Measuring the Impact of Free Goods on Real Household Consumption

Erik Brynjolfsson, Avinash Collis, W. Erwin Diewert, Felix Eggers and Kevin J. Fox[1]

January 10, 2020

Discussion Paper 20-01
Vancouver School of Economics
The University of British Columbia
Vancouver, Canada V6T 1L4

Abstract

A puzzling development over the past 15 years is decline in Total Factor Productivity in many advanced economies. Part of this decline may be due to the rapid growth of free digital goods.[2] Statistical agencies have no reliable way to measure the benefits of the introduction of free goods. This is true even when the provision of the goods is paid for via advertising. Yet these free goods are enormously popular and surely create substantial utility for households. In this paper, we suggest a methodology which will allow statistical agencies to form rough approximations to the benefits that flow to households from new free goods. The present paper draws heavily on the contributions of Brynjolfsson, Collis, Diewert, Eggers and Fox (2019) (subsequent references will be to BCDEF) and Diewert, Fox and Schreyer (2019). In section I, we outline how the reservation price methodology introduced by Hicks (1940; 114) can be used to measure the consumption benefits to households of new products that are provided at zero cost or costs that are close to zero. This Hicksian approach relies on normal index number theory but requires the estimation of reservation prices. In section II, we show how choice experiments about compensation for product withdrawals can be used to estimate these reservation prices. Section III concludes with a summary and implications.

Key Words: Welfare measurement, GDP, Productivity, mismeasurement, productivity slowdown, new goods, free goods, online choice experiments, GDP-B.

Journal of Economic Literature Classification Numbers: C43, D60, E23, O3, O4

[1] Brynjolfsson: MIT Sloan School of Management, 100 Main Street, Cambridge MA 02142, and NBER (e-mail: erikb@mit.edu); Collis: MIT Sloan School of Management, 100 Main Street, Cambridge, MA 02142 (e-mail: avinashg@mit.edu); Diewert: UBC, UNSW Sydney and NBER, Vancouver School of Economics, University of British Columbia, 6000 Iona Drive, Vancouver, B.C., Canada, V6T 1L4 (e-mail: erwin.diewert@ubc.ca); Eggers: University of Groningen, Faculty of Economics and Business, Nettelbosje 2, 9747 AE Groningen, The Netherlands (e-mail: f.eggers@rug.nl); Fox: UNSW Sydney, School of Economics, Sydney 2052, Australia (e-mail: k.fox@unsw.edu.au). We thank David Byrne for very helpful comments.

[2] In this paper, we will use the word "goods" to include both goods and services.

1. Basic Index Number Theory and Hicksian Reservation Prices

Suppose that we are given nonnegative, nonzero price and quantity data for T+1 periods for a representative utility maximizing household. The period t price vector is (p_0^t,p^t) and the period t quantity vector is (q_0^t,q^t) for $t = 0,1,...,T$ where p^t and q^t are N dimensional price and quantity vectors and p_0^t and q_0^t are the period t price and quantity for product 0.

The true (fixed base) Fisher (1922) quantity or volume index for period t relative to period 0 is defined as follows, for $t = 1,...,T$:[3]

$$(1)\quad Q_F^t \equiv \{[p_0^0q_0^t + p^0\cdot q^t]/[p_0^0q_0^0 + p^0\cdot q^0]\}^{1/2}\{[p_0^tq_0^t + p^t\cdot q^t]/[p_0^tq_0^0 + p^t\cdot q^0]\}^{1/2}.$$

The ratio of period t nominal consumption to period 0 nominal consumption for the representative household is:

$$(2)\quad V^t/V^0 = [p_0^tq_0^t + p^t\cdot q^t]/[p_0^0q_0^0 + p^0\cdot q^0].$$

If the household has certain preferences,[4] then the Fisher quantity index defined by (1) is equal to the household's utility ratio between periods t and 0; i.e., $Q_F^t = f(q_0^t,q^t)/f(q_0^0,q^0)$.

Suppose that commodity 0 is a new product that was not available in period 0 but is available in periods 1 to T. There is a Hicksian reservation price for product 1 in period 0 that will induce consumers to demand 0 units of the product ($q_0^0 = 0$). Denote this reservation price by $p_0^{0*} > 0$.[5] Suppose further that in periods 1 to T that the new product is *free* so that the prices p_0^t equal 0 for $t = 1,...,T$. Thus, the product 0 prices and quantities satisfy the following assumptions, for $t = 1,...,T$:

$$(3)\quad p_0^{0*} > 0;\ q_0^0 = 0;\ p_0^t = 0;\ q_0^t > 0.$$

[3] Notation: $p^0\cdot q^t \equiv \Sigma_{n=1}^N p_n^0q_n^t$ where $p^0 \equiv [p_1^0,...,p_N^0]$ and $q^t \equiv [q_1^t,...,q_N^t]$.
[4] See Diewert (1976). The utility function must be equal to $[\Sigma_{i=0}^N \Sigma_{k=0}^N a_{ik}q_iq_k]^{1/2}$ or the dual unit cost function must be equal to $[\Sigma_{i=0}^N \Sigma_{k=0}^N b_{ik}p_ip_k]^{1/2}$ where the parameter matrices $A \equiv [a_{ik}]$ and $B \equiv [b_{ik}]$ must satisfy certain restrictions.
[5] See Hicks (1940; 114).

Now substitute (3) into (1) and (2). We find that the true Fisher quantity index for period t relative to period 0, Q_F^t, and the corresponding nominal GDP value ratio, V^t/V^0, are equal to the following expressions, for t = 1,...,T:

$$(4)\quad Q_F^t \equiv \{[p_0^{0*}q_0^t+p^0\cdot q^t]/p^0\cdot q^0\}^{1/2}\{p^t\cdot q^t/p^t\cdot q^0\}^{1/2}$$

$$(5)\quad V^t/V^0 = p^t\cdot q^t/p^0\cdot q^0.$$

Because the price is zero, nominal household consumption growth simply ignores product 0. However, product 0 does play a role in the growth of real household consumption (measured at final demand prices) as equations (4) above indicate. The problem with Q_F^t defined by (4) is that the period 0 reservation price p_0^{0*} is not directly observable. We need a way to estimate the period 0 reservation price p_0^{0*} in order to evaluate (4).[6]

1.1 Bias in Measurement

The *incorrect volume measure* for household consumption in period t that simply ignores product 0 is the fixed base Fisher index Q_F^{t*} defined as follows, for t = 1,...,T:

$$(6)\quad Q_F^{t*} \equiv \{p^0\cdot q^t/p^0\cdot q^0\}^{1/2}\{p^t\cdot q^t/p^t\cdot q^0\}^{1/2}.$$

The ratio of the true volume index Q_F^t defined by (4) to the incorrect volume index Q_F^{t*} defined by (6) is:[7]

$$\begin{aligned}(7)\quad Q_F^t/Q_F^{t*} &= \{[p_0^{0*}q_0^t + p^0\cdot q^t]/p^0\cdot q^t\}^{1/2}\ ;\\ &= [1 + (p_0^{0*}q_0^t/p^0\cdot q^t)]^{1/2}\\ &\approx 1 + (½)(p_0^{0*}q_0^t/p^0\cdot q^t) > 1.\end{aligned}$$

[6] Feenstra (1994) developed a way to implement methods for estimating reservation prices in the context of consumers having CES preferences. Diewert and Feenstra (2019) developed methods for estimating reservation prices in the context of preferences that are exact for the Fisher price and quantity index but their methods are far from easy to implement. Hausman (1996, 1999) also estimated reservation prices and in addition, he used consumer surplus concepts to measure the benefits of new products.

[7] A first order Taylor series approximation to $g(x) \equiv (1+x)^{1/2}$ around x = 0 is 1 + (½)x which is the approximation used in (7).

If *the hybrid share of product 0 in period t*, $p_0^{0*}q_0^t/p^0\cdot q^t$, grows over time (e.g., due to the growth of free goods, q_0^t) the downward bias in the incorrect Fisher index Q_F^{t*} will grow.

The inequalities defined by (7) are very simple and easy to explain if the Hicksian reservation price methodology is accepted.

It is straightforward to modify the methodology to deal with the situation where the price of the new product in periods 1 to T is not zero: replace assumptions (3) with the following assumptions, t = 1,...,T:

$$(8)\quad p_0^{0*} > 0;\ q_0^0 = 0;\ p_0^t > 0;\ q_0^t > 0.$$

The new value ratio and the new true Fisher quantity index Q_F^{t**} are defined as follows:

$$(9)\quad V^t/V^0 = [p_0^tq_0^t + p^t\cdot q^t]/p^0\cdot q^0; \quad t = 1,...,T;$$

$$\begin{aligned}(10)\quad Q_F^{t**} &\equiv \{[p_0^{0*}q_0^t + p^0\cdot q^t]/p^0\cdot q^0\}^{1/2}\{[p_0^tq_0^t + p^t\cdot q^t]/p^t\cdot q^0\}^{1/2} \\ &= [1 + (p_0^tq_0^t/p^t\cdot q^t)]^{1/2}Q_F^t,\end{aligned}$$

where Q_F^t is defined by (4). Assume that the statistical agency uses a fixed base Fisher price index to deflate the true expenditure ratio defined by (9). Since the period 0 reservation price is not available to the statistical agency, a fixed base matched products Fisher index could be used to deflate the true value ratio defined by (9). In this case, the incorrect Fisher quantity index for period t, Q_F^{t***}, will be defined as:

$$\begin{aligned}(11)\quad Q_F^{t***} &\equiv \{[p_0^tq_0^t + p^t\cdot q^t]/p^0\cdot q^0\}/[p^t\cdot q^tp^t\cdot q^0/p^0\cdot q^tp^0\cdot q^0]^{1/2} \\ &= \{[p_0^tq_0^t + p^t\cdot q^t]/p^t\cdot q^t\}Q_F^{t*} \\ &= [1 + (p_0^tq_0^t/p^t\cdot q^t)]Q_F^{t*}\end{aligned}$$

where Q_F^{t*} is defined by (6). Thus, the ratio of the new true Fisher volume index, Q_F^{t**}, to the new incorrect Fisher volume index, Q_F^{t***}, is

$$
\begin{aligned}
(12)\quad Q_F^{t**}/Q_F^{t***} &= [1 + (p_0^t q_0^t/p^t\cdot q^t)]^{1/2} Q_F^t/[1 + (p_0^t q_0^t/p^t\cdot q^t)] Q_F^{t*} && \text{using (10) and (11)} \\
&= [1 + (p_0^t q_0^t/p^t\cdot q^t)]^{-(1/2)} Q_F^t/Q_F^{t*} \\
&= [1 + (p_0^{0*} q_0^t/p^0\cdot q^t)]^{1/2}/[1 + (p_0^t q_0^t/p^t\cdot q^t)]^{1/2} && \text{using (7)} \\
&\approx [1 + (½)(p_0^{0*} q_0^t/p^0\cdot q^t)]/[1 + (½)(p_0^t q_0^t/p^t\cdot q^t)].
\end{aligned}
$$

The last line of (12) is again straightforward to interpret. If the new product is free in periods $t \geq 1$, so that $p_0^t = 0$, then it can be seen that (12) collapses down to the simpler formula (7).

As was the case with the bias formula (7), the bias formula (12) cannot be implemented without an estimate for the period 0 reservation price p_0^{0*} for the new product. Therefore, a key contribution of the present paper is to show (in the following section) how choice experiments (including the online surveys implemented by BCDEF and Brynjolfsson, Collis and Eggers, 2019) can be used to provide an approximation to the missing reservation price p_0^{0*}.

1.2 The Role of Advertising

At this point, it is worthwhile considering some of the implications of the bias estimates given by (7) or (12) above. The Hicksian reservation price methodology casts some light on the role of advertising. The Hicksian methodology assumes that the consumer has latent preferences over new products before they enter the marketplace.[8] Advertising can cause consumers to discover new products and hence advertising can have beneficial welfare effects of the type illustrated by the above algebra. These benefits are due to the expansion of the consumer's effective consumption set. It is important to note that the business costs of producing advertising will in general not equal the consumer benefits of advertising.[9] Advertising expenses in the US increased by about 4% each year in the last five years on average.[10] However, a meta-analysis by Sethuraman, Tellis and Briesch (2011) shows that the mean long-run (short-run) advertising

[8] A weakness of the Hicksian approach is that the discovery of a new product does not materially change the consumer's preferences over previously discovered products; i.e., before a new product is discovered by the consumer, the consumer's preferences are given by $f(0,q)$; after discovery of product 0, the preferences are given by $f(q_0,q)$. However, a revolutionary new product may cause the consumers preferences to change from $f(0,q)$ to a *new* utility function, $f^*(q_0,q)$.

[9] For this reason (and other reasons, including the fact that the present international methodology for measuring GDP excludes most household production), it would also be useful to introduce a new version of GDP that would be household oriented.

[10] Advertising expenses in the US increased from 64.8 to 76.0 billion US$ from 2014 to 2018. See: https://www.statista.com/statistics/185466/estimated-expenses-in-us-advertising-and-related-services-since-2005/

elasticity is 0.24 (0.12; median values are lower), which implies that 4% more advertising spending only implies 1% higher ad-induced sales in the long run. Considering the magnitude of advertising expenses relative to the size of the economy the welfare effect to consumers due to increased consumption is likely to be rather small. Moreover, it can be seen that advertising itself typically has a *direct negative* effect on consumer surplus due to being intrusive or raising privacy concerns. For example, Papies, Eggers and Wlömert (2011) show the negative effect of several ad types on consumer utility for free music streaming. These direct negative welfare effects counteract the indirect positive effects due to increased sales.

Business expenditures on advertising are motivated by profit maximization considerations in a monopolistic competition framework. They will not be proportionate to the benefits of these new products for consumers except by coincidence.[11] Therefore, even when free goods are supported by advertising, simply measuring expenditures on advertising will not provide a meaningful estimate of the welfare gains from the free goods. Another aspect of the Hicksian methodology is that a "new" product need not be new to the marketplace; i.e., a "new" product is one that is *just discovered* by the household.

Of course, there are some problems with the above simple computations:

- Comparisons of real consumption cannot be made going from period $t-1$ to period t; instead, we have to compare periods t and $t-1$ with period 0 and so the resulting comparisons are indirect.
- To really model the benefits of a new free product, we need to also bring the allocation of time into the picture (e.g. Brynjolfsson and Oh, 2012). Integrating these approaches is left to the future.

2. Compensation for Product Withdrawals and Reservation Prices

Define the *household's conditional cost function*, $c(u,p,q_0)$, as follows:[12]

[11] For many products, cost of advertising is zero or a trivial fraction of benefits, while for other products, especially unsuccessful products, it may be that advertising expenses greatly exceed consumer benefits. Advertising expenditure may even be entirely fruitless; see Blake, Nosko and Tadelis (2015) for evidence that millions of dollars on online advertising by eBay had no effect.

[12] Conditional cost or expenditure functions were introduced into the economics literature by Pollak (1975). This section draws heavily on BCDEF and Diewert, Fox and Schreyer (2019).

(13) $c(u,p,q_0) \equiv \min_q \{p \cdot q : f(q_0,q) \geq u\}$.

This cost function minimizes the cost of consuming commodities q conditional on having q_0 units of the new commodity 0 in order to achieve the target level of utility u. The household's *regular cost function*, $C(u,p_0,p)$ is defined as follows:

$$
\begin{aligned}
(14) \qquad C(u,p_0,p) &\equiv \min_{q_0,q} \{p_0q_0 + p \cdot q : f(q_0,q) \geq u\} \\
&= \min_{q_0} \{[\min_q \{p \cdot q : f(q_0,q) \geq u\}] + p_0q_0\} \\
&= \min_{q_0} \{c(u,p,q_0) + p_0q_0\}.
\end{aligned}
$$

We assume (q_0^t,q^t) is a solution to the cost minimization problem defined by $C(u^t,p_0^t,p^t)$ where $u^t = f(q_0^t,q^t)$ and q^t is a solution to the conditional cost minimization problem defined by $c(u^t,p^t,q_0^t)$ for some period $t \geq 1$.

Thus, using definition (14) for (u^t,p_0^t,p^t), we have the following equalities:

$$
\begin{aligned}
(15) \qquad p^t \cdot q^t + p_0^tq_0^t = C(u^t,p_0^t,p^t) &= \min_{q_0} \{c(u^t,p^t,q_0) + p_0^tq_0\} \\
&= c(u^t,p^t,q_0^t) + p_0^tq_0^t.
\end{aligned}
$$

We assume that $c(u^t,p^t,q_0)$ is differentiable with respect to q_0 at $q_0 = q_0^t > 0$. Thus, the first order necessary condition for the cost minimization problem in (15) implies the following equality:

(16) $\partial c(u^t,p^t,q_0^t)/\partial q_0 = -p_0^t$.

Note that equation (15) also implies the following equality:

(17) $c(u^t,p^t,q_0^t) = p^t \cdot q^t$.

Choice experiments come into play at this point by asking households in period t: how much money will it take for the household to give up its use of the new commodity?[13] The answer is the following conditional cost minimization problem:

$$(18) \quad c(u^t,p^t,0) \equiv \min{}_q \{p^t \cdot q : f(q,0) = u^t\} > c(u^t,p^t,q_0^t)$$

where the inequality follows from the assumptions that $q_0^t > 0$ and that f is increasing in its arguments and hence $c(u,p,q_0)$ is decreasing in q_0. Define the *monetary compensation* m^t that is additional to $p^t \cdot q^t$ that is required to keep the household at the period t utility level u^t without using q_0 as follows:

$$(19) \quad m^t \equiv c(u^t,p^t,0) - p^t \cdot q^t$$
$$= c(u^t,p^t,0) - c(u^t,p^t,q_0^t)$$

where we have used (17) to derive the second equality.[14] Note that utility and the prices of continuing commodities are held constant on the right hand sides of (19).[15] Assuming that m^t can be estimated through choice experiments or other methods, it can be seen that estimates for $c(u^t,p^t,0) = p^t \cdot q^t + m^t$ can be determined.[16] We convert m^t into a period t *average compensation price per unit of q_0 foregone* by setting m^t equal to $w_0^t q_0^t$:

$$(20) \quad w_0^t \equiv m^t/q_0^t .$$

Using (19) and (20), we can write the cost difference, $c(u^t,p^t,0) - c(u^t,p^t,q_0^t)$, as follows:

$$(21) \quad c(u^t,p^t,0) - c(u^t,p^t, q_0^t) = w_0^t q_0^t .$$

[13] Put another way: what is the income required for the household to achieve its period t utility level u^t using commodities that are available in period t but excluding the use of the new commodity?
[14] This is equation (27) of BCDEF (2018), which they describe as a global willingness to accept function.
[15] Thus, the right-hand side of (19) does not equal either a Hicksian price or quantity variation; it is a Hicksian like *mixed variation*. See Diewert and Mizobuchi (2009) for a discussion of Hicksian variations.
[16] In the context of free digital goods, this is what BCDEF (2018) called "total income": actual income ($p^t \cdot q^t$) plus the additional income (m^t) required to achieve the same level of actual period t utility $u^t \equiv f(q_0^t,q^t)$.

At this point, we assume that $c(u^t,p^t,q_0)$ is also differentiable with respect to q_0 at $q_0 = 0$; i.e., we assume a one sided derivative exists at this point. Thus, we can form the following two first-order Taylor series approximations:

$$(22) \quad c(u^t,p^t,0) \approx c(u^t,p^t,q_0^t) + [\partial c(u^t,p^t,q_0^t)/\partial q_0][0 - q_0^t]$$
$$= c(u^t,p^t,q_0^t) - p_0^t[0 - q_0^t] \quad \text{using (16)}$$
$$= c(u^t,p^t,q_0^t) + p_0^t q_0^t.$$

$$(23) \quad c(u^t,p^t,q_0^t) \approx c(u^t,p^t,0) + [\partial c(u^t,p^t,0)/\partial q_0][q_0^t - 0]$$
$$= c(u^t,p^t,0) - p_0^{t*}[q_0^t - 0]$$
$$= c(u^t,p^t,0) - p_0^{t*} q_0^t ,$$

where p_0^{t*} is the *Hicksian reservation price for the new product 0* in period t which is defined as $-\partial c(u^t,p^t,0)/\partial q_0$. This reservation price is not directly observable, but we will be able to solve for it shortly using the estimate from the choice experiments w_0^t defined by (20). The approximate equalities (23) can be rewritten as:

$$(24) \quad c(u^t,p^t,0) \approx c(u^t,p^t,q_0^t) + p_0^{t*} q_0^t .$$

A more accurate approximation to $c(u^t,p^t,0) - c(u^t,p^t,q_0^t)$ can be obtained if we take the following arithmetic average of the two first order approximations (22) and (24):

$$(25) \quad c(u^t,p^t,0) - c(u^t,p^t,q_0^t) \approx ½(p_0^t + p_0^{t*})q_0^t.$$

The approximation given by (25) will be an exact one if $c(u^t,p^t,q_0)$ is a quadratic function of q_0 between 0 and q_0^t; see the quadratic approximation lemma in Diewert (1976).

Note that the left-hand side of (25) is equal to the left-hand side of (21) for each period t. Thus, the right-hand sides are approximately equal to each other and we obtain the following approximate equalities:

$$(26) \quad w_0^t q_0^t \approx ½(p_0^t + p_0^{t*})q_0^t .$$

Recall that q_0^t, w_0^t and p_0^t are observable. Thus, we can use (26) to solve for the unknown period t reservation price p_0^{t*}. The approximate solution is:

(27) $p_0^{t*} \approx 2w_0^t - p_0^t$.

If households are reluctant to surrender their units of q_0, so that the average period t compensation price w_0^t is greater than the period t market price p_0^t, then from (27) the period t reservation price for product 0, p_0^{t*}, will be greater than the observed period t price for a unit of q_0, p_0^t. Note that if the commodity 0 is a free good or service in period t, then $p_0^t = 0$ and the approximate reservation price is twice the estimated compensation price from the choice experiment, $p_0^{t*} \approx 2w_0^t$.

We have found a reservation price, p_0^{t*}, for a period t indifference surface but what we want is a reservation price for period 0. In order to obtain this reservation price, we temporarily restrict ourselves to the case where N = 1, so that $q = q_1$ is now a scalar. For period 0, the consumer's utility maximizing vector when commodity 0 is not available is $(q_0^0,q_1^0) = (0,q_1^0)$. The slope of the indifference curve through this point is $-\, p_0^{0*}/p_1^0$. When N = 1, equation (21) becomes $p_1^tq_1^{t*} = p_1^tq_1^t + w_0^tq_0^t$, which can be solved for q_1^{t*} where $p_1^tq_1^{t*} = c(u^t,p_1^t,0)$. The slope of the period t indifference curve through the point $(0,q_1^{t*})$ is $-\, p_0^{t*}/p_1^t$. But when N = 1, $f(q_0,q)$ becomes $f(q_0,q_1)$ and because $f(q_0,q_1)$ is assumed to be linearly homogeneous, all of the indifference curves that pass through the q_1 axis will have the same slope. We will have $-p_0^{0*}/p_1^0 = -p_0^{t*}/p_1^t$ and thus, using (27):

(28) $p_0^{0*}= p_0^{t*}/(p_1^t/p_1^0) \approx [2w_0^t - p_0^t]/[p_1^t/p_1^0]$

Thus, the period 0 reservation price for product 0 is the *inflation-adjusted carry backward period t reservation price*; the period t reservation price p_0^{t*} for the new commodity is deflated by an index of inflation for the continuing commodity q_1 between periods 0 and t, p_1^t/p_1^0.

We return to the case of a general N. We replace the one commodity price index for continuing commodities, p_1^t/p_1^0, by the fixed base Fisher index P_F^{t*} that compares the prices for continuing commodities in period t with their counterparts in period 0; i.e., define $P_F^{t*} \equiv$

$[p^t \cdot q^t p^t \cdot q^0/p^0 \cdot q^t p^0 \cdot q^0]^{1/2}$. Replace $p_1{}^t/p_1{}^0$ in (28) by $P_F{}^{t*}$ and we obtain the following approximate equality:

$$(29) \quad p_0{}^{0*}q_0{}^t/p^0 \cdot q^t \approx q_0{}^t[2w_0{}^t - p_0{}^t]/[P_F{}^{t*}p^0 \cdot q^t]$$
$$= 2(m^t/p^t \cdot q^t)(p^t \cdot q^t/p^0 \cdot q^t)(1/P_F{}^{t*}) - (p_0{}^t q_0{}^t/p^0 \cdot q^t)(1/P_F{}^{t*}) \text{ using (20)}$$
$$= 2(m^t/p^t \cdot q^t)(P_P{}^t/P_F{}^{t*}) - (p_0{}^t q_0{}^t/P_F{}^{t*}p^0 \cdot q^t)$$

where $P_P{}^t$ is the fixed base Paasche price index defined over continuing commodities; i.e., $P_P{}^t \equiv p^t \cdot q^t/p^0 \cdot q^t$. Now substitute (29) into (12) to get an approximation to the bias term, $Q_F{}^{t**}/Q_F{}^{t***}$, in terms of m^t instead of in terms of the reservation price $p_0{}^{0*}$. If $p_0{}^t$ is zero or tiny and if $P_P{}^t$ is approximately equal to $P_F{}^t$, then the right-hand side will be approximately equal to $2m^t/p^t \cdot q^t$ and $Q_F{}^{t**}/Q_F{}^{t***}$, the ratio of the true Fisher volume index to the incorrect Fisher volume index, will be approximately equal to $1 + m^t/p^t \cdot q^t$.

3. Conclusion

We have demonstrated how estimates of the amount of money consumers have to be paid to give up free digital goods can be translated into estimates of reservation prices and then to adjusted estimates of real household consumption that take into account household consumption of free products. Several approximations were required to accomplish this translation but in the end, we are simply applying normal index number theory to the problem. In other words, we did not use consumer surplus arguments to derive our final estimates for real household consumption. There are additional approximations required to extend our analysis from a single household to cover aggregation over consumers. Thus, there are many weaknesses in our analysis but we think that it is preferable to have a rough estimate of the benefits of free digital products than to have no estimate whatsoever.

Two important points emerged from our analysis. First, the present production-oriented GDP measures are not satisfactory for measuring real household consumption. In particular, they will be increasingly inaccurate as free goods, such as those made possible by the digital revolution, become more important. A new measure of aggregate output that changes the present production boundary and focuses on household welfare is required. A second important point that emerged is that advertising expenditures are not an adequate substitute for measuring the benefits of new goods to the household sector. Instead, we need to draw on estimates such as those provided by choice experiments.

REFERENCES

Blake, T., C. Nosko, and S. Tadelis (2015), "Consumer Heterogeneity and Paid Search Effectiveness: A large-scale field experiment", *Econometrica* 83, 155-174.

Brynjolfsson, E., A. Collis, and F. Eggers (2019) "Using Massive Online Choice Experiments to Measure Changes in Well-Being" *Proceedings of the National Academy of Sciences*.

Brynjolfsson, E., A. Collis, W.E. Diewert, F. Eggers and K.J. Fox (2019), "The Digital Economy, GDP and Household Welfare: Theory and Evidence" NBER Working Paper 25695, Cambridge, MA.

Brynjolfsson, E., & Oh, J. (2012). "The Attention Economy: Measuring the Value of Free Digital Services on the Internet" *Proceedings of International Conference on Information Systems*.

Diewert, W.E. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 114-145.

Diewert, W.E. and R. Feenstra (2019), "Estimating the Benefits of New Products: Some Approximations", Discussion Paper 19-02, Vancouver School of Economics, University of British Columbia, Vancouver, B.C., Canada.

Diewert, W.E., K.J. Fox and P. Schreyer (2019), "Experimental Economics and the New Commodities Problem", Discussion Paper 19-03, Vancouver School of Economics, University of British Columbia, Vancouver B.C., Canada, V6T 1L4.

Diewert, W.E. and H. Mizubuchi (2009), "Exact and Superlative Price and Quantity Indicators", *Macroeconomic Dynamics* 13 (Supplement 2), 335–380.

Feenstra, R.C. (1994), "New Product Varieties and the Measurement of International Prices", *American Economic Review* 84:1, 157-177.

Fisher, I. (1922), *The Making of Index Numbers*, Boston: Houghton-Mifflin.

Hausman, J.A. (1996), "Valuation of New Goods under Perfect and Imperfect Competition", pp. 20-236 in *The Economics of New Commodities*, T.F. Bresnahan and R.J. Gordon (eds.), Chicago: University of Chicago Press.

Hausman, J.A. (1999), "Cellular Telephone, New Products and the CPI", *Journal of Business and Economic Statistics* 17:2, 188-194.

Hicks, J.R. (1940), "The Valuation of the Social Income", *Economica* 7, 105-124.

Papies, D., F. Eggers and N. Wlömert (2011), "Music for free? How free ad-funded downloads affect consumer choice", *Journal of the Academy of Marketing Science* 39:5, 777-794.

Pollak, R.A. (1975), "Subindexes in the Cost of Living Index", *International Economic Review* 16, 135-150.

Sethuraman, R., G.J Tellis and R.A. Briesch (2011), "How Well Does Advertising Work? Generalizations From Meta-analysis of Brand Advertising Elasticities", *Journal of Marketing Research* 48:3, 457-471.