Equilibrium Effects of Policy Adjustments

Before analyzing specific policies, it is useful to understand the equilibrium forces that are at play when the government adjusts its policy. For this purpose, consider an abstract policy adjustment, captured by a change in a parameter ψ.[19] We would like to understand how a change in ψ affects the ex-ante probability of default, which, for a given interest rate r, is given by $\Pr(A < A^*)$. This preliminary abstract analysis has additional advantages: (1) It highlights how dispersed beliefs and endogenous expectations affect the impact of government policies, and (2) it helps us understand how and when predictions of the model with dispersed beliefs will differ from the predictions of the models in which defaults are driven only by fundamentals.

Let A^{**} denote households' and lenders' belief regarding the default threshold (where, in equilibrium, we have $A^* = A^{**}$, as agents' beliefs have to be correct). We have the following Proposition.

Proposition 2 *The change in default threshold implied by the adjustment in a policy parameter ψ is given by*

$$(2) \qquad \frac{dA^*}{d\psi} = \underbrace{\frac{1}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}} di}}_{\textit{beliefs effect } (\boldsymbol{B})} \times \underbrace{\left(\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi} di\right)}_{\textit{Direct effect } (\boldsymbol{D})}$$

The beliefs effect is always strictly greater than 1 *so that* $sgn(dA^*/d\psi) = sgn(D)$.

The above Proposition establishes that the effect of an adjustment in any parameter ψ on A^* can be decomposed into the direct effect and the beliefs effect. To understand the intuition behind Equation (2), consider a change in ψ, but first keep households' and lenders' beliefs about A^* constant. Then, a change in ψ affects the government's incentive to default by changing the difference between the values of repaying and defaulting on the debt. This effect works through the government's indifference condition, which I denote by $\partial A^*/\partial \psi$, since it corresponds to the partial effect of a change in

[18] While Assumption 5 simplifies the comparative statics analysis, it does not affect its underlying logic. In particular, Proposition 2 holds in the same form, regardless of whether we impose Assumption 5. For a more detailed discussion of the consequences of this assumption, see Section E of the Appendix.

[19] For concreteness, one can think of this policy as an increase in taxes, in which case $\psi = \tau$.

policy, keeping the strategies of households and lenders fixed. Moreover, the policy change potentially affects households' and lenders' decision problems, thereby leading households and lenders to adjust their strategies and, in turn, bringing about a further change in the government's incentive to default (these effects are captured by terms $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}$ and $\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}$, respectively). Thus, the "direct effect" is equal to the change in the default threshold, keeping households' and lenders' expectations fixed.

The households' and lenders' expectations, however, are not fixed. In response to this initial change in the default threshold, the households and lenders adjust their expectations and, thus, their strategies, which leads to a further change in A^*, inducing another round of adjustment in the households' and lenders' expectations and so on. Thus, "beliefs effect" captures the change in the default threshold driven by the adjustment in households' and lenders' expectations.

Proposition 2 leads to three important implications. First, whether a change in a government policy increases or decreases the probability of default is determined by the "direct effect." Thus, to establish whether a given policy decreases or increases the likelihood of a debt crisis, one can focus on understanding how the policy affects the government's incentive to default, holding agents' beliefs. Second, adjustments in endogenous expectations always amplify the initial impact of any policy adjustments and, thus, are key to quantifying the impact that any policy has on the probability of default (see Section 6 for the analysis when this effect is particularly strong). Third, the presence of dispersed beliefs affects the qualitative predictions of the model: Even though the "direct effect" captures intuitive forces that are present in standard models, these forces are distorted by the presence of dispersed information. Intuitively, the direct effect of a given policy depends on the agents' behavior without the policy change, as well as on their response to a change in a policy, both of which are distorted by the presence of dispersed information (see Section 7 for a more detailed analysis).

4.2 Overview of Policies

Using the above insights, I now analyze two policy measures that received a lot of attention in policy debates during the recent sovereign debt crisis in Europe: (1) austerity (increase in taxes) and (2) a fiscal stimulus (financed with debt). The European debt crisis generated a lively debate about the ability of the above policies to prevent debt crises (see Brunnermeier et al., 2016). Below, I describe how each of these policies is introduced into the model.

Increase in Taxes In the model, a rise in the tax rate is captured by an increase in τ, the fraction of output that the government takes away from households. Below, I consider the case where once adjusted, τ is kept constant across periods and is the same regardless of whether the government defaults. This fits a scenario in which the government finds it difficult to change tax laws once they have been enacted (for example because of the lengthy political process it involves). In Section B of the Appendix, I consider the situation in which higher τ is implemented only if the government repays the debt, a case that is relevant when policymakers are willing to increase taxes only to avoid default, and once default occurs, they are likely to abandon this idea. The results are similar for both cases.

Fiscal Stimulus I model fiscal stimulus as an increase in the initial capital stock of each household from k_1 to $(1+s)\,k_1$ financed by the government, where s measures the size of the stimulus as a percentage of the initial capital stock. Thus, if the government decides to engage in a stimulus, the total output of the economy will increase.[20] I do not explicitly model the government's financing decision. Instead, I assume that to finance a stimulus, the government issues additional debt at the end of the period preceding period 1. I consider separately the case where this additional debt matures at the end of period 1 together with B_1 (short-term debt financing with interest rate $r^{ST} \geq 0$) or in period 2 (long-term debt financing with interest rate $r^{LT} \geq 0$).

4.3 Increase in Taxes

As explained above, to understand the effect of an increase in the tax rate τ on the default threshold, it is enough to focus on its direct effects. A higher tax rate leads to a change in the government's incentives to repay debt equal to

$$\begin{aligned}(3)\qquad &\underbrace{Y_1^R\left(u_{g_1}^R - u_{g_1}^D\right) + Y_2^R\left(u_{g_2}^R - u_{g_2}^D\right)}_{\text{Concavity effect}} + \underbrace{Y_1^R\,(1-Z)\,u_{g_1}^D + Y_2^R\,(1-Z)\,u_{g_2}^D}_{\text{Differential increase in tax revenues}} \\ &\underbrace{-\frac{\alpha}{1-\tau}\tau Y_2^R\left(u_{g_2}^R - Z u_{g_2}^D\right)}_{\text{Investment distortion}},\end{aligned}$$

[20] This is a simple way to model a fiscal stimulus in the current framework. One should interpret the increase in k_1 not as an increase in physical capital owned by households but, rather, as an increase in government spending on public goods and services that enhance production (e.g., an increase in expenditure on infrastructure or on the maintenance of the rule of law). An alternative way to model stimulus would be to explicitly allow government spending to enter the production function; that is, to write the household production function as $y_t^i = e^{A_i} f\left(k_t^i, h_t\right)$, where h_t explicitly captures the government expenditure that is important for production. However, the qualitative conclusions would remain unchanged.

where $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utilities from government spending in period t in repayment and default, respectively, and Y_t^R is the total output of the economy in period t in repayment, all evaluated at the threshold productivity level A^*. If the expression in (3) is positive, then the government's incentive to repay its debt increases following an increase in τ.[21]

The expression in (3) tells us that an increase in the tax rate affects the government's default incentives through three channels. First, a higher τ implies higher tax revenues. Since at A^*, the government's spending is lower in repayment than in default, the concavity of the utility function implies that a given increase in government spending leads to a greater increase in the value of repaying than in the value of defaulting, thus decreasing the government's default incentive (the "concavity effect"). Second, since the total output is higher in repayment, a given increase in the tax rate translates into a greater increase in tax revenues in repayment than in default, further decreasing the government's default incentives (the "differential increase in tax revenues"). The last term captures the negative effect of higher taxes on households' investment decisions, where $\alpha/(1-\tau)$ is the rate at which output decreases with higher taxes, and $u_{g_2}^R - Zu_{g_2}^D$ measures how "painful" this decrease in spending is to households in repayment compared to default (the "investment distortion").

Proposition 3 *There exists $\underline{\tau} > 0$ such that for all $\tau \leq \underline{\tau}$, an increase in taxes decreases the probability of default. Moreover, if $\sigma_x \to 0$ and $r \times b < B_1$, then $\underline{\tau} > 1/(1+\alpha)$.*

The above proposition states that if the initial tax rate is not "too high" (i.e., $\tau \leq \underline{\tau}$), then an increase in the tax rate will decrease the probability of default. This result follows from the observation that the "investment distortion" $\alpha/(1-\tau)$ is a convex function of τ, and for high values of τ, it dominates the positive effect of higher tax revenues. The second part of Proposition 3 states that if the supply of funds in the bond market (which, when lenders have precise information, is bounded from above by rb) is lower than B_1, then an increase in τ decreases the default threshold for all $\tau \leq 1/(1+\alpha)$. In other words, if the government is unable to roll over all of its debt, then an increase in taxes necessarily decreases the probability of default for all $\tau \leq 1/(1+\alpha)$.

[21] The expression in (3) corresponds to $\frac{\partial}{\partial\tau}\Delta V(A^*, k_2, x^*; \psi)$. The direct effect is equal to $\frac{\partial}{\partial\tau}\Delta V(A^*, k_2, x^*; \psi)$ divided by $-\frac{\partial}{\partial A^*}\Delta V(A^*, k_2, x^*; \psi) < 0$. In particular, the sum of the concavity effect and the differential increase in tax revenues divided by $-\frac{\partial}{\partial A^*}\Delta V(A^*, k_2, x^*; \psi)$ is equal to $\frac{\partial A^*}{\partial\psi}$, while the expression for investment distortion divided by $-\frac{\partial}{\partial A^*}\Delta V(A^*, k_2, x^*; \psi)$ corresponds to $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial\psi}$ in Equation (2).

How likely is this last condition to be satisfied in reality? Note that, in the model, α can be interpreted as the capital share of output, and, thus, $\alpha \approx 0.33$. The average ratio of government tax revenues to GDP in the Eurozone in 2011, was according to Eurostat, about 0.4 (translating into $\tau \approx 0.4$ in the model). This implies that the sufficient conditions for austerity to decrease the probability of default during the recent European debt crisis were likely satisfied.

The next result further strengthens the case for austerity. It shows that when the initial expectations about the current economic situations (as captured by A_{-1}) are low, then an increase in the tax rate will decrease the probability of default even if τ is already very high.

Corollary 1 *For any $\tau \in (0,1)$, there exists $\underline{A}_{-1}(\tau)$ such that if $A_{-1} < \underline{A}_{-1}(\tau)$, then $dA^*/d\tau < 0$.*

While this result might seem surprising at first, it is intuitive: When A_{-1} is low, lenders are unwilling the supply the funds to the bond market unless they receive very high signals, which implies that the total amount of funds available in the bond market is low. As a consequence, for low enough A_{-1}, the government is able to borrow very little, and the only way it can repay the debt and avoid default is by increasing its revenues. An increase in τ is one way to achieve this.

4.4 Fiscal Stimulus

Now consider the effect of a fiscal stimulus on the probability of default. A fiscal stimulus leads to a change in the government's incentives to repay debt equal to

$$\begin{aligned}
&\underbrace{\tau\frac{\partial Y_1^R}{\partial s}\left(u_{g_1}^R - u_{g_1}^D\right) + \tau\frac{\partial Y_2^R}{\partial s}\left(u_{g_2}^R - u_{g_2}^D\right)}_{\text{Concavity effect}} + \underbrace{\left[\frac{\partial Y_1^R}{\partial s}u_{g_1}^D + \frac{\partial Y_2^R}{\partial s}u_{g_2}^D\right]\tau\left(1-Z\right)}_{\text{Differential increase in tax revenues}} \\
&\underbrace{-u_{g_t}^R\left(1+r^{stim}\right)k_1}_{\text{Increase in debt}}, \qquad (4)
\end{aligned}$$

where $r^{stim} \in \left\{r^{ST}, r^{LT}\right\}$ is the interest rate on the debt issued to finance the stimulus, $\partial Y_t^R/\partial s$ is the increase in output in period t resulting from the stimulus, and where $u_{g_t}^R$, $u_{g_t}^D$ and Y_t^R are defined as in Section 4.3.

The expression in (4) tells us that a fiscal stimulus affects the government's default incentive through three channels: (1) the "concavity effect"; (2) a differential increase in government tax revenues in repayment and default (both of which were also present in the case of a tax increase); and (3) a negative effect due to an increase in the government's debt burden (equal to $u_{g_1}^R\left(1+r^{ST}\right)k_1$ if the stimulus is financed with short-term debt, or to $u_{g_2}^R\left(1+r^{LT}\right)k_1$ if financed with long-term debt).

Proposition 4 *Consider a stimulus financed with short-term debt. There exists* $\overline{B}_1$ *such that the stimulus decreases the probability of debt crisis if and only if* $B_1 > \overline{B}_1$*. Moreover,* $\overline{B}_1/k_1 > (1+r^{ST})\frac{1}{\alpha}$*.*

Proposition 4 establishes that a stimulus decreases the probability of default if and only if the debt to capital stock ratio is high. The intuition behind this observation is simple: A higher B_1 implies a higher marginal benefit from an increase in output in repayment, while a higher k_1 implies a higher cost of increasing capital stock by a given percentage. Proposition 4 provides also a necessary condition for the stimulus to work: The ratio of debt to capital has to be larger than $\frac{1}{\alpha}$.

It is important to stress that, even though the above proposition identifies conditions under which fiscal stimulus financed with short-term debt can work, these conditions are unlikely to hold in practice. Since α can be interpreted as the capital share of output so that $\alpha \approx 0.33$, the above proposition suggests that in order for a fiscal stimulus financed with short-term debt to work, one needs a capital to debt ratio in excess of 3. This is unlikely to be the case for most countries. For example, this ratio is less than 1 for Eurozone countries, suggesting that stimulus was not a valid option for the governments during the recent European debt crisis.[22]

When a stimulus is financed with long-term debt the necessary condition for the stimulus to work becomes $\overline{B}_1/k_1 > (1+r^{LT})\frac{1}{\alpha}(u^R_{g1}/u^R_{g2})$. Since $u^R_{g1}/u^R_{g2} < 1$,[23] as long as r^{LT} is not significantly higher than r^{ST}, the condition under which a fiscal stimulus financed with long-term debt decreases the probability of default is less stringent compared to that with short-term debt financing. However, even this condition is unlikely to hold since it would require an implausibly large drop in government spending in period 1 compared to period 2.[24]

5 Economic Policy Uncertainty and Its Consequences

Above, I considered a situation in which a policy change was expected by both households and lenders. In this section, I investigate how the above results change if the households and lenders are uncertain as to whether the government will adjust its

[22] The capital-output ratio for most Eurozone countries is above 3 (see Penn World Tables, Feenstra et al., 2015) while the debt-to-GDP ratio is smaller than 2.

[23] In equilibrium, the government expenditure in period 1 is always lower than in period 2 in repayment, as the government is unable to smooth debt repayment over time.

[24] Given that, for most countries, $\frac{1}{\alpha} \approx 3$ and $B_1/k_1 \leq 1$, we would need the government spending in period 2 to be three times higher than in period 1 in order for this condition to be satisfied.

policies. The analysis is motivated by the observation that, often, there is a strong disagreement among policymakers regarding the political and economic desirability of given economic policies, thereby giving rise to a substantial policy uncertainty. Indeed, as discussed in the introduction (Figure 1), there was a large spike in such an uncertainty during the European debt crisis.[25] Thus, it is important to understand if and how such uncertainty distorts the effectiveness of austerity and stimulus.

I consider two cases. First, I investigate the model's predictions when a policy change is unexpected by lenders and households. This case describes a situation in which either government announcements have no credibility (so that agents do not believe there will be any policy change), or the government decides to do an unexpected U-turn on its economic policy. Second, I analyze a situation in which households and lenders expect that the government to adjust its policy with probability $p \in (0,1)$. Otherwise, there are no changes compared to Section 4.

5.1 Unexpected Policy Adjustment

Proposition 5 *Suppose that a policy change is unexpected. Then,*

$$\frac{dA^*}{d\psi} = \frac{\partial A^*}{\partial \psi}.$$

Moreover, $dA^*/d\psi \to 0$ *as* $\varepsilon, \sigma_x \to 0$.

Proposition 5 tells us that when a policy change is unexpected, the change in the default threshold is equal to the direct effect that the policy has on the government's incentives to default. Since agents expect no policy adjustment, their strategies are unchanged, implying that the beliefs effect and the part of the direct effect that operates through households' and lenders' choices are absent. Moreover, in the limit, an

[25] Policy uncertainty played an important role in Greece, where after winning the unexpected early elections in January 2015, the Syriza-led coalition stopped implementation of reforms, only to suddenly change its mind six months later, but not until after pushing Greece to the verge of default. This issue also played an important role in Italy. In response to the crisis, the Italian parliament formed a technocratic government, with Mario Monti as prime minister, to implement a package of structural reforms. Lacking political support, the government was less successful than expected in passing the reforms.

unexpected policy change becomes completely ineffective as the direct effect converges to 0.[26]

This last result provides a strong warning against unexpected policy U-turns so that agents are not surprised by the government actions. It also worth emphasizing that the same logic applies to policy announcements that agents view as not credible, and, hence, governments should strive to communicate their policy plans not only in advance, but also in a credible manner.

Corollary 2 *Suppose that a policy change is unexpected and that $\varepsilon, \sigma_x > 0$.*

1. *An increase in the tax rate τ always decreases the government's incentives to default.*

2. *A fiscal stimulus financed with short-term debt decreases the government's incentives to default if and only if*

$$\varsigma_{ST}^{unexp} = \frac{\alpha\left(B_1 - B_2\right)}{\tau Y_1 - B_1 + B_2} - \frac{\left(1 + r^{ST}\right) k_1}{\tau Y_1 - B_1 + B_2} > 0,$$

while in case of long-term debt financing the relevant condition is

$$\varsigma_{LT}^{unexp} = \frac{\alpha\left(B_1 - B_2\right)}{\tau Y_1 - B_1 + B_2} - \frac{\left(1 + r^{LT}\right) k_1}{\tau Y_2 - \left(1 + r\right) B_2} > 0.$$

The above corollary implies that, as long as $\varepsilon, \sigma_x > 0$, an unexpected increase in the tax rate always leads to a decrease in the probability of default. This is because the negative effect of higher taxes on households' investment choices is now absent (no investment distortion). On the other hand, a fiscal stimulus, if unexpected, leads only to an expansion of output in period 1; households keep their investment strategies constant, as they do not expect any change in the economy. As a consequence, a fiscal stimulus is now more likely than before to increase the probability of default. It follows that if

[26] To understand this, consider lender j who can observe the actual A. Lender j would lend to the government if and only if $A > A^{**}$, where A^{**} corresponds to households' and lenders' beliefs about the default threshold. Thus, lender j will not respond to any policy change unless it also leads to a change in A^{**}; that is, it leads to a change in other agents' beliefs. But, since a policy change is unexpected, agents' beliefs are fixed and A^{**} is unchanged. This implies that lender j does not adjust his behavior following the policy change. While in the model, lenders cannot observe true A, as $\sigma_x \to 0$, the uncertainty about A disappears and we converge to the case described above. Similar logic applies to the behavior of households.

the government lacks credibility or if it suddenly decides to act, austerity is a better option than stimulus. However, it should be kept in mind that, in light of Proposition 5, the overall effect of these policies on the probability of default will be rather small, especially when households' and lenders' private information is precise.

5.2 Uncertainty about Reforms

Next, I consider a case in which agents expect the government to implement a given reform with probability $p \in (0,1)$. Let $dA^*/d\psi\,(p)$ denote the total change in the default threshold when the agents expect the policy to be implemented with probability p and the government does implement the announced policy. It can be shown that, in this case, we have:[27]

$$\frac{dA^*}{d\psi}(p) = p\frac{dA^*}{d\psi}(1) + (1-p)\frac{\partial A^*}{\partial \psi}, \tag{5}$$

Thus, a change in the default threshold is a weighted average of the change in the default threshold when there is no uncertainty ($dA^*/d\psi\,(1)$) and when the policy change is unexpected ($\partial A^*/\partial\psi$). Intuitively, when agents expect that the policy will be implemented with probability p, their response to the prospect of the policy adjustment is proportionately less than in the case of no economic policy uncertainty. This results in an adjustment of the default threshold equal to $p\frac{dA^*}{d\psi}(1)$. On the other hand, with probability $1-p$, households and lenders do not expect the adjustment, in which case, if the policy adjustment happens, it is driven by the direct change in the government's default incentive (and, hence, the adjustment in A^* is equal to the change in the default threshold when the policy adjustment is unexpected).

Proposition 6 *Suppose that agents attach probability $p \in (0,1)$ to the announced policy being implemented.*

1. *Then, an increase in τ decreases the probability of default for a wider range of initial conditions than in the case of no uncertainty ($p = 1$); that is,*

$$\frac{dA^*}{d\tau}(1) < 0 \implies \frac{dA^*}{d\tau}(p) < 0 \text{ but not vice versa.}$$

2. *Then, a fiscal stimulus decreases the probability of default for a more limited range of initial conditions than in the case of no uncertainty ($p = 1$); that is,*

$$\frac{dA^*}{ds}(p) < 0 \implies \frac{dA^*}{ds}(p) < 0 \text{ but not vice versa.}$$

[27] For more details behind the derivations of Equation (5), see Section C of the Appendix.

3. If ε and σ_x are small then $\left|\frac{dA^*}{d\psi}(p)\right| < \left|\frac{dA^*}{d\psi}(1)\right|$.

Proposition 6 shows that the conclusions obtained in the case of unexpected policy changes extend to the case when policies are implemented with positive probability. In particular, Part 1 establishes that, in the presence of uncertainty as to whether the government will implement announced policies, an increase in taxes is an effective way to decrease the likelihood of a crisis for a wider range of initial conditions. The intuition behind this result is the same as before: Uncertain as to whether higher taxes will be implemented, households do not decrease their investment as much as they would otherwise. Similarly, Part 2 establishes that, in the presence of such uncertainty, the range of conditions under which fiscal stimulus decreases the likelihood of a crisis shrinks. Thus, the presence of policy uncertainty strengthens the appeal of austerity compared to stimulus. However, as shown in Part 3, in both cases, economic policy uncertainty decreases the overall effect that both policies have on the default threshold.

Proposition 6 leads to two conclusions. First, economic policy uncertainty is undesirable, as it decreases the overall effectiveness of government policies. Second, in the presence of economic policy uncertainty austerity is relatively more preferred option compared to stimulus.

6 Numerical Analysis and the Role of the Beliefs Effect

Above, I analyzed analytically how fiscal stimulus and an increase in taxes affect the government incentives to default and how these effects depends on the degree of economic policy uncertainty. In this section, I complement the above analytical results with a numerical investigation. In particular, I investigate numerically: (1) whether for reasonable parameter values, the government policies considered above tend to decrease or increase the probability of default; and (2) when the effect of expectations is particularly important (i.e., when is the beliefs effect large?).

6.1 The Role of the Beliefs Effect

Since the beliefs effect captures the role of expectations, we should expect that it plays an important role if changes in households' and lenders' expectations have a relatively strong impact on the value to the government of repaying its debt or defaulting. Below, I argue that households' and lenders' beliefs have a strong impact on the government's decisions when households tend to invest a high fraction of their income and the government's desired borrowing is high.

Households' expectations are important if the difference between the investments of a pessimistic household and an optimistic household (holding the productivity level constant) is large since, then, an adjustment in households' expectations will lead to a large change in total output and, hence, in tax revenues. Since this difference is equal to

$$k_2^R - k_2^D = (1 - Z)(1 - \tau) e^{A_i} k_1^{\alpha} \frac{\alpha}{1 + \alpha},$$

one should expect that households' beliefs play an important role when $k_2^R - k_2^D$ is large, which is the case when τ, Z are low and α, k_1 are high.

Lenders' beliefs affect the government default decision by determining how much the government can borrow. However, if the government's desired borrowing is low, then the quantity of funds supplied to the market matters relatively little since the government would not want to borrow much anyway. Therefore, one should expect that the role of lenders' expectations is large when the government's desired borrowing is high. From the government's problem, it follows that the government's desired borrowing is equal to

$$B_2^{R,u}(A) = \frac{(1 + r) B_1 + \tau Y_2^R(A) - (1 + r) \tau Y_1^R(A)}{2(1 + r)},$$

where $Y_t^R(A)$ is the aggregate output at time t if the government repays its debt when the average productivity is A. The desired borrowing tends to be high when τ is low (a high τ decreases investment and, hence, decreases Y_2), k_1 is low and α is high (since, then, Y_2 is relatively high compared to Y_1) or B_1 is high.

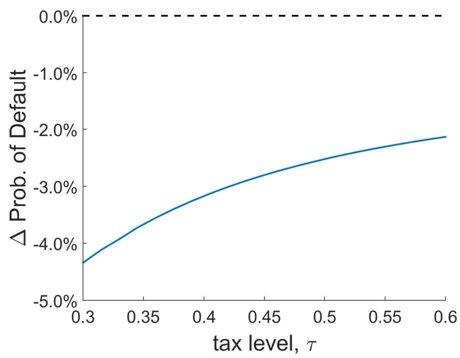
6.2 Numerical Analysis

The next goal is to understand: (1) whether for a reasonable parameter choice, an increase in tax and fiscal stimulus tend to decrease or increase the probability of default; and (2) how important the beliefs effect is in driving these results.

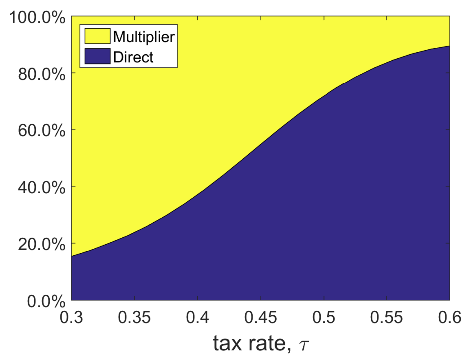
I choose a reference set of parameters so that, in a stylized way, the model resembles the GIIPS economies (i.e., Greece, Ireland, Italy, Portugal, and Spain) at the onset of the European debt crisis in 2008. I then vary key parameters from this reference point, one at a time, to see how the effectiveness of the government policies and the importance of the beliefs effect vary with the parameters.[28] To make the results comparable across

[28] From the perspective of the analysis, the most important parameters are τ, the tax rate; Z, the output costs of default; k_1, the initial the capital stock; and α, the capital share of output, since these parameters directly determine the costs and benefits of both policies considered above. I set $\tau = 0.4$, the average ratio of governments' tax revenue to GDP in the Eurozone in 2011, as reported by Eurostat, and $Z = 0.92$, implying that in the case of a debt crisis, output declines by 8% (the observed output decline in Greece after it defaulted in 2010). I choose $k_1 = 1.31$ to match the average growth of the net capital stock of 2% in the GIIPS economies in the run-up to the crisis (period 2004-2008), and $\alpha = 0.4$

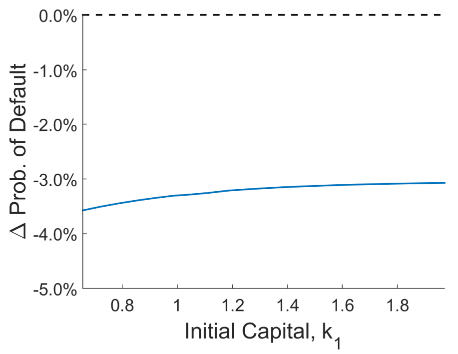
different parameter values, following each change in a parameter of the model, I adjust the mean of the prior belief so that the ex-ante probability of default, before a new policy is implemented, is equal to 10%. Due to space considerations, I report below only results where I vary the tax rate τ and the initial level of capital k_1.[29]



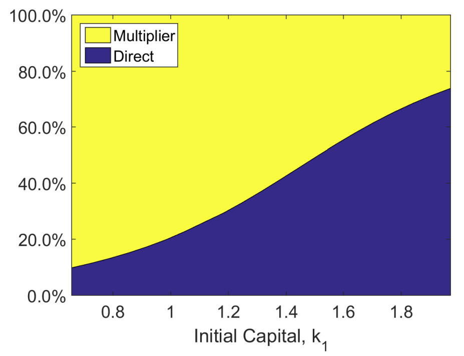
(a) The change in the probability of default as the initial τ varies.



(b) The contribution of the beliefs effect as the initial τ varies.



(c) The change in the probability of default as the initial k_1 varies.



(d) The contribution of the beliefs effect as the initial k_1 varies.

Figure 6: The effect of a 1% increase in the tax rate.

Increase in Taxes I consider, first, the effect of a 1% increase in taxes for different initial values of the tax rate τ and the capital stock k_1. Panel A of Figure 6 shows how the effect of this policy varies with the initial tax level, while Panel B depicts how much of the change in the default threshold is driven by the beliefs effect. We see that an

(see Arpaia et al. (2009)). The information parameters are $\sigma_x = 1/20$, $\varepsilon = \sqrt{3}\sigma_x$, and $\sigma = 1/12$. Mean of prior, A_{-1}, is set to imply a 10% probability of default. The initial debt is $B_1 = 1$, and the total wealth of the lenders is four times the maturing debt, implying the ratio $b/B_1 = 4$ (twice the average bid-to-cover ratio in the debt auctions in Germany and Italy, as reported in Beetsma et al., 2013).

[29] Additional results can be found in the "Additional Results" document available on the author's website `https://econ.sites.olt.ubc.ca/files/2016/01/pdf_szkup_debt_crises_additional.pdf`.

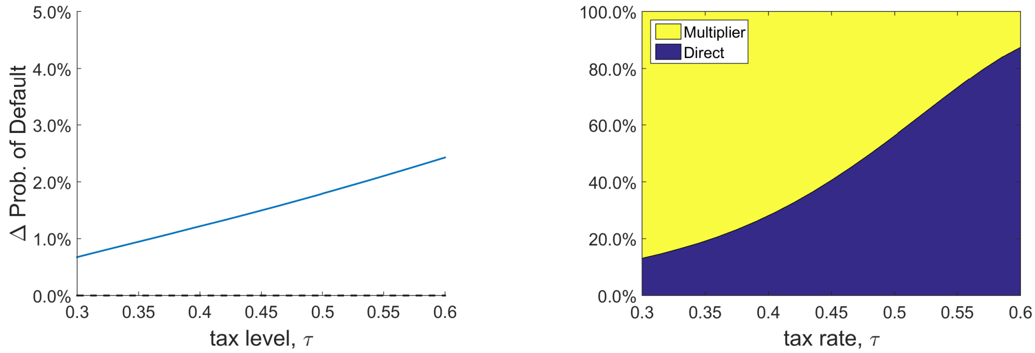
increase in the tax rate has a larger positive effect when taxes are initially low. This is because at low τ, the distortive effect of a tax increase is small, while the beliefs effect is large. Panel B shows that the relative importance of the beliefs effect decreases as τ increases: When the initial tax rate is low, the majority of the adjustment in the default threshold A^* is driven by the adjustment in households' and lenders' beliefs, but as initial τ increases, the importance of beliefs decreases. This is in line with the intuition provided in Section 6.1.

Panels C and D of Figure 6 depict the corresponding results of a 1% increase in the tax rate τ for different values of k_1. We see that varying the initial level of capital has relatively little effect on the efficacy of an increase in taxes. However, the initial level of capital stock does affect the importance of the beliefs effect, with the beliefs effect being stronger for low values of k_1. To understand why this is the case, note that, as explained in Section 6.1, as k_1 increases, the importance of the households' beliefs tends to increase, while the importance of the lenders' beliefs tends to decrease. For the parameters considered here, the latter effect dominates (as the difference between k_2^R and k_2^D is relatively small), and the importance of the beliefs effect declines as k_1 increases.

Fiscal Stimulus Next, I report the effects of a fiscal stimulus for different values of the initial tax rate τ and capital stock k_1. I consider a fiscal stimulus with size equal to 1% of the initial capital stock and financed with short-term debt (with $r^{ST} = 0$).[30] Panels A and C of Figure 7 show that engaging in fiscal stimulus when a crisis is likely is not a good idea, as a fiscal stimulus tends to increase the probability of default. Moreover, we see that this negative effect is stronger when the initial tax rate is high (since at higher τ, households invest less, leading to a lower positive effect of a stimulus on future output) and when k_1 is high (since, then, the marginal value of an extra unit of capital is low, while the cost of such a policy is high). Moving our attention to Panels B and D, we observe that, as in the case of an increase in τ, the beliefs effect is an important driver of the adjustment in the probability of default when k_1 or τ are relatively low, and its role diminishes as k_1 and τ increase.

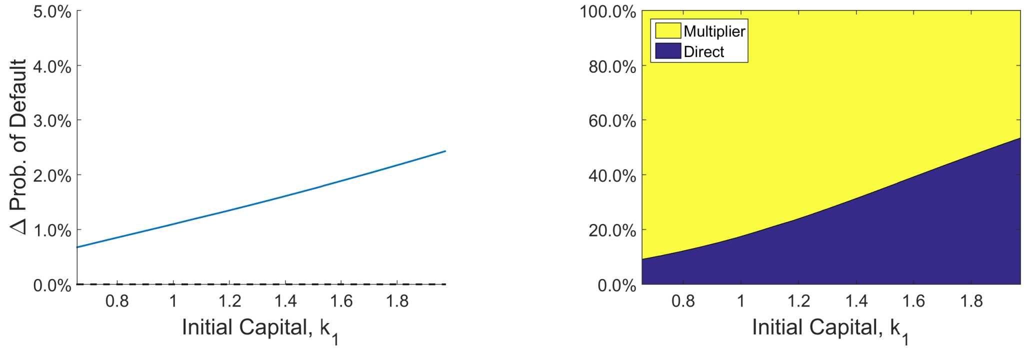
Summary The above results indicate that an increase in the tax rate is an effective policy for decreasing the probability of default for a wide range of parameters, while the opposite is true for a fiscal stimulus. They also support the intuition provided above that endogenous adjustments in expectations play an important role in determining the total change in the default threshold A^*.

[30] The results for a fiscal stimulus financed with long-term debt are similar.



(a) The change in the probability of default as the initial τ varies.

(b) The contribution of the beliefs effect as the initial τ varies.



(c) The change in the probability of default as the initial k_1 varies.

(d) The contribution of the beliefs effect as the initial k_1 varies.

Figure 7: The effect of a 1% stimulus.

7 Model with fundamental crises only

One may wonder how the predictions of the model with dispersed information and endogenous expectations differ from predictions of a model in which crises are driven purely by fundamentals and agents have common beliefs. To answer this question, I consider the model of Section 2, but allow agents to observe A and coordinate their beliefs on repayment equilibrium whenever A belongs to the "fragility region." In this case, the government defaults only when fundamentals are poor enough, which happens when $A < \underline{A}$ (i.e., below the lower bound of the fragility region). I refer to this version of the model as "the model with only fundamental crises." I then explore how the effects of adjusting a policy parameter ψ in the model with fundamental crises differ from the effect in the model with dispersed information. In other words, I compare $d\underline{A}/d\psi$ with $dA^*/d\psi$.

7.1 No policy uncertainty

In the model with only fundamental crises, a change in $\underline{A}$ in response to a change in policy ψ is simply equal to the direct effect that ψ has on the government's incentives to default; that is,

$$\frac{d\underline{A}}{d\psi} = \underbrace{\frac{\partial \underline{A}}{\partial \psi} + \frac{\partial \underline{A}}{\partial k_2}\frac{\partial k_2}{\partial \psi}}_{\text{Direct Effect } (D^{Fun})}, \tag{6}$$

Comparing Equation (6) with Equation (2), which provides a decomposition of $dA^*/d\psi$, we see that the signs of both $dA^*/d\psi$ with $d\underline{A}/d\psi$ are determined by the associated direct effects, which capture the fundamental economic forces that are at play in the models.[31] Since the two models share the same fundamental structure and differ only in terms of the assumed belief systems (i.e., common versus dispersed information), it follows that the two models will deliver intuitively similar predictions.

However, this does not mean that the predictions of the two models will coincide quantitatively or qualitatively. First, in the model with only fundamental crises, all agents have common and correct beliefs, and, thus, the "amplification effect" that is present in the model with dispersed information is missing. As such, even if the two models deliver the same qualitative predictions, their quantitative predictions will differ.

Second, the presence of dispersed information distorts the direct effect of a given policy change. On the one hand, the presence of dispersed information makes agents more pessimistic about the prospect of the government repaying its debt. This translates into a lower supply of funds in the bond market and lower investment, implying that $A^* > \underline{A}$. As such, the government revenues at the default threshold tend to be higher in the model with dispersed information. Given the concavity of households' utility, this decreases the benefit of policies that increase government's revenues, holding everything else constant. On the other hand, under dispersed information, the government is unable to roll over its maturing debt since lenders who receive low signals do not lend their funds to the government. As a consequence, if the government repays its debt, then its expenditure in period 1 is substantially lower than in period 2, which is suboptimal given the concavity of the households' utility function. This, in turn, implies that policies that result in a larger increase in government's revenues in period 1 than in period 2 (such as a fiscal stimulus) or policies whose negative effect falls in period 2 (such as an increase in taxes) tend to decrease the government's incentives to default by more under dispersed information. In contrast, in the model with fundamental crises only, the government can

[31] Recall that the direct effect consists of the effect that a change in ψ has on government default incentives directly ($\partial\underline{A}/\partial\psi$ in Equation (6) and $\partial A^*/\partial\psi$ in Equation (2)) and through its effect on the households' investment strategies ($\int_{i=0}^{1}\left(\partial\underline{A}/\partial k_2^{R,i}\right)\left(\partial k_2^{R,i}/\partial\psi\right) di$ in Equation (6) and $\int_{i=0}^{1}\left(\partial A^*/\partial k_2^{i}\right)\left(\partial k_2^{i}/\partial\psi\right) di$ in Equation (2)).

always smooth repayment perfectly across the two periods so that such considerations are not important.

Proposition 7 *Let $\psi \in \{\tau, s\}$. Then, there exists $\bar{A}_{-1}(\psi)$ such that for all $A_{-1} < \bar{A}_{-1}(\psi)$, we have*

$$d\underline{A}/d\psi < 0 \implies dA^{*}/d\psi < 0$$

but not vice versa.

The above proposition establishes that when the past productivity level is low, if a change in $\psi \in \{\tau, s\}$ decreases the probability of default in the model with only fundamental crises, then it also does so in the model with dispersed information, but not vice versa. In other words, for low levels of A_{-1}, government policies decrease the probability of default for a wider range of parameters in the model with dispersed information. This results follows from the observation that when the past productivity level is low, then in the model with dispersed information, the supply of funds in the bond market is low. In this case, if the government decides to repay its debt then its spending will be particularly low in period $t = 1$ compared to period $t = 2$, providing the government with strong incentives to adjust its policies.

7.2 Policy uncertainty

Next, I compare how the effects of political uncertainty differ across the two models. As in Section 5, let $p \in [0, 1)$ denote the probability with which a policy ψ is implemented. Then,

$$\frac{d\underline{A}}{d\psi}(p) = p\frac{d\underline{A}}{d\psi}(1) + (1-p)\frac{\partial \underline{A}}{\partial \psi} \tag{7}$$

The intuition underlying this result is similar to that underlying Equation (5). As agents expect the policy to be implemented with probability p, they respond to the announced change in ψ proportionally less.[32] As a consequence, the presence of political uncertainty decreases the distortionary effects of higher taxes on households' investment but also decreases the expansionary effect of a stimulus. Therefore, as in the model with dispersed beliefs, political uncertainty relaxes the conditions under which an increase in taxes leads to a decrease in the probability of default but tightens the conditions for a fiscal stimulus.

[32] With probability $1 - p$, households and lenders do not expect the adjustment, in which case a change in $\underline{A}$ is driven by the direct change in the government's default incentive.

However, the two models differ in the impact that political uncertainty has on the overall effect of government policies on the probability of default. This difference stems from the fact that in the model with dispersed information, political uncertainty decreases the magnitude of the amplification effect. Since the beliefs effect is absent in the model with only fundamental crises, this negative effect of political uncertainty is absent in this model.

Proposition 3 *Suppose that in the model with dispersed information, $\varepsilon, \sigma \to 0$. Then,*

$$\left|\frac{d\underline{A}/d\psi\,(p)}{d\underline{A}/d\psi\,(1)}\right| > \left|\frac{dA^*/d\psi\,(p)}{dA^*/d\psi\,(1)}\right|,$$

as long as $sgn\,(dA^*/d\psi\,(p)) = sgn\,(dA^*/d\psi\,(1)) \neq 0$ and $sgn\,(d\underline{A}/d\psi\,(p)) = sgn\,(d\underline{A}/d\psi\,(1)) \neq 0$.[33]

8 Conclusions

In this paper, I investigated how a government can prevent a self-fulfilling debt crisis. To answer this question I developed a model of self-fulfilling sovereign default with endogenous expectations and dispersed information. I then used this model to determine how fiscal policies, such as an increase in taxes or a fiscal stimulus, affect the probability of a crisis and how these effects are perturbed by the presence of endogenous expectations and dispersed beliefs. Finally, I studied how uncertainty about government's economic policies changes the effect of government policies.

The findings of this paper contribute to the debate over government's proposed actions when facing a looming debt crisis that took place during European debt crisis: Should the government engage in austerity or in a fiscal stimulus. My results provide support for the choice of austerity and suggest that austerity is particularly preferable to fiscal stimulus in an environment in which there is high uncertainty about future economic policies, as often is the case during debt crises. Thus, the results provide support for the policies adopted during European debt crises, while suggesting that they would have been substantially more effective in the absence of policy uncertainty that accompanied their implementation.

A few words of caution are needed regarding the interpretation of the results. In this paper, I analyzed the particular situation in which the government finds itself on a verge of a debt crisis. Indeed, the main question that this paper addresses is how to

[33] These conditions ensure that a change in ψ has the same effect on the probability of default both in the absence and in the presence of political uncertainty and that the ratios below are well-defined.

avoid a debt crisis when such a crisis is likely in the near future. For that purpose, that fact that the model presented above is two-period is a minor issue. However, the fact that the model is not dynamic becomes key when trying to answer questions regarding medium-term policies. A question of particular importance is what the government should do to avoid facing another debt crisis in the future once the current debt crisis has been averted. This remains an important question for the future research.

Appendix
(For Online Publication)

This appendix contains the proofs of the results that have been stated in the paper. In Section A, I solve the main model, while Section B contains derivations of the direct and beliefs effects and the proofs of Propositions 2 to 4 and Corollary 1 from the paper, as well as the analysis of an increase in τ when it is implemented only in repayment. Section D contains brief derivations of the total change in the default threshold when the agents expect the policy to be implemented with probability p — $dA^*/d\psi\,(p)$ — as well as proofs of Propositions 5 and 6 and Corollary 2. In Section E, I briefly discuss how the results would change if Assumption 5 were not imposed. Section F contains a discussion of the effect of an adjustment in the interest rate on the effects of policy changes, while Section G contains several technical claims invoked in proofs throughout the Appendix.[34]

A Global Game model

A.1 Uniqueness Result

To prove uniqueness of equilibrium (Proposition 1), I first characterize the optimal households' and lenders' strategies in response to a monotone default strategy by the government. Then, I show that in response to these households' and lenders' strategies the government indeed finds it optimal to follow a monotone default strategy. Finally, I show that there exists a unique fixed-point of this argument.

Notation 1 *I will use the following notation throughout the Appendix:*

1. *A^* denotes the default threshold used by the government.*

2. *A^{**} denotes the default threshold expected by the households and lenders.*

[34] The solution to the complete information version of the model, as well as detailed derivations of the beliefs and direct effects when agents are uncertain whether announced policies will be implemented, can be found in the "Additional Results" document available on the author's website `https://econ.sites.olt.ubc.ca/files/2016/01/pdf_szkup_debt_crises_additional.pdf`.

A.1.1 Households

Suppose that households expect the government to repay its debt if and only if $A \geq A^{**}$. Household i's optimal investment then solves the households' problem specified in Section 3.4. Each household receives a productivity shock A_i, where $A_i = A + \varepsilon_i$ and $\varepsilon_i \in [-\varepsilon, \varepsilon]$.

If $A_i > A^{**} + \varepsilon$, then household i expects no default; in that case,

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \frac{\alpha}{1+\alpha}.$$[35]

If household i receives productivity $A_i < A^{**} - \varepsilon$, then household i believes that the government will always default, and

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \frac{\alpha Z}{1+\alpha}.$$

Finally, when $A_i \in (A^{**} - \varepsilon, A^{**} + \varepsilon)$, the household is uncertain as to whether the government will default. In that case,

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \Lambda(A_i; \varepsilon, A^{**}),$$

where

$$\begin{aligned} \Lambda(A_i; \varepsilon, A^{**}) &= \frac{\alpha(1+Z) + P(A^{**}|A_i) + Z(1 - P(A^{**}|A_i))}{2(1+\alpha)} \\ &\quad - \frac{\sqrt{[\alpha(1+Z) + P(A^{**}|A_i) + Z(1 - P(A^{**}|A_i))]^2 - 4\alpha Z(1+\alpha)}}{2(1+\alpha)} \end{aligned}$$

and $P(A^{**}|A_i) \equiv \Pr(A < A^{**}|A_i)$. It is straightforward to show that $\Lambda(A_i; \varepsilon, A^{**})$ is increasing in A_i and decreasing in A^{**}. This establishes Lemma 1 in the paper.

Next, I perform a change of variables $\kappa = \frac{\varepsilon_i}{\varepsilon}$, where $\varepsilon_i \in [-\varepsilon, \varepsilon]$ so that $\kappa \in [-1, 1]$. This change of variables turns out to be useful for computing the output in the limiting case, as $\varepsilon \to 0$, and in general, when analyzing the effect of changes in ε. Define

$$\Lambda(A + \kappa\varepsilon; \kappa, A^{**}) \equiv \begin{cases} \frac{\alpha}{(1+\alpha)} & \text{when } A_i = A + \kappa\varepsilon > A^{**} + \varepsilon \\ \Lambda(A_i; \varepsilon, A^{**}) & \text{when } A_i = A + \kappa\varepsilon \in (A^{**} - \varepsilon, A^{**} + \varepsilon) \\ \frac{\alpha Z}{(1+\alpha)} & \text{when } A_i = A + \kappa\varepsilon < A^{**} - \varepsilon \end{cases},$$

In what follows, I will denote the optimal choice of capital as $k_2^*(A, \kappa, A^{**})$ to emphasize its dependence on A, κ and households' belief about the default threshold A^{**}.

[35] It is here that the assumption of full depreciation of households' capital simplifies the model. When the capital depreciates fully each period, the optimal choice of capital is linear. As we will see below, this will make the government's default condition near linear in e^A.

A.1.2 Lenders

Denote by $p_x = 1/\sigma_x^2$ and $p_A = 1/\sigma_A^2$ the precisions of the lenders' private signals and the prior, respectively. As usual, it is more convenient to work with precisions rather than with standard deviations or variances.

Let $u(1, A; x^{**}, A^{**})$ be the expected payoff to lender j from lending to the government when the average productivity is equal to A, the government uses a threshold strategy with cutoff A^{**}, and the other lenders use monotone strategies with cutoff x^{**}. Similarly, denote by $u(0, A; x^{**}, A^{**})$ the payoff to lender j from investing in the risk-free asset. Then,

$$\begin{aligned} u(1, A; x^{**}, A^{**}) &= \begin{cases} 1 + r \min\left\{\frac{B_2^{R,u}(A)}{S(A;x^{**})}, 1\right\} & \text{if } A \geq A^{**} \\ 0 & \text{otherwise} \end{cases} \\ u(0, A; x^{**}, A^{**}) &= 1, \end{aligned}$$

Define $\Delta u(A; x^{**}, A^{**}) \equiv u(1, A; x^{**}, A^{**}) - u(0, A; x^{**}, A^{**})$.

It is immediate to see that for any pair (A^{**}, x^{**}), and regardless of the government's desired borrowing function $B_2^{R,u}$, the function $\Delta u(A; x^{**}, A^{**})$ satisfies a weak single crossing property in A.[36] Moreover, it is well-known that a family of normal density functions parameterized by x_j

$$\left\{(p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x_j + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}}\right)\right\}_{x_j \in \mathbb{R}}$$

satisfies the strict monotone likelihood ratio (MLR) property, implying that the above density function is strictly log-supermodular in (A, x_j) (see Athey, 1996). By Theorem 3.2 in Athey (1996),

$$\Delta U(x_j; x^*, A^{**}) \equiv \int_{A^{**}}^{\infty} \Delta u(A; x^{**}, A^{**}) (p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x_j + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}}\right) dA$$

satisfies the strict single-crossing property in A^{**}. Thus, in response to monotone strategies by the government and the other lenders, lender j finds it optimal to follow a monotone strategy.

Consider $\Delta U(x^*; x^*, A^{**})$, the expected utility difference from supplying the funds to the market versus not supplying them, evaluated at x^*, and let $L(A^{**}, x^*) \equiv \Delta U(x^*; x^*, A^{**})$. I want to show that for each A^{**}, there exists a unique x^* such that $L(A^{**}, x^*) = 0$. First note that $\Delta u(A; x^*, A^{**})$, as defined above, is increasing in x^*. This is because $S(A; x^*) = b\left(1 - \Phi\left(\frac{x^* - A}{p_x^{-1/2}}\right)\right)$ is decreasing in x^*. Moreover, for all $A \geq A^{**}$, $B_2^{R,u}(A)$ is differentiable in A, and, therefore, $\Delta u(A; x^*, A^{**})$ is piecewise continuous. Second, note that the product of $\Delta u(A; x^*, A^{**})$ and $(p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}}\right)$ is different than 0, at least for all $A < A^{**}$.

[36] A function $f(x)$, where $f\colon \mathbb{R} \to \mathbb{R}$, satisfies a weak single-crossing property in x if for all $x_H > x_L$, $f(x_L) > 0$ implies $f(x_H) \geq 0$.

Then, by Theorem 3.4 in Athey (1996), it follows that $L(A^{**}, x^*)$ satisfies a strict single-crossing condition in x^*. This proves Lemma 2 in the text.

A.1.3 The Government's Monotone Default Strategy

Suppose that the households follow investment strategies as characterized above, and the lenders use monotone strategies with a common threshold x^*. I show that $\Delta V(A, \mathbf{k}_2^*, S)$ is strictly increasing in A.

Define $\mathbf{k}_2^*(A, A^{**}) \equiv \{k_2(A, \kappa, A^{**})\}_{\kappa \in [-1,1]}$; that is, $\mathbf{k}_2^*(A, A^{**})$ denotes the households' investment choices when the average productivity is equal to A and when all households expect that the default threshold is A^{**}. Note that if the lenders follow monotone strategies, then $S = b\left[1 - \Phi\left(\frac{x^* - A}{\sigma_x}\right)\right]$. Thus, with a slight abuse of notation, I will write $\Delta V(A, \mathbf{k}_2^*(A, A^{**}), S)$ as $\Delta V(A; \mathbf{k}_2^*(A, A^{**}), x^*)$. Finally, let $B_2^{R,u}$ denote the government's optimal unconstrained borrowing.

Using the definition of $\Delta V(A, \mathbf{k}_2^*(A, A^{**}), x^*)$, substituting for $\mathbf{k}_2^*(A, A^{**})$ the expression found in Section A.1.1 and rearranging, we get

$$
\begin{aligned}
\Delta V(A, \mathbf{k}_2^*(A, A^{**}), x^*) \;=\; & \int_{-1}^{1} \frac{1}{2} \log\left(\frac{1 - \Lambda(A + \kappa\varepsilon, \kappa, A^{**})}{Z - \Lambda(A + \kappa\varepsilon, \kappa, A^{**})}\right) d\kappa + \log\left(\frac{\tau Y_1^R - B_1 + B_2^{R^*}}{\tau Z Y_1^R + (1-\xi) B_2^{D^*}}\right) \\
& + \log\left(\frac{1}{Z}\right) + \log\left(\frac{\tau Y_2^R - (1+r) B_2^{R^*}}{Z \tau Y_2^R}\right),
\end{aligned}
$$

where

$$
B_2^{R^*} = \begin{cases} B_2^{R,u}(A) & \text{if } B_2^{R,u} \leq S(A, x^*) \\ S(A, x^*) & \text{if } B_2^{R,u} > S(A, x^*) \end{cases}.
$$

Differentiating with respect to A, simplifying, and taking the limit as $\xi \to 1$, we get

$$
\frac{\partial \Delta V(A; \mathbf{k}_2^*(A, A^{**}), x^*, A^*)}{\partial A} \geq \frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{(1+\alpha) B_2^{R^*} (1+r)}{\tau Y_2^R - (1+r) B_2^{R^*}}, \tag{8}
$$

where I used the observation that if $B_2^{R^*} = B_2^{R,u}(A)$, then by the optimality of the government borrowing choices, the terms containing $\partial B_2^{R^*}/\partial A$ add up to 0, while otherwise their sum is strictly positive.

Add the above fractions on the right-hand side of Equation (8.) The resulting numerator can be written as

$$
2(1+r)\left(B_2^{R^*}\right)^2 - B_2^{R^*}\left(\tau Y_2^R + 2(1+r) B_1 - (1+r)\tau Y_1^R\right) + B_1 \tau Y_2^R
$$

This expression is a quadratic in $B_2^{R^*}$. Let $B_2^{R^*,1}(A)$ and $B_2^{R^*,2}(A)$ be its two roots. Whether these roots are real or not depends on the parameters of the model. For all $A \in [\underline{A}, \overline{A}]$, define $\bar{b}(A) = \min\left\{B_2^{R^*,1}(A), B_2^{R^*,2}(A)\right\}$ if the roots are real, and $\bar{b}(A) = \infty$ if they are complex. Let $\bar{b} = \min_{A \in [\underline{A}, \overline{A}]} \bar{b}(A)$. It follows that if $b < \bar{b}$, then the government's best response to monotone strategies is itself monotone. I assume that the lenders' wealth b satisfies this constraint (Assumption 3 in the paper).[37]

A.1.4 Uniqueness of Equilibrium

In light of the above results, to establish uniqueness, it is enough to show that

$$\Delta V\left(A^*, \mathbf{k}_2^*\left(A^*, A^*\right), x^*\left(A^*\right)\right)$$

is monotone in A^*, where $\mathbf{k}_2^*\left(A^*\right) \equiv \left\{k_2\left(A^*, \kappa, A^*\right)\right\}_{\kappa \in [-1,1]}$ is a vector whose components are the individual households' investment strategies when the households have the correct expectations about the default threshold (i.e., $A^{**} = A^*$), and x^* is the common signal threshold used by the lenders when households and lenders expect the default threshold to be A^*. I denote the optimal lender's threshold by $x^*\left(A^*\right)$, to emphasize that it depends on A^*.

Fix $\eta > 0$, where η is a small positive number. Differentiating $\Delta V\left(A^*; \mathbf{k}_2^*\left(A^*\right), x^*\left(A^*\right)\right)$ with respect to A^* and taking the limit as $\xi \to 1$, we get

$$\begin{aligned} \frac{d\Delta V}{dA^*} &= \int_{-1}^{1} \frac{-\frac{\partial \Lambda}{\partial A^*}\left[Z - \Lambda\right] + \left[1 - \Lambda\right] Z \frac{\partial \Lambda}{\partial A^*}}{\left[1 - \Lambda\right]\left[Z - \Lambda\right]} d\kappa \\ &\quad + \frac{\frac{dB_2^{R^*}}{dA^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} - \frac{\left(1 + r\right) \frac{\partial B_2^{R^*}}{\partial A^*}}{\tau Y_2^R - \left(1 + r\right) B_2^{R^*}} \\ &\quad + \frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{\left(1 + \Psi\right)\left(1 + r\right) B_2^{R^*}}{\tau Y_2^R - \left(1 + r\right) B_2^{R^*}}, \end{aligned}$$

where

$$\Psi \equiv \frac{\int_{-1}^{1} \frac{1}{2} \frac{\partial}{\partial A^*} f\left(k_2\left(A^* + \kappa\varepsilon; \varepsilon, A^*\right)\right) d\kappa}{Y_2^R} \to \alpha \text{ as } \varepsilon \to 0.$$

Since

$$\lim_{\varepsilon \to 0} \frac{\partial \Pr\left(A^* | A^* + \kappa\varepsilon\right)}{\partial A^*} \to 0,$$

there exists $\bar{\varepsilon}$ such that for all $\varepsilon \in (0, \bar{\varepsilon}]$ we have

$$\int_{-1}^{1} \frac{-\frac{\partial \Lambda}{\partial A^*}\left[Z - \Lambda\right] + \left[1 - \Lambda\right] Z \frac{\partial \Lambda}{\partial A^*}}{\left[1 - \Lambda\right]\left[Z - \Lambda\right]} d\kappa < \frac{\eta}{2}$$

[37] One may wonder how restrictive this assumption is. The answer is that it depends on the parameters. However, numerical simulations suggest that unless α or Z is very close to 1, both roots are complex, which means that the bound can be made arbitrarily large (though it has to be finite). In particular, this is the case for the calibration used in the paper.

Next, since $\frac{\partial S(A^*)}{\partial A^*} > -b\frac{p_A}{p_x^{1/2}}\frac{1}{\sqrt{2\pi}} \to 0$ as $p_x \to \infty$, it follows that there exists a large enough $\overline{p}_x$ such that for all $p_x > \overline{p}_x$ we have

$$\frac{\frac{dB_2^{R^*}}{dA^*}}{\left[\tau Y_1^R - B_1 + B_2^{R^*}\right]} - \frac{(1+r)\frac{\partial B_2^{R^*}}{\partial A^*}}{\left[\tau Y_2^R - (1+r)\, B_2^{R^*}\right]} > -\frac{\eta}{2}.^{38}$$

Finally, following the same argument as in Section $A.1.3$, one can show that there exists $\overline{\overline{b}}(\varepsilon)$ such that for all $b < \overline{\overline{b}}(\varepsilon)$ we have

$$\frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{(1+r)\, B_2^{R^*}}{\left[\tau Y_2^R - (1+r)\, B_2^{R^*}\right]} > \eta.$$

Therefore, for all ε with $0 < \varepsilon < \overline{\varepsilon}$ and all $p_x > \overline{p}_x$, we have

$$\frac{d\Delta V}{dA^*} > -\frac{\eta}{2} - \frac{\eta}{2} + \eta = 0,$$

implying that there exists a unique default threshold A^* that satisfies all the equilibrium conditions.

The above analysis applies to a fixed value of A^*. However, since $A^* \in \left[\underline{A}, \overline{A}\right]$, which is a compact interval, there exist bounds $\overline{\varepsilon}$ and $\overline{p}_x$, which are independent of A^*, such that if $\varepsilon < \overline{\varepsilon}$ and $p_x < \overline{p}_x$, then $d\Delta V/dA^*$ is strictly positive for all $A^* \in \left[\underline{A}, \overline{A}\right]$. This completes the proof.

B Policy Analysis

This Section contains proofs of all the claims made in Section 4 of the paper.

Proof of Proposition 2. Let ψ denote a parameter of the model (for concreteness, one can think of the tax rate, in which case $\psi = \tau$). Then, for given r, the equilibrium conditions can be written as

$$I\left(A^* + \kappa\varepsilon, A^{**}, k_2^*(\kappa), \psi\right) = 0,$$

which is the equilibrium condition for a household with productivity $A^* + \kappa\varepsilon$ and which determines the capital choice for a household with productivity shock $\kappa\varepsilon$;

$$L\left(A^{**}, x^*, \psi\right) = 0,$$

which is the equilibrium condition that describes the lenders' behavior and determines x^*; and, finally,

$$\Delta V\left(A^*, \{k_2^*(\kappa)\}_{\kappa\in[-1,1]}, x^*, \psi\right) = 0,$$

which is the equilibrium condition that describes the government's default decision and determines A^*.

[38] If $\partial B_2^{R^*}/\partial A^* = \partial B_2^{R,u}/\partial A^*$, then the sum of these terms is 0.

Note that, for each $\kappa \in [-1,1]$, the equation $I(A^* + \kappa\varepsilon, A^*, k_2^*(\kappa), \psi) = 0$ specifies $k_2^*(\kappa)$ as a function of households' productivity $A^* + \kappa\varepsilon$, household's belief about the default threshold A^{**}, and the policy parameter ψ for each $\kappa \in [-1,1]$. Similarly, the equation $L(A^*, x^*, \psi) = 0$ determines x^* as a function of the lenders' belief about the default threshold A^{**} and ψ. In equilibrium, $A^{**} = A^*$; that is, the households and lenders hold correct beliefs about the government's default decision. However, to derive the effect of a change in the households' and lenders' beliefs on the default threshold, we have to differentiate between the belief about the threshold held by the households and lenders and the actual default threshold, where the latter is defined as the level of productivity at which the government defaults.

(Derivations of the beliefs and the direct effect) To compute the equilibrium change in A^* due to a change in ψ, I compute the total derivatives of the expressions on both sides of the equilibrium conditions and solve the resulting linear system of equations for $dA^*/d\psi$:

$$I_1(\kappa)\frac{dA^*}{d\psi} + I_2(\kappa)\frac{dA^{**}}{d\psi} + I_3(\kappa)\frac{dk_2^*(\kappa)}{d\psi} + I_4(\kappa) = 0 \tag{9}$$

$$L_1\frac{dA^{**}}{d\psi} + L_2\frac{dx^*}{d\psi} + L_3 = 0 \tag{10}$$

$$\Delta V_1\frac{dA^*}{d\psi} + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{dk_2^*(\kappa)}{d\psi}d\kappa + \Delta V_3\frac{dx^*}{d\psi} + \Delta V_4 = 0, \tag{11}$$

where I_n is the partial derivative of $I(A^* + \kappa\varepsilon, A^{**}, k_2^*(\kappa), \psi)$ with respect to its nth argument and similarly for L_n and ΔV_n. $dA^{**}/d\psi$ is the total change in agents' beliefs regarding the government default threshold implied by a change in ψ. In equilibrium, $dA^{**}/d\psi = dA^*/d\psi$, but, for now, it is important to keep the distinction between the two objects.

Solving for $dx^*/d\psi$ and $dk_2^*/d\psi$ using Equations (10) and (9) and recognizing that $\partial x^*/\partial A^{**} = -L_1/L_2$, $\partial k_2^*(\kappa)/\partial A^* = -I_1(\kappa)/I_3(\kappa)$, $\partial k_2^*(\kappa)/\partial A^{**} = -I_2(\kappa)/I_3(\kappa)$, and $\partial k_2^*(\kappa)/\partial\psi = -I_4(\kappa)/I_3(\kappa)$ we obtain:

$$\frac{dx^*}{d\psi} = \frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial x^*}{\partial\psi} \quad \text{and} \quad \frac{dk_2^*(\kappa)}{d\psi} = \frac{\partial k_2^*(\kappa)}{\partial A^*}\frac{dA^*}{d\psi} + \frac{\partial k_2^*(\kappa)}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial k_2^*(\kappa)}{\partial\psi}$$

Substituting the above expressions into Equation (11) and rearranging, we get

$$\begin{aligned}\left[\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa\right]\frac{dA^*}{d\psi} &= \\ -\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\left[\frac{\partial k_2^*(\kappa)}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial k_2^*(\kappa)}{\partial\psi}\right]d\kappa - \Delta V_3\left[\frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial x^*}{\partial\psi}\right] - \Delta V_4,\end{aligned} \tag{12}$$

where $\left[\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa\right]$ captures the effect of an increase in the productivity on the government's incentives to default.

Recall that an individual household's investment strategy is a function that maps the individual productivity into an investment choice; that is, it is a map $k_2^* : A_i \rightarrow \mathbb{R}$. Thus, a change in the household's strategy is defined as a shift in this mapping, that is a change in k_2^* for each A_i. On the other hand, holding household strategies constant, a change in A_i also affects household i's investments: It is simply a movement along the curve $k_2 : A_i \rightarrow \mathbb{R}$. Thus, the term $\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa$ captures the effect of a change in productivity on the government's incentives to default holding households' and lenders' strategies constant.

Using the above observation, divide Equation (12) by $\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa$ to obtain

$$
\begin{aligned}
\frac{dA^*}{d\psi} \quad = \quad & \frac{-\int\limits_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa}{\Delta V_1 + \int\limits_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa} - \frac{\Delta V_3 \frac{\partial x^*}{\partial \psi}}{\Delta V_1 + \int\limits_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*}} - \frac{\Delta V_4}{\Delta V_1 + \int\limits_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*}} \\
& - \frac{\int\limits_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa}{\Delta V_1 - \int\limits_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*}} \frac{dA^{**}}{d\psi} - \frac{\Delta V_3 \frac{\partial x^*}{\partial A^{**}}}{\Delta V_1 - \int\limits_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*}} \frac{dA^{**}}{d\psi}
\end{aligned}
$$

The first three terms capture the direct effects of a change in ψ on the equilibrium strategies of the households', the lenders', and the government, respectively, holding households' and lenders' beliefs about the default threshold constant (i.e., holding A^{**} constant). The two remaining terms capture the effect of a change in ψ on the households' and lenders' beliefs. Next, define

$$
\frac{\partial A^*}{\partial \psi} \equiv - \frac{\Delta V_4}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa},
$$

so that the third term captures the partial effect of a change in ψ on the government's default incentives, holding households' and lenders' strategies and beliefs constant. Similarly,

$$
\frac{\partial A^*}{\partial x^*} \frac{\partial x^*}{\partial A^{**}} \equiv \frac{\Delta V_3}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*}} \frac{\partial x^*}{\partial A^{**}},
$$

and, slightly abusing notation,

$$
\int\limits_{-1}^{1} \frac{1}{2} \frac{\partial A^*}{\partial k_2^*(\kappa)} \frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa \equiv - \int\limits_{-1}^{1} \frac{\frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa}{\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*}},
$$

where this term captures the effect of a change in the households' beliefs on the government's incentives to default. In a similar fashion,

$$\int_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi}d\kappa \equiv -\int_{-1}^{1} \frac{\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial \psi}d\kappa}{\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa},$$

where $\int_{-1}^{1}\frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi}d\kappa$ captures the effect of a change in the households' strategies caused by a change in ψ, holding the households' beliefs about the default threshold, A^{**}, constant.

Using the above notation, recognizing that, in equilibrium, $\partial A^{**}/\partial\psi = dA^*/d\psi$, and rearranging, we obtain

$$\frac{dA^*}{d\psi} = \frac{\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_{-1}^{1}\frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi}d\kappa}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*} - \int_{-1}^{1}\frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}}d\kappa} \tag{13}$$

Since $\int_{-1}^{1}\frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi}d\kappa$ corresponds simply to $\int_0^1 \frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial \psi}di$, while the term $\int_{-1}^{1}\frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}}d\kappa$ corresponds to $\int_0^1 \frac{1}{2}\frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial A^{**}}di$, we arrived at Equation (2) in the paper.

(Establishing that B>1) Recall from the proof of uniqueness that the government default condition, after taking into account the dual role of A^* as the average value of productivity in the economy and the default threshold, is strictly increasing in A^*. Thus,

$$\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\left.\frac{\partial k_2^*(\kappa)}{\partial A^{**}}\right|_{A^{**}=A^*}d\kappa + \Delta V_3\left.\frac{\partial x^*}{\partial A^{**}}\right|_{A^{**}=A^*} > 0,$$

where the third and fourth terms capture the effect of a change in the households' and lenders' beliefs, respectively. Dividing the above expression by $\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa$ establishes the non-negativity of the beliefs effect.

Under Assumption 4, we have $\frac{B_2^{R,u}(A)}{S(A;x^{**})} = 1$ for all A, and, hence, it can be shown that $x^* = \frac{p_x+p_A}{p_x}A^{**} - \frac{p_A}{p_x}A_{-1} + \frac{\sqrt{p_x+p_A}}{p_x}\Phi^{-1}\left(\frac{1}{1+r}\right)$, implying that $\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} > 0$. Similarly, it is straightforward to show that $\partial k_2^*/\partial A^{**} < 0$. Since a higher investment by all households decreases the government's incentives to default ($\int_{-1}^{1}\frac{1}{2}\partial A^*/\partial k_2^*(\kappa)\,d\kappa < 0$), we have $\int_{-1}^{1}\frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}}d\kappa > 0$. It follows that the denominator of the beliefs effect is less than 1, so that the beliefs effect is greater than 1. ■

To establish Propositions 3 and 4, recall that the default threshold is determined by the following condition:

$$\begin{aligned} 0 &= \Delta V\left(A^*, \mathbf{k}_2^*, x^*\right) \\ &= \int_{-1}^{1}\log\left(c_1^R\right)d\kappa + \log\left(\tau Y_1^R - B_1 + B_2^{R^*}\right) + \int_{-1}^{1}\log\left(c_2^R\right)d\kappa + \log\left(\tau Y_2^R - (1+r)B_2^{R^*}\right) \\ &\quad - \int_{-1}^{1}\log\left(c_1^D\right)d\kappa - \log\left(\tau Z Y_1^R\right) - \int_{-1}^{1}\log\left(c_2^D\right)d\kappa - \log\left(\tau Z Y_2^R\right), \end{aligned}$$

where c_t^R and c_t^D are the consumption in period t in repayment and default, respectively, Y_t^R is the total output of the economy in period t, and $B_2^{R^*}$ is the equilibrium borrowing by the government, all evaluated at the threshold productivity A^*. Before proceeding further, note that $\int \log\left(\frac{c_t^R}{c_t^D}\right) d\kappa$ is independent of τ and k_1 for $t = 1, 2$, and, thus, policy change will affect the government's incentive to default only through its effect on government spending in repayment and in default.[39]

Proof of Proposition 3 **and Corollary** 1. Differentiate $\Delta V\left(A^*, \mathbf{k}_2^*, x^*\right)$ with respect to τ to obtain

$$u_{g_1}^R Y_1^R + u_{g_2}^R Y_2^R + u_{g_2}^R \tau \frac{\partial Y_2^R}{\partial \tau} - u_{g_1}^D Z Y_1^R + u_{g_2}^D Z Y_2^R + u_{g_2}^D Z \tau \frac{\partial Y_2^R}{\partial \tau},$$

where $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utility from government spending in period t in repayment and default, respectively, and Y_1^R the total output of the economy in period t in repayment, all evaluated at A^*.[40] Given households' equilibrium investment choices, $\partial Y_2^R / \partial \tau = \frac{\alpha}{1-\tau}$. Thus, rearranging the terms in the above expression, we obtain

$$\underbrace{Y_1^R (1-Z) u_{g_1}^D + Y_2^R (1-Z) u_{g_1}^D}_{\text{Differential increase in tax revenues}} + \underbrace{Y_1^R \left[u_{g_1}^R - u_{g_1}^D\right] + Y_2^R \left[u_{g_2}^R - u_{g_2}^D\right]}_{\text{Concavity effect}} - \underbrace{\frac{\partial Y_2^R}{\partial \tau} \tau \left[u_{g_2}^R - Z u_{g_2}^D\right]}_{\text{Investment distortion}},$$

which corresponds to the expression (3) in the paper. Note that $u_{g_t}^D = 1/\left(Z\tau Y_t^R\right)$, $u_{g_1}^R = 1/\left(\tau Y_1 - B_1 + B_2\right)$ and $u_{g_2}^R = 1/\left(\tau Y_2 - (1+r) B_2\right)$. Thus, one can write the above condition as

$$\frac{(B_1 - B_2)}{\tau Y_1 - B_1 + B_2} + \frac{(1+r) B_2}{\tau Y_2 - (1+r) B_2} - \frac{\alpha \tau}{1-\tau} \frac{(1+r) B_2}{\tau Y_2 - (1+r) B_2} \tag{14}$$

The first part of Proposition 3 follows from the observation that, according to the proof of equilibrium uniqueness, $\frac{(B_1-B_2)}{\tau Y_1 - B_1 + B_2} + \frac{(1+r)B_2}{\tau Y_2 - (1+r)B_2}$ is bounded away from 0, while $\frac{\alpha\tau}{1-\tau} \to 0$ as $\tau \to 0$. The second part of Proposition 3 follows from the observation that $\lim_{\tau_x \to \infty} S = b\left[1 - \Phi\left(\frac{x^* - A^*}{\sigma_x}\right)\right] = b\frac{r}{1+r}$. Thus, if $B_1 > rb$, then the first term in Equation 14 is positive. It follows that as long as $1 > \frac{\alpha\tau}{1-\tau}$, an increase in τ will decrease the probability of default. Rearranging this inequality, we arrive at the inequality stated in the text.

The proof of Corollary 1 follows from the observation that $\lim_{A_{-1} \to -\infty} S = 0$, in which case expression (14) converges to $\frac{B_1}{\tau Y_1 - B_1} > 0$. ■

Proof of Proposition 4. The proof of Proposition 4 is similar to the proof of Proposition 3. I consider only a stimulus financed with short-term debt. The case of a stimulus financed

[39] This is because $c_2^D = Z c_2^R$, $c_1^D = Z(1-\tau) e^{A_i} f(k_1) - k_2$, $c_1^R = (1-\tau) e^{A_i} f(k_1) - k_2$, and k_2 is linear in $f(k_1)$ and τ.

[40] There is no effect of a change of τ on B_2^R, the equilibrium level of borrowing, since under Assumption 4, $B_2^R = S(A, x^*)$ and $\partial S(A, x^*) / \partial \tau = 0$. If Assumption 4 were relaxed, there would be an additional term capturing the potential impact of a change in taxes on government borrowing in equilibrium (via the competition effect among lenders).

with long-term debt is analogous. Note, first, that when the government engages in a fiscal stimulus financed with short-term debt that matures at the end of period 1, government spending in repayment in period 1 becomes $\tau Y_1^R - B_1 + B_2^R - \left(1 + r^{ST}\right) sk_1$, where sk_1 is the size of the stimulus. The positive effect of such a stimulus is that it leads to expansion of output. Differentiating both sides of the government indifference condition with respect to s, and rearranging, we get

$$\underbrace{\frac{\partial Y_1}{\partial s}\tau(1-Z)u_{g_1}^D + \frac{\partial Y_2}{\partial s}\tau(1-Z)u_{g_2}^D}_{\text{Differential increase in tax revenues}} + \underbrace{\tau\frac{\partial Y_1}{\partial s}\left[u_{g_1}^R - u_{g_1}^D\right] + \tau\frac{\partial Y_2}{\partial s}\left[u_{g_2}^R - u_{g_2}^D\right]}_{\text{Concavity effect}} - \underbrace{\left(1 + r^{ST}\right)k_1 u_{g_1}^R}_{\text{Increase in debt}}$$

When the government engages in a stimulus, $k_2^i = (1-\tau)e^{A_i}f(k_1(1+s))\Lambda(A_i;\varepsilon,A^*)$, and, thus,

$$\frac{\partial Y_1}{\partial s} = \frac{\alpha}{1+s}Y_1^R \text{ and } \frac{\partial Y_2}{\partial s} = \frac{\alpha^2}{1+s}Y_2^R$$

Following steps similar to those in the proof of Proposition 3, one can simplify the above equation for the effect of the stimulus to

$$\frac{\alpha(B_1 - B_2)}{\tau Y_1^R - B_1 + B_2} + \frac{\alpha^2(1+r)B_2}{\tau Y_2^R - (1+r)B_2} - \frac{\left(1 + r^{ST}\right)k_1}{\tau Y_1^R - B_1 + B_2} \tag{15}$$

To establish the proposition, note, first, that $u_{g_1}^R \geq (1+r)u_{g_2}^R$ (with equality only if the government can borrow the unconstrained optimal amount) and, thus,

$$\alpha(B_1 - B_2)u_{g_1}^R + \alpha^2(1+r)B_2 u_{g_2}^R - \left(1 + r^{ST}\right)k_1 u_{g_1}^R < \left[\alpha B_1 - \left(1 + r^{ST}\right)k_1\right]u_{g_1}^R,$$

where the right-most expression is negative as long as $B_1/k_1 < \left(1 + r^{ST}\right)(1/\alpha)$. Moreover,

$$\alpha(B_1 - B_2)u_{g_1}^R + \alpha^2(1+r)B_2 u_{g_2}^R - \left(1 + r^{ST}\right)k_1 u_{g_1}^R > u_{g_1}^R\left[\alpha(B_1 - B_2) - \left(1 + r^{ST}\right)k_1\right],$$

where the last term is positive for sufficiently high B_1 (as $B_2 < b \in \mathbb{R}$). Thus, for sufficiently high B_1, a stimulus increases the government's incentives to repay its debt. Finally, by continuity of the expression in 15, we know that there exists B_1 such that this expression is equal to 0 and, hence, $\partial A^*/\partial s = 0$. It is easy to see that at such B_1, the derivative of expression in 15 is positive, which implies that there exists a unique $\overline{B}_1$ such that if $B_1 < \overline{B}_1$, then, stimulus decreases the government's incentives to repay its debt, while the opposite is true when $B_1 > \overline{B}_1$. Finally, note that since $\alpha < 1$ and $\tau Y_1^R - B_1 + B_2 < \tau Y_2^R - (1+r)B_2$, it follows that the expression in 15 is necessarily negative if $\alpha B_1 < \left(1 + r^{ST}\right)k_1$. This establishes the proposition.

When the stimulus is financed with long-term debt, then the last term of the expression in 15 becomes $\left(1 + r^{LT}\right)k_1/\left(\tau Y_2^R - (1+r)B_2\right)$. It follows that in this case, expression 15 is necessarily negative when $\alpha B_1 u_{g_1}^R < \left(1 + r^{ST}\right)k_1 u_{g_2}^R$. ■

B.1 Higher tax rate in repayment only

In this section, I consider the case in which the government implements an increase in taxes only if it repays the debt. Let τ^R denote the tax rate in repayment and τ^D the tax rate in default where, initially, $\tau^R = \tau^D = \tau$. An increase in the tax rate only in repayment is captured by an increase in τ^R holding τ^D constant.

An increase in τ^R can be analyzed the same way as an increase in τ considered above. A higher τ^R leads to a change in the government's incentives to repay the debt equal to

$$\underbrace{Y_1^R u_{g_1}^R + Y_2^R u_{g_2}^R}_{\substack{\text{Increase in the government's} \\ \text{revenue in repayment}}} \underbrace{- \int_{i=0}^{1} \left(u_{c_1}^R \frac{\partial c_1^R}{\partial \tau^R} + u_{c_2}^R \frac{\partial c_2^R}{\partial \tau^R} - u_{c_1}^D \frac{\partial c_1^D}{\partial \tau^R} - u_{c_2}^D \frac{\partial c_2^D}{\partial \tau^R} \right) di}_{\substack{\text{Differential decrease} \\ \text{in private consumption}}} \underbrace{-\tau^R \frac{\partial Y_2^R}{\partial \tau^R} \left(u_{g_2}^R - Z u_{g_2}^D \right)}_{\text{Investment distortion}}, \tag{16}$$

where, u_{gt}^R and u_{gt}^D are the marginal utilities from the government spending in period t in repayment and default, respectively, u_{ct}^R and c_t^R are household i's marginal utility from the private consumption and private consumption at time t in repayment, and Y_t^R is the total output of the economy in period t in repayment, all evaluated at the threshold A^*. If the expression in (16) is positive, then the government's incentives to repay its debt increase following an increase in τ^R.

There are three noticeable differences compared to the case when the tax rate is increased in both repayment and default. First, a higher τ^R increases government tax revenues only in repayment, which tends to increase the government's incentives to repay the debt more than in the earlier case. On the other hand, a higher τ^R decreases the government's incentives to repay by decreasing private consumption in repayment by more than in default (private consumption in default is affected indirectly through the change in households' investment strategies). Finally, the investment distortion effect, while still present, is now smaller since, at the time that they make their investment decisions, the households are uncertain whether the announced tax increase will be implemented.

While a choice whether to increase the tax rate only in repayment or both in repayment and in default is most likely determined by the political constraints, it is of interest to compare the effect of increasing τ^R against increasing the tax rate in both repayment and default. The following proposition establishes that when the initial tax rate is low, an increase only in τ^R leads to a larger increase in the government's incentives to repay then does an increase in both τ^R and τ^D, while the opposite is true when the initial tax rate is high.[41]

[41] The Proof of Proposition B.1 can be found in the "Additional Results" document available on

Proposition B.1 *Let* $\frac{\partial \Delta V}{\partial \tau^R}$ *and* $\frac{\partial \Delta V}{\partial \tau}$ *denote the effect on the government incentives of increasing the tax rate only in repayment and both in repayment and in default, respectively. Then, there exists* $\underline{\underline{\tau}}$ *and* $\overline{\overline{\tau}}$*, with* $0 < \underline{\underline{\tau}} < \overline{\overline{\tau}} < 1$ *such that*

1. *If* $\tau > \overline{\overline{\tau}}$ *then* $\frac{\partial \Delta V}{\partial \tau^R} < \frac{\partial \Delta V}{\partial \tau}$.

2. *If* $\tau < \underline{\underline{\tau}}$ *then* $\frac{\partial \Delta V}{\partial \tau^R} > \frac{\partial \Delta V}{\partial \tau}$.

To understand this result, note that when the tax rate is initially low, households' private consumption is relatively high while government spending is relatively low. Thus, the negative effect of higher τ^R on the utility from the private consumption in repayment is small, while the the positive effect of higher τ^D on the utility from the government spending would be high. It follows that at if, initially, both $\tau^R = \tau^D = \tau$ where τ is low, then increasing only τ^R has a larger effect on government's incentives to repay than does increasing both τ^R and τ^D at the same time; the opposite is true when the initial tax level is low.

C Policy Adjustments under Uncertainty

To derive the change in the default threshold when households and lenders are uncertain as to whether the policy change will be implemented, I start by considering a situation in which, with probability $(1-p)$, the policy parameter takes value ψ (which I associate with the case when the policy change is not implemented) and, with probability p, the policy parameter takes value ψ' (which I associate with the new level of the policy parameter if the policy is implemented). I then follow the same steps as in the proof of Proposition 2 to compute the effect of a further change in ψ'. Finally, I impose the condition that, initially, $\psi' = \psi$. By following these steps, I obtain the effect of an announcement of a change in the policy parameter when such a change will take place with probability p.

Let A^* be the threshold if the policy parameter takes value ψ (i.e., the policy change is not implemented) and $A^{*\prime}$ the policy threshold when the policy parameter takes value ψ' (i.e., the policy change is implemented).[42] Then, the equilibrium conditions can be written as

$$
\begin{aligned}
&(17) \qquad & (1-p)\, I\left(A^* + \kappa\varepsilon, A^*, k_2^*(\kappa), \psi\right) + pI\left(A^* + \kappa\varepsilon, A^{*\prime}, k_2^*(\kappa), \psi'\right) &= 0 \\
&(18) \qquad & (1-p)\, I\left(A^{*\prime} + \kappa\varepsilon, A^*, k_2^{*\prime}(\kappa), \psi\right) + pI\left(A^{*\prime} + \kappa\varepsilon, A^{*\prime}, k_2^{*\prime}(\kappa), \psi'\right) &= 0 \\
&(19) \qquad & (1-p)\, L\left(A^*, x^*, \psi\right) + pL\left(A^{*\prime}, x^*, \psi'\right) &= 0 \\
&(20) \qquad & \Delta V\left(A^*; \{k_2^*(\kappa)\}_{\kappa\in[-1,1]}, x^*, A^*, \psi\right) &= 0 \\
&(21) \qquad & \Delta V\left(A^{*\prime}; \{k_2^*(\kappa)\}_{\kappa\in[-1,1]}, x^*, A^{*\prime}, \psi'\right) &= 0,
\end{aligned}
$$

the author's website `https://econ.sites.olt.ubc.ca/files/2016/01/pdf_szkup_debt_crises_additional.pdf`.

[42] For example, if the relevant policy parameter is a tax rate τ, and the government contemplates increasing the tax rate to $\tau' > \tau$, then $\psi = \tau$ while $\psi' = \tau'$.

where $k_2^*(\kappa)$ denotes an individual household's equilibrium investment when that household's productivity is equal to $A^* + \kappa\varepsilon$, while $k_2^{*\prime}$ denotes the individual household's equilibrium investment when that household's productivity is equal $A^{*\prime} + \kappa\varepsilon$.

In the presence of political uncertainty, there are additional equilibrium equations compared to the case considered in Section B of this appendix. This is because we need to determine the default threshold both when the policy in implemented and when it is not (the possibility of a policy change also affects the threshold even if, in the end, the policy is not implemented). In particular, to compute the equilibrium default threshold when the policy parameter takes value ψ, we need both the government's default condition and the household investment decisions evaluated at ψ (Equations 17 and 20). Similarly, to compute the equilibrium default threshold when the policy parameter takes value ψ', we need both the government's default condition and the household investment conditions evaluated at ψ' (Equations 18 and 21).

To compute $dA^*/d\psi$ and $dA^{*\prime}/d\psi'$, one can follow an approach similar to the one in Section B of this appendix; that is, consider the total derivatives of all equilibrium conditions with respect to ψ'. Solving the resulting system of equations for $dA^*/d\psi$ and $dA^{*\prime}/d\psi'$ and evaluating all derivatives at $\psi = \psi'$ (since we consider a small policy change from its initial level at ψ) yield the desired result.[43]

Proof of Proposition 5. That $dA^*/d\psi = \partial A^*/\partial\psi$ when a policy change in unexpected is immediate from the discussion of Proposition 5 in the main text. Thus, it remains to show that $\lim_{\sigma_x,\varepsilon\to 0} \partial A^*/\partial\psi = 0$. For simplicity, I consider the case when only $\varepsilon \to 0$.

Note that $\lim_{\varepsilon\to 0} \frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^*}\Big|_{A^{**}=A^*} = \infty$ (see Claim 6 in Section G of this appendix), and, thus, from the expression for $k_2^*(A^* + \kappa\varepsilon, \kappa, A^{**})$, we obtain $\lim_{\varepsilon\to 0} \frac{\partial k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})}{\partial A^*}\Big|_{A^{**}=A^*} = \infty$. From this, it follows that $\Delta V_1 + \int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa \to \infty$ as $\varepsilon \to 0$. Now, recall from the proof of Proposition 2 that

$$\frac{\partial A^*}{\partial \psi} = -\frac{\Delta V_4}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa}$$

$\Delta V_4 = \partial\Delta V/\partial\psi$ is well-defined for each parameter of the model and, hence, finite. It follows that $\lim_{\varepsilon\to 0} \partial A^*/\partial\psi = 0$. ■

Proof of Corollary 2. Corollary 2 follows from Propositions $3-5$. ■

Proof of Proposition 6. Proposition 6 follows from Equation (5) in the paper, which states that $\frac{dA^*}{d\psi}(p) = p\frac{dA^*}{d\psi}(1) + (1-p)\frac{\partial A^*}{\partial\psi}$, Proposition 5 and Corollary 2. ■

[43] The detailed derivations can be found in the "Additional Results" document available on the author's website `https://econ.sites.olt.ubc.ca/files/2016/01/pdf_szkup_debt_crises_additional.pdf`.

D Discussion of Assumption 5

The above analysis was conducted under Assumption 5, stated in the text. To determine a bound on B_1, which is assumed implicitly in Assumption 5, assume that interest rate r is less than $\widehat{r}$ for some arbitrarily high $\widehat{r}$. Note that the unconstrained optimal borrowing by the government in repayment is given by

$$B_2^{R,u} = \frac{(1+r)\,B_1 + \tau Y_2^R - (1+r)\,\tau Y_1^R}{2\,(1+r)}$$

For a fixed $r < \widehat{r}$, a higher B_1 increases B_2, not only directly, but also indirectly by shifting the lower bound of the fragility region, $\underline{A}\,(r)$, upwards. For sufficiently high $\underline{A}\,(r)$, we have $Y_2^R\,(\underline{A}\,(r)) > Y_1^R\,(\underline{A}\,(r))$, where $Y_t^R\,(A)$ denotes the total output at time t when average productivity is A. Moreover, $\partial Y_2^R/\partial A = (1+\alpha)\,Y_2^R$ and $\partial Y_1^R/\partial A = Y_1^R$, implying that once $\underline{A}\,(r)$ is high enough so that $Y_2^R\,(\underline{A}\,(r)) > Y_1^R\,(\underline{A}\,(r))$, a further increase in $\underline{A}\,(r)$ leads to an increase in $\tau Y_2^R - (1+r)\,\tau Y_1^R$ and, hence, in the desired borrowing. It follows that for a fixed b and a fixed r, there exists a high enough B_1 such that $B_2^{R,u} > b$. Since $[0, \widehat{r}]$ is a compact interval there exists a high enough B_1, call it $\widehat{B}_1$, such that if $B_1 > \widehat{B}_1$, then $B_2^{R,u} > b$ for all $r \in [0, \widehat{r}]$.

Assumption 5 simplifies the lender's problem. The difficult part of the lender's problem is the competition effect: Ceteris paribus, a higher supply of funds in the bond market decreases the lenders' expected return from lending. This effect, however, is not present when $B_2^{R,u} > b$, in which case there exists a closed-form solution for x^*. In particular, under Assumption 5, it is easy to show that

$$x^* = \frac{p_x + p_A}{p_x} A^{**} - \frac{p_A}{p_x} A_{-1} + \frac{\sqrt{p_x + p_A}}{p_x} \Phi^{-1}\left(\frac{1}{1+r}\right)$$

This, in turn, substantially simplifies the analysis presented in Sections 4 and 5 of the paper.[44]

D.1 Policy Analysis without Assumption 5

Without Assumption 5, a change in households' investment strategies will affect the lenders' equilibrium behavior. This is because the government's desired unconstrained borrowing, $B_2^{R,u}$, depends on Y_2, and a change in $B_2^{R,u}$ translates into a change in x^*. Thus, the lenders' indifference condition has to be written as

$$L\,(A^{**}, x^*, \psi, k_2) = 0,$$

[44] Assuming that lenders ignore the competition effect would have the same implications.

rather than as $L(A^{**}, x^*, \psi) = 0$. This is the only change compared to the case when Assumption 5 is imposed. Following the same steps, one can show that

$$\frac{dA^*}{d\psi} = \frac{\frac{\partial A^*}{\partial \psi} + \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi}di + \frac{\partial A^*}{\partial x^*}\left[\frac{\partial x^*}{\partial \psi} + \int_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi}di\right]}{1 - \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}}di - \frac{\partial A^*}{\partial x^*}\left[\frac{\partial x^*}{\partial A^{**}} + \int_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}}di\right]}$$

Compared to the case in which Assumption 5 holds, there is an additional term in the expression for the direct effect, $\frac{\partial A^*}{\partial x^*}\int\limits_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi}di$. This is because a change in ψ leads to an adjustment in the households' investment, which affects the government's desired borrowing. Without Assumption 5, there is a "competition effect": A higher supply of funds to the bond market tends to mean less lending per lender. Thus, a change in the households' investment strategies leads to an adjustment in x^*. Similarly, the beliefs effect has an additional term equal to $\frac{\partial A^*}{\partial x^*}\int\limits_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}}di$ since, now, a change in households' expectations affects the lenders' behavior through its impact on the government's desired borrowing.

There are two main reasons why, in the paper, I consider a case in which Assumption 5 holds. First, Assumption 5 substantially simplifies the subsequent analysis. This is particularly true when considering effects of an increase in taxes and of a fiscal stimulus, or when deriving an expression for $dA^*/d\psi$, since $\int_0^1 \left[\partial x^*/\partial k_2^i\right]\left[\partial k_2^*/\partial \psi di\right]$ is a complicated object and can be computed only implicitly. Second, numerical simulations suggest that the competition effect, which is assumed away when Assumption 5 is imposed, plays only a minor role in determining the desirability of a particular policy.

E The Effect of the Interest Rate on Policy Adjustments

Above, I analyzed the case in which the policy change takes place after the interest rate has been set, and, thus, the change in the policy and the resulting change in the default threshold A^* do not affect the interest rate r. In this section, I analyze what happens when the policy change is announced before the government chooses the interest rate, in which case I have to take into account how a policy change affects the choice of interest rate and how this change in the interest rate affects the default threshold.

Recall that the government chooses the interest rate to maximize the ex-ante welfare. The optimal interest rate is, then, the solution to the first-order condition associated with this problem, which can be written as

$$R(A^*, k_2, x^*, \psi, r^*) = 0$$

Here, we recognize that r^* depends on the government's future decisions, households' investment choices, and lenders' supply decisions. The choice of r^* is also affected by the policy parameters, since ψ affects the gains and costs associated with a higher r.

Following the same approach as in Section B of this appendix, I find that the total effect of a change in policy ψ on the default threshold is given by

$$\frac{dA^*}{d\psi}=M_{Total}\left[1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right]\left[\frac{\frac{\partial A^*}{\partial \psi}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2^i}{\partial \psi}di}{1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right]$$
$$+M_{Total}\left[1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right]\left[\frac{\left(\frac{\partial A^*}{\partial r}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi}+\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)}{1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right],$$

where M_{Total} is the (total) beliefs effect that is present in the model when r can adjust; it is given by

$$M_{Total}=\frac{1}{1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di-\frac{\left(\frac{\partial A^*}{\partial r}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)}{1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial r}}\left(\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}+\int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right)}.[45]$$

To understand the above expression, note first that $\left[1-\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the beliefs effect in the case when we hold the interest rate constant, and $\left[1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}-\int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the beliefs effect when the government's default decision is affected by the change in households' beliefs only through an implied adjustment in the interest rate. Then, the first term in the expression for $dA^*/d\psi$ captures the change in the default threshold implied by a change in the policy, holding the interest rate constant (the expression in the square brackets), weighted by the relative importance of the "partial" beliefs effect (i.e., beliefs effect when r is kept constant, as in Section 4 of the paper) compared to the total beliefs effect, M_{Total}. This effect is familiar from the earlier analysis. The second term captures the total change in the default threshold implied by the adjustment in r^*. Here, $\left(\frac{\partial A^*}{\partial r}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi}+\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)$ captures the effect that an adjustment in ψ has on r^* (and, hence, on A^*), holding households' and lenders' expectations constant: A change in ψ leads to a change in r^*, which then affects A^*. This effect is then reinforced by the associated beliefs effect that results from the initial adjustment in r^* and is adjusted by the relative importance of its partial beliefs effect.

How does an adjustment in r^* alter the effectiveness of various government policies compared to the case when r^* is constant? While it is difficult to answer this question analytically, intuition suggests that an adjustment in r^* tends to decrease the magnitude of the change in A^* implied by ψ as long as the default threshold A^* is lower than A_{-1}, the prior of the mean belief about A. To understand this, note that a decrease in A^* decreases the benefit of a higher r (since a lower A^* means that a further decrease in A^* due to the choice of higher r translates into a lower decrease in the probability of default) and increases the cost of a higher r (since a fall in A^* implies that the government has to incur the cost of a higher r for a larger set of productivity values). The opposite is true when A^* increases. This suggests that a policy

[45] The above expression can be derived by following the same steps as in Section $B.1$.

change that leads to a decrease in A^* is accompanied by a decrease in r^*, which decreases the positive effect of the policy adjustment. On the other hand, a policy change that leads to an increase in A^* is accompanied by an increase in r^*, which tends to partially offset the negative effect that such a policy has on the probability of default.

F Proofs for Section 7

Derivations of Equation (6). The lower bound for the fragility region is the unique solution to

$$\Delta V_1^R\left(\underline{A}, \mathbf{k}_2\left(\underline{A}\right), b\right) \equiv V_1^R\left(\underline{A}, \mathbf{k}_2^R\left(\underline{A}\right), b\right) - V_1^D\left(\underline{A}, \mathbf{k}_2^R\left(\underline{A}\right), b\right) = 0,$$

where $V_1^R\left(A, \mathbf{k}_2, S\right)$ and $V_1^D\left(A, \mathbf{k}_2, S\right)$ are defined as in Section 3.2, $\mathbf{k}_2^R\left(\underline{A}\right) = \left\{k_2^{R,i}\left(\underline{A}\right)\right\}_{i\in[0,1]}$ is the profile of households' investment when $A = \underline{A}$ with $k_2^{R,i}\left(\underline{A}\right) = (1-\tau)\, e^{\underline{A}+\kappa\varepsilon_i} f\left(k_1\right) \frac{\alpha}{1+\alpha}$, and b is the total wealth of lenders. Applying the implicit function theorem to the above equation and defining

$$\frac{\partial \underline{A}}{\partial \psi} \equiv -\frac{\frac{\partial \Delta V_1^R\left(\underline{A}, \mathbf{k}_2^R(\underline{A}), b\right)}{\partial \psi}}{\frac{\partial \Delta V_1^R\left(\underline{A}, \mathbf{k}_2^R(\underline{A}), b\right)}{\partial \underline{A}} + \int_{i=0}^{1} \frac{\partial V_1^R\left(\underline{A}, \mathbf{k}_2^R(\underline{A}), b\right)}{\partial k_2^{R,i}} \frac{\partial k_2^{R,i}}{\partial \underline{A}} di}$$

and

$$\int_{i=0}^{1} \frac{\partial \underline{A}}{\partial k_2^{R,i}} \frac{\partial k_2^{R,i}}{\partial \psi} di \equiv -\frac{\int_{i=0}^{1} \frac{\partial \Delta V_1^R\left(\underline{A}, \mathbf{k}_2^R(\underline{A}), b\right)}{\partial k_2^{R,i}} \frac{\partial k_2^{R,i}}{\partial \psi} di}{\frac{\partial \Delta V_1^R\left(\underline{A}, \mathbf{k}_2^R(\underline{A}), b\right)}{\partial \underline{A}}},$$

we obtain Equation (6) in the text.[46] ■

Derivations of Equation (7). Suppose that households are uncertain whether the government will implement the announced policy. Let $k_2^{R,i}\left(\underline{A}, \psi; p\right)$ be the households' investment when agents believe that a new policy will be implemented with probability p. Then, using the households' problem and following the same approach as in the model with dispersed information, it is straightforward to show that

$$\frac{\partial k_2^{R,i}\left(\underline{A}, \psi; p\right)}{\partial \psi} = p \frac{\partial k_2^{R,i}\left(\underline{A}, \psi; 1\right)}{\partial \psi}$$

The decomposition in Equation (7) follows immediately from this observation. ■

Proof of Proposition 7. Consider, first, the model with dispersed information. An increase in taxes and fiscal stimulus decrease the probability of default if the expressions in Equation

[46] Note that $\partial \underline{A}/\partial \psi$ and $\int_{i=0}^{1}\left(\partial \underline{A}/\partial k_2^{R,i}\right)\left(\partial k_2^{R,i}/\partial \psi\right)$ are defined in an analogous way as in the derivation of Equation (2).

(6) and Equation (7) are satisfied, respectively. Taking the limit as $A_{-1} \to 0$, these expressions converge to

$$\frac{B_1}{\tau Y_1 - B_1} \quad \text{and} \quad \frac{\alpha B_1 - \left(1 + r^{ST}\right) k_1}{\tau Y_1 - B_1},$$

respectively, since as $A_{-1} \to 0$, lenders' optimal signal threshold converges to infinity, implying that the supply of funds in the bond market converges to 0.

It remains to derive the relevant conditions in the model with only fundamental crises. Note that the derivations of Equations (3) and (4) are also applicable in the model with fundamental crises, with the difference that all endogenous variables are evaluated at $A = \underline{A}$ rather than at $A = A^*$. Furthermore, note that in the model with only fundamental crises, the government can always borrow the desired amount so that

$$\frac{1}{\tau Y_1 \left(\underline{A}\right) - B_1 + B_2^* \left(\underline{A}\right)} - \frac{(1 + r)}{\tau Y_2 \left(\underline{A}\right) - (1 + r) B_2^* \left(\underline{A}\right)} = 0$$

This implies that the relevant conditions under which an increase in taxes decreases the probability is

$$\frac{B_1}{\tau Y_1 \left(\underline{A}\right) - B_1 + B_2^* \left(\underline{A}\right)} - \frac{\alpha \tau}{1 - \tau} \frac{(1 + r) B_2^* \left(\underline{A}\right)}{\tau Y_1 \left(\underline{A}\right) - (1 + r) B_2} = \frac{B_1 - \frac{\alpha \tau}{1 - \tau} B_2^* \left(\underline{A}\right)}{\tau Y_1 \left(\underline{A}\right) - B_1 + B_2^* \left(\underline{A}\right)},$$

while the relevant condition for fiscal stimulus is

$$\frac{\alpha B_1 - \left(1 + r^{ST}\right) k_1 - \left(\alpha - \alpha^2\right) B_2^* \left(\underline{A}\right)}{\tau Y_1 \left(\underline{A}\right) - B_1 + B_2^* \left(\underline{A}\right)}$$

The proposition follows by comparing the conditions under which fiscal policies decrease the probability of default in both models. ■

Proof of Proposition 3. This follows immediately from Propositions 5 and 6 and Equation (7). ■

G Intermediate and Technical Claims

In this section, I provide proofs of several results that have been invoked throughout this appendix. First, I show that $\partial x^* / \partial A^* < \frac{p_x + p_A}{p_x}$. Then, I compute limits of several expressions as $\varepsilon, \sigma_x \to 0$ and which where used in the proof of Proposition 2.

Lemma 4 *The derivative of x^* with respect to A^* is bounded from above by $\frac{\sigma_x^2 + \sigma_A^2}{\sigma_A^2}$.*

Proof. Applying the implicit function theorem to the lenders' indifference condition, we get

$$\frac{dx^*}{dA^*} = -\frac{(-1)\left(1 + r \min\left\{1, \frac{B_2^u(A^*)}{S(A^*, x^*)}\right\}\right) f\left(A^* | x^*\right)}{\frac{\partial}{\partial x^*}\left[\int\limits_{A^*}^{\infty} \left(1 + r \min\left\{1, \frac{B_2^u(A)}{S(A.x^*)}\right\}\right) f\left(A | x^*\right) dA\right]},$$

where $f(A|x)$ is the conditional density of A given lender j observed signal $x_j = x^*$. Define $\mathcal{A}^u = \left\{A \geq A^* | B_2^{R,u}(A) < S(A)\right\}$ and $\mathcal{A}^c = \left\{A \geq A^* | B_2^{R,u}(A) \geq S(A)\right\}$, and note that $B_2^{R,u}(A)$ and $S(A)$ intersect at most finitely many times. Without loss of generality, I assume that $B_2^{R,u}(A)$ and $S(A)$ intersect at least once (otherwise, the result follows immediately). Then we can write $\mathcal{A}^u$ and $\mathcal{A}^c$ as $\mathcal{A}^u = \cup_{i=1}^{N_u}\left[A_{i0}^u, A_{i1}^u\right]$ and $\mathcal{A}^c = \cup_{i=1}^{N_c}\left(A_{i0}^c, A_{i1}^c\right)$, where $N_u, N_c \in \mathbb{N}$, $\left\{A_{i0}^u\right\}_{i=1}^{N_u}$ are the values of the productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from above and $\left\{A_{i1}^u\right\}_{i=1}^{N_u}$ are the values of productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from below.[47] With these definitions, we can write the above derivative as

$$\frac{dx^*}{dA^*} = \frac{\left(1 + r\min\left\{1, \frac{BB_2^{R,u}(A)}{S(A^*,x^*)}\right\}\right) f(A^*|x^*)}{\sum \frac{\partial}{\partial x^*}\int\limits_{A_{i_0}^c}^{A_{i_1}^c}(1+r) f(A|x^*)\, dA + \sum \frac{\partial}{\partial x^*}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\left\{1 + r\frac{B_2^{R,u}(A)}{S(A,x^*)} f(A|x^*)\right\} dA}$$

Consider the case where at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. Then the denominator becomes:

$$\begin{aligned}
&\sum_{i=1}^{N_u}\int\limits_{A_{i_0}^c}^{A_{i_1}^c}\frac{\partial}{\partial x^*}(1+r) f(A|x^*)\, dA + \sum_{i=1}^{N_c}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\frac{\partial}{\partial x^*}\left\{1 + r\frac{B_2^{R,u}(A)}{S(A,x^*)} f(A|x^*)\right\} dA \\
= &\int\limits_{A^*}^{\infty}\frac{\partial}{\partial x^*}(1+r) f(A|x^*)\, dA + \sum_{i=1}^{N_c}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\frac{\partial}{\partial x^*}\left\{r\left(\frac{B_2^{R,u}(A)}{S(A,x^*)} - 1\right) f(A|x^*)\right\} dA
\end{aligned}$$

It remains to show that the second of the above terms is positive. Intuitively, that is what we expect, since a higher x^* makes high values of A more likely and $B_2^u(A)$ is increasing in A. The remainder of this proof is devoted to establishing it analytically.

The idea of the next few steps is to change differentiation with respect to x^* with the differentiation with respect to A. First, note that, since $f(A|x^*) = (p_A + p_x)^{1/2}\phi\left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{(p_A + p_x)^{-1/2}}\right)$, we have

$$\int\limits_{A^*}^{\infty}\frac{\partial}{\partial x^*}(1+r) f(A|x^*)\, dA = -\frac{p_x}{p_x + p_A}\int\limits_{A^*}^{\infty}\frac{\partial}{\partial A}(1+r) f(A|x^*)\, dA = \frac{p_x}{p_x + p_A}(1+r) f(A^*|x^*)$$

Next, let $H(A, x^*) = \left(\frac{B_2^u(A)}{S(A,x^*)} - 1\right) f(A|x^*)$. Then,

$$\frac{\partial}{\partial x^*}H(A,x^*) = -\frac{p_x}{p_x + p_A}\frac{\partial}{\partial A}H(A,x^*) + \frac{\partial B_2^{R,u}(A)}{\partial A}\frac{1}{S(A,x^*)} f(A|x^*) - \frac{p_A}{p_x + p_A}\frac{B_2^{R,u}(A)}{S(A,x^*)}\frac{\frac{\partial}{\partial x^*}S(A,x^*)}{S(A,x^*)},$$

[47] If at A^* we have $S(A,x^*) > B_2^{R,u}(A)$, then $A_{1_0}^u = A^*$, $A_{1_1}^u = A_{1_0}^c$, $A_{i_1}^c = A_{2_0}^u$, and so on. If at A^* we have $S(A,x^*) < B_2^{R,u}(A)$ then $A_{1_0}^c = A^*$, $A_{1_1}^c = A_{1_0}^u$, $A_{i_1}^u = A_{2_0}^c$, and so on.

where, since $\partial B_2^{R,u}(A)/\partial A > 0$ and $\frac{\partial}{\partial x^*} S(A, x^*) < 0$, the last two terms are strictly positive. Moreover, note that for $i = 1, ..., N_c$ we have $H\left(A_{i_1}^u, x^*\right) = H\left(A_{i_0}^u, x^*\right) = 0$.

Therefore,

$$
\begin{aligned}
& \sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} \frac{\partial}{\partial x^*} \left\{ r \left(\frac{B_2^{R,u}(A)}{S(A, x^*)} - 1 \right) f(A|x^*) \right\} dA \\
> & \sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} -\frac{p_x}{p_x + p_A} \frac{\partial}{\partial A} H(A, x^*) \, dA \\
= & -\frac{p_x}{p_x + p_A} \sum_{i=1}^{N_c} \left[H\left(A_{i_1}^u, x^*\right) - H\left(A_{i_0}^u, x^*\right) \right] = 0
\end{aligned}
$$

This establishes the claim for the conclusion of the Lemma when at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. The case when $B_2^u(A^*) < S(A^*, x^*)$ is established in an analogous way. ■

Claim 5 $\lim_{\varepsilon \to 0} \frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} = 0$

Proof. Note that

$$
\begin{aligned}
& \frac{\partial \Pr(A^*|A^* + \kappa\varepsilon)}{\partial A^*} \\
= & \frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)} + \\
& + \frac{\left[\frac{1}{\sigma_A}\phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]\left[\Phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]^2}
\end{aligned}
$$

Taking the limit as $\varepsilon \to 0$ and using l'Hôpital's rule one can show that the first term converges to $\frac{A^*-A_{-1}}{\sigma_A}\frac{(1-\kappa)}{2}$ while the second term converges to $-\frac{A^*-A_{-1}}{\sigma_A}\frac{(1-\kappa)}{2}$. It follows that $\lim_{\varepsilon \to 0} \frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} = 0$. ■

The next two claims have been used in the proof of Proposition 2.

Claim 6 $\lim_{\varepsilon \to 0} \left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*} = \infty$ *and* $\lim_{\varepsilon \to 0} \frac{\left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^*}\right|_{A^{**}=A^*}}{\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}} = -1$

Proof. If $A^{**} \in (A^* - (1-\kappa)\varepsilon, A^* + (1+\kappa)\varepsilon)$, then

$$
\Pr(A^{**}|A^* + \kappa\varepsilon) = \frac{\Phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}
$$

Differentiating with respect to A^{**}, we get

$$
\frac{\partial \Pr(A^{**}|A^* + \kappa\varepsilon)}{\partial A^{**}} = \frac{\phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}
$$

Taking the limit as $\varepsilon \to 0$ at $A^* = A^{**}$, we get

$$\lim_{\varepsilon \to 0} \frac{\partial \Pr\left(A^{**}|A^{**}+\kappa\varepsilon\right)}{\partial A^{**}} = \infty$$

Next, consider

$$\frac{\left.\frac{\partial \Pr(A^{**}|A^{*}+\widehat{\kappa}\varepsilon)}{\partial A^{*}}\right|_{A^{**}=A^{*}}}{\left.\frac{\partial \Pr(A^{**}|A^{*}+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^{*}}} = -\frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}} - \frac{\frac{\left[\Phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]\left[\frac{1}{\sigma_A}\phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]^2}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}}$$

Using l'Hôpital's rule, one can establish that

$$\lim_{\varepsilon \to 0} -\frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\frac{\phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}} = -1$$

Similarly, using l'Hôpital's rule, one can show that the second term converges to 0. ■

Claim 7 $\lim_{\sigma_x \to 0} \frac{\partial}{\partial A} \left. S\left(A, x^*\right)\right|_{A=A^*} = \infty$

Proof. Note that

$$S\left(A, x^*\right) = b\left[1 - \Phi\left(\frac{x^* - A}{\sigma_x}\right)\right]$$

Taking the derivative with respect to A, we get

$$\frac{\partial S\left(A, x^*\right)}{\partial A} = \frac{1}{\sigma_x}\phi\left(\frac{x^* - A}{\sigma_x}\right).$$

Under Assumption 4, we have $x^* = \frac{\sigma_x^2+\sigma_A^2}{\sigma_A^2}A^* - \frac{\sigma_x^2}{\sigma_A^2}A_{-1} + \sigma_x^2\sqrt{\frac{1}{\sigma_x^2}+\frac{1}{\sigma_A^2}}\Phi^{-1}\left(\frac{1}{1+r}\right)$, and thus $\lim_{\sigma_x \to 0} \frac{x^*-A}{\sigma_x} = \phi\left(\Phi^{-1}\left(\frac{1}{1+r}\right)\right)$. The Claim follows immediately from this observation. ■

Claim 8 *We have* $dA^*/dA_{-1} < 0$.

Proof. In light of Proposition 2, it is enough to consider the direct effect of increasing A_{-1}. By inspection, we see that A_{-1} does not directly affect the government incentives to default. Next, note that A_{-1} affects household i's investment choice only through its impact on $P\left(A^{**}|A_i\right) \equiv \Pr\left(A < A^{**}|A_{-1}, A_i\right)$, and hence on $\Lambda\left(A_i; \varepsilon, A^{**}\right)$. Since $\Lambda\left(A_i; \varepsilon, A^{**}\right)$ is decreasing in $P\left(A^{**}|A_i\right)$ and $P\left(A^{**}|A_i\right)$ is decreasing in A_{-1} it follows that $\partial\Lambda\left(A_i; \varepsilon, A^{**}\right)/\partial A_{-1}$

is increasing in A_{-1}. Thus, an increase in A_{-1} leads to a higher investment by households. Since a higher investment strictly decreases governments' incentives to default we have

$$\int_{i=0}^{1} \frac{\partial A^*}{\partial k_2} \frac{\partial k_2}{\partial A_{-1}} di < 0$$

Next, consider lenders. Recall that the signal threshold above which lenders supply their funds to the bond market is defined implicitly by

$$\int_{A^{**}}^{\infty} \Delta u \left(A; x^*, A^{**}\right) \left(p_x + p_A\right)^{1/2} \phi \left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{\left(p_x + p_A\right)^{-1/2}} \right) dA = 0$$

where A^{**} denote expected default threshold by the lenders. Then $\partial x^* / \partial A_{-1} = -p_x / p_A < 0$. Thus, an increase in A_{-1} leads to a decrease in x^* implying that lenders supply more funds to the bond market for any given A. Since a higher supply of funds weakly decreases the government's default incentives we have

$$\frac{\partial A^*}{\partial x^*} \frac{\partial x^*}{\partial A_{-1}} \leq 0.^{48}.$$

The result follows then from Proposition 2. ■

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