A biographical sketch of Walter Erwin Diewert

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Walter Erwin Diewert began life in the winter of 1941 in Vancouver, British Columbia. He completed a B.A. degree in 1963, and an M.A. in mathematics in 1964, both at the University of British Columbia. He completed a Ph.D. at the University of California at Berkeley in 1968. He is currently a full professor at the University of British Columbia, where he has been a member of the faculty since 1970.

His parents had both fled the Ukraine, and had met in Canada. His father managed to establish himself as a building contractor and his mother as a seamstress. But it is far from certain that Erwin would have gone to college were it not for the relatively affordable University of British Columbia in Vancouver. As a UBC student, Erwin bagged peas and worked in a plywood mill to earn his tuition. Erwin is an economist with feelings and opinions about labour unions that are rooted in personal experience: he *was* a union guy, having found he could earn substantially more in union jobs. It was not until his third year at UBC when he was awarded a small merit scholarship that he was finally able to devote full time to his studies, and began to think he might go on to graduate school.

Erwin is an economic theorist whose research, from his graduate student days on, has been empirically motivated. His career has been a personal quest to develop tools for economic modeling and econometrics that meet the needs of applied analysis for real world economics problems.

His real world concerns are rooted in ongoing and deepening involvement with empirical researchers, with government agencies, and with private sector and public enterprises. In addition to the publications in economics journals for which he has become so well known, he has produced a substantial body of consulting-related reports (including the report to the New Zealand Business Roundtable on which his contribution to this special issue, with his former Ph.D. student Denis A. Lawrence, is based). The problems tackled in these reports have been posed by government officials, officials at international organizations like the World Bank, and practitioners and analysts in railway, power and telecommunications companies.

Erwin's four main areas of research are: (1) flexible functional forms, (2) the implications of maximizing behaviour, (3) welfare economics and applied general equilibrium modeling, and (4) index numbers. We attempt to sketch out the evolution of his work in each of these main areas, all of which involve duality theory in some way.

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I. FLEXIBLE FUNCTIONAL FORMS

His interests in flexible functional forms began with a summer job. Erwin told us that while working with the Department of Manpower and Immigration in Ottawa in the summer of 1967, he was struck by the restrictiveness of the Leontief no substitution production functions which were being used to model the demands for different types of labour. He returned to Berkeley in the fall and gave an econometrics seminar on an approach to modeling labour demands that used relative wage rates as explanatory variables. His thesis advisor, Dan McFadden, pointed out an error in his formulation: his demand functions were not integrable into a cost function that would generate the assumed demand functions. He referred Erwin to his own work on duality (McFadden 1966) and to that of Ronald Shephard (1953) who was then a professor of industrial engineering at Berkeley. Erwin modified his original functional form into the Generalized Leontief cost function, which resulted in a first publication in 1971 in the *Journal of Political Economy* titled "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function."

Not only is the Generalized Leontief cost function integrable, but it also has an important second order approximation property: any twice continuously differentiable unit cost function can be approximated to the second order by the Generalized Leontief unit cost function. In his 1974a paper, Erwin termed a functional form "flexible" if it has this sort of a second order approximation property. As Erwin had previously pointed out in his first *JPE* paper, if the functional form adopted in an applied study is not flexible, a priori restrictions will be imposed on the elasticities of substitution simply by the choice of functional form. These restrictions may be unwarranted, and applied researchers have not always been fully aware of them.

In his first published paper, Erwin also explains why duality theory is so useful in applied production studies. He contrasts the duality approach to obtaining reduced form demand functions with the traditional approach that makes use of Lagrangian or programming techniques:

> One method is to postulate a functional form for the production function f and then use Lagrangian or programming techniques in order to obtain the derived demand functions $x_i(y; p_1, \ldots, p_n)$. Alternatively, one can postulate a functional form for the cost function and obtain derived demand functions simply by partially differentiating the cost function with respect to input prices, provided that the cost function satisfies the regularity conditions of the Shephard duality theorem. Our approach will be to apply the second method of obtaining the system of derived demand equations.
>
> (Diewert 1971, p. 483)

The use of Shephard's Lemma as a relatively painless means of deriving systems of demand functions that are linear in the unknown parameters, and hence can be conveniently estimated by regression techniques, is the first of the major applications of duality theory that Erwin stressed in his research papers such as his 1974a survey article and his 1982 treatise on duality theory which appears in full (including substantial portions omitted in the 1982 version) as Chapter 6 in the first volume of his collected papers titled *Essays in Index Number Theory* (Diewert and Nakamura 1993).

Erwin's interest in applied econometrics measurement problems was already evident in his first published paper, and even in his Masters thesis, "Analysis of Variance Estimators for the Seasonal Adjustment of Economic Time Series." This interest was bolstered by interactions with his Berkeley teacher, Dale Jorgenson, and interactions as well with fellow Jorgenson students Lau Christensen, Mike Denny, Mel Fuss and Larry Lau. The interactions within this student-professor group proved fruitful for the others too! Christensen, Jorgenson and Lau (1971) developed the translog functional form, Denny (1974) developed a functional form that generalized Erwin's Generalized Leontief form as well as the CES, and Fuss (1977) developed other generalizations that have been useful in modeling putty-clay problems.

In recent years, Erwin has continued to work on flexibility problems, mainly with his UBC colleague Terry Wales. Estimated flexible functional forms may still fail to satisfy desired curvature conditions.

For example, there are theoretical reasons why a cost or expenditure function should be concave in prices, and these concavity conditions will not necessarily be satisfied by an estimated cost or expenditure function even when a flexible functional form has been used. A 1987 Diewert-Wales article shows how curvature conditions can be imposed globally without loss of flexibility. A proof of the surprising finding that this is possible for only one class of functional forms is presented. In a 1988 Diewert-Wales paper, the authors call this class the normalized quadratic functional form: a functional form originally proposed by McFadden (1978b). It is this class of functional forms that is used by Erwin and Denis Lawrence in their taxation study for the New Zealand economy, published in this issue.

One drawback to the use of flexible functional forms is that the number of parameters required to model an N commodity production or preference function is roughly $N^2/2$: an unworkably large number if the number of commodities is large. Erwin's response to this problem was to introduce the concept of a semiflexible functional form in the 1988 Diewert-Wales paper. For the Diewert-Wales semiflexible, normalized quadratic functional form, the correct curvature can be imposed prior to estimation, but instead of allowing for a substitution matrix of full rank (which would require N(N-1)/2 parameters), a restricted substitution matrix is used of rank K less than N (resulting in a more modest (N-K) (N-K-1)/2 parameters). The 1988 Diewert-Wales paper predicts that semiflexible functional forms will have a promising future in the construction of econometric applied general equilibrium models where limitations on the length of time series would typically prevent the estimation of fully flexible functional forms.

In many applications involving large data sets, researchers would like their chosen production or utility functions to have the capacity to approximate an arbitrary function at more than one point. To allow for this, the Diewert-Wales normalized quadratic functional forms were combined with spline functions in 1992 and 1993a Diewert-Wales contributions. In the 1993a paper, the authors compare the resulting normalized quadratic spline model with three popular semi-nonparametric families of functional forms and find the latter lacking in that either they are not fully flexible, or curvature conditions cannot be imposed globally without destroying flexibility. This leads them to conclude that the Diewert-Wales normalized quadratic spline model has important advantages over existing semi-nonparametric models.

In a 1993b Diewert-Wales paper, the problems involved in the nonparametric estimation of a time series or in the smoothing of a function of one variable are tackled. A nonparametric class of approximating functions is recommended: a class of piece-wise concave and convex functions with at most k changes in curvature. This paper also offers an interesting definition of smoothness. The approximating function is deemed to be smoother the lower the value is for the positive integer k (i.e. the smaller the number of changes in curvature). Methods are given for choosing k, and for determining approximating functions by solving quadratic programming problems. (A new programming algorithm is provided as well.)

A final flexibility research problem that Erwin has tackled is the search for empirically tractable models that generalize the usual expected utility approach to choice under risk, but at the same time are sufficiently general to avoid the paradoxes associated with the expected utility model. In a 1993a paper that was first published as Chapter 14 in *Essays in Index Number Theory*, he uses a contingent commodity approach (instead of the usual preferences over lotteries framework) to derive a class of nonexpected utility models that is more flexible than the usual expected utility model. Erwin's 1995d paper, "Functional Form Problems in Modeling Insurance and Gambling," extends this analysis still further.

II. IMPLICATIONS OF MAXIMIZING BEHAVIOUR

Erwin's second major research area is concerned with observable implications of maximizing behaviour, in the tradition of Hicks (1946) and Samuelson (1983) whose work Erwin greatly admires. Papers by his thesis advisor (McFadden 1966, 1978a) led him to realize that dual functions embody the restrictions of maximizing behaviour and hence can be used to derive comparative statics results:

> The second principal advantage of duality theory is that it enables us to derive in an effortless way the `comparable statics' theorems originally deduced from maximizing behaviour.

(Diewert 1974a, p. 107)

Erwin uses duality theory to derive comparative statics results in his own 1974a, 1981a, 1982, and 1983a papers and in a 1977 paper with his former UBC colleague Alan Woodland. His duality approach has been widely adopted by others. (Erwin is also the originator of another non-duality approach to establishing comparative statics theorems for a general inequality constrained nonlinear programming problem; see his 1984 paper.)

Erwin's duality theorems for determining the properties of dual functions have also been widely used. Duality theorems basically state that if a primal function satisfies certain regularity conditions, then the corresponding dual function will satisfy certain conditions, and vice versa. His first 1971 paper contained a duality theorem between cost and production functions. Statements of other widely used duality theorems of his can be found in his 1974a and 1982 papers.

The maximizing (and minimizing) results of economic theory imply that cost functions must be concave in prices. Utility functions are usually assumed to be quasiconcave. Because of uses such as these, Erwin became interested in generalized concave functions. His contributions on this topic include his 1981b and 1981c papers as well as a 1981 Diewert-Avriel-Zang paper. In his 1981c paper, Erwin found it useful to prove a version of the Mean Value Theorem from elementary calculus that applies to non-differentiable functions. Applied mathematicians working on nonsmooth optimization theory have cited and used this result, so Erwin's contributions are not limited to economics.

Erwin's interests in yet another topic having to do with observable implications of maximizing behaviour date back to an experience while he was an assistant professor at the University of Chicago in 1968-70. He was asked to referee a paper by Sydney Afriat for the *JPE*. According to Erwin, he had to work on this paper for two or three weeks, including mastering an earlier 1967 paper of Afriat's, before he could finally write his referee report. The report was lengthy, and recommended acceptance subject to revisions to make the paper more understandable. Afriat did not revise his paper for the *JPE* but it was eventually published elsewhere (see Afriat 1973). Since Erwin believed Afriat's work was important and since it took him so long to understand it, he thought it would be useful to revise his referee report and publish this interpretation of Afriat's work, which he did (see Diewert, 1973a; 1981a, pp. 198-199). Erwin recognized the power of Afriat's results: in particular, Afriat (1967) provided necessary and sufficient conditions for a given set of price and quantity vectors to be consistent with utility maximizing behaviour. He showed how Afriat's conditions could be checked by solving a linear programming problem. If the optimized objective function of the problem turned out to be zero, Erwin showed how the solution to the linear program could be used to construct a nonparametric approximation to the decision maker's preferences that would be consistent with the observed data. He also showed how the program could be adapted to check for consistency with homothetic preferences. Varian subsequently followed up on Erwin's programming approach by providing a more efficient algorithm in papers Varian published in 1982 and 1983. This nonparametric approach to preference estimation was adapted to the production context by Afriat (1972a) and Hanoch and Rothschild (1972), and Erwin and his Ph.D. student Celik Parkan generalized their approaches in a 1983 Diewert-Parkan paper. (In the management science literature, this nonparametric approach to technology estimation has come to be known as data envelopment analysis; see Charnes and Cooper, 1985.)

III. WELFARE ECONOMICS AND APPLIED GE MODELS

Erwin says that his third main area of research -- applied general equilibrium models -- was stimulated by Dale Jorgenson's interest in this topic. Since his graduate student days, Erwin has aspired to provide econometric applied general equilibrium models like those described by Jorgenson (1984) and Lau (1984): GE models for which the parameters were estimated econometrically rather than being assumed without any suitable empirical basis. (Often GE model parameter values are simply picked up from applied research undertaken for other purposes -- and often based on data for other places and time periods, and with equation specifications that are not fully consistent one to another or with the GE model context.)

However, it is only with the completion of his work with Denis Lawrence on the New Zealand economy (briefly summarized in this issue) that he finally feels he is beginning to achieve this objective. In earlier research, Erwin did contribute substantially to theoretical aspects of GE modeling. In his 1973b paper, he proved the existence of a general equilibrium by using a fixed point theorem over the space of utility weights rather than over the space of commodity prices, following an approach of Negishi (1960). As he points out, this procedure has important computational advantages if the number of consumer classes is less than the number of commodities. Also, in a 1977 paper, Erwin established one of the first theorems for the existence of a temporary equilibrium.

Erwin's research efforts in the applied GE area have focused mainly on the use of duality theory to simplify the derivation of comparative statics results:

> We shall find it convenient to formulate our general equilibrium model of an open economy in terms of dual functions; i.e., in terms of expenditure functions and unit profit functions. Our reason for doing this is that these dual functions already contain the restrictions implied by utility or profit maximizing behaviour and hence the characterization of a competitive equilibrium in the economy reduces to the solution of a certain (relatively small) system of equations or inequalities.
>
> (Diewert 1985a, p. 45)

Basically, the use of duality theory in GE models dramatically reduces the number of endogenous variables and the sizes of matrices that have to be inverted in order to obtain derivatives of endogenous variables with respect to exogenous variables.

Erwin's use of duality theory to obtain comparative statics results dates back to two early papers on the effects of unionization (see Diewert 1974b and 1974c). His subsequent 1978a and 1983b papers and two of his papers with his former Ph.D. student Arja Turunen-Red and with Alan Woodland (Diewert, Turunen-Red and Woodland 1989 and 1991) are more complex studies and use the idea of obtaining Pareto improvements by manipulating policy instruments. Erwin has also produced a series of papers (1981e, 1983c, 1985b, 1985c) that develop second order approximations to the welfare or efficiency costs of distortions (a distortion is considered to be anything that causes the price of a homogenous commodity to differ between economic agents). In one of these papers, Erwin developed an interesting approximate relationship between static distortions and growth:

> In this model, optimal capital stocks are chosen in order to maximize discounted profits. We adapted our static producer loss methodology to this dynamic model and we eventually derived a formula for the approximate dynamic loss due to the distortions, L_D. We then considered another quasidynamic model where producers' investment decisions were hypothetically frozen at the optimal levels that corresponded to no distortions. We calculated a quadratic approximation to the output loss L_{QD} due to distortions in this ‘capital frozen’ economy. Finally, we showed that $L_D \geq L_{QD}$, i.e. to the second order, the true dynamic loss generally exceeds the quasidynamic loss. Thus we have shown that growth can *never* ameliorate the effect of any kind of production distortion; growth can only augment the harmful effects of distortions (at least to the second order). Thus the usual static loss measures due to distortions may be seriously biased downwards.
>
> (Diewert 1985b, p. 237)

The above result may help to explain why the deadweight costs of taxation are invariably much higher in dynamic models (such as the model on which Jorgenson's paper in this issue is based) compared with static models that do not model capital accumulation decisions.

In all of Erwin's papers on applied GE modeling discussed so far, the difficult problems involved in making interpersonal utility comparisons are avoided by either deriving conditions for Pareto improving changes in policy instruments or by utilizing the Allais (1977)-Debreu (1951, 1954) approach to the measurement of the costs of distortions. In the latter approach, consumers' utility levels are frozen at their distorted equilibrium levels so there is no problem of interpersonal comparisons. It is only in a more recent 1985a paper that he does address some of the problems involved in finding "reasonable" social welfare functions that allow for utility tradeoffs.

IV. INDEX NUMBER RESEARCH

Erwin's last major area of research, index numbers, also dates back to his Berkeley graduate student days. He says he began thinking about problems of index numbers and aggregation over commodities after reading a 1956 book by Richard Stone. Erwin's attempt at refereeing the paper Afriat had submitted to the *JPE* had led to an initial friendship with Sydney Afriat, and as a result, Afriat had sent Erwin a copy of his 1972b paper. This paper of Afriat's was on the theory of exact index numbers; it showed that under the assumption of utility maximizing behaviour, certain functional forms for a utility function were consistent with certain index number formulae. This interested Erwin, since his Ph.D. thesis was concerned with functional forms for utility and production functions. While visiting Stanford during the summer of 1973, Erwin decided to look up some of the more obscure articles that were listed in the references for Afriat (1972b). Thanks to the large collection of Russian material in the Hoover Library at Stanford, this led Erwin to find an article by Konüs and Byushgens (1926) which had not been translated into English or referred to in the English language literature on index numbers. Erwin found that the techniques of proof in that article could be used to establish a large class of exact index number formulae. His Stanford technical report written in the summer of 1973 was submitted to the *Review of Economic Statistics* and then to the *Quarterly Journal of Economics*. Both rejected it. He then submitted it to the *Journal of Econometrics* where he was an associate editor and where it was published in 1976 under the title "Exact and Superlative Index Numbers." That paper (Diewert 1976) is reprinted as Chapter 8 in *Essays in Index Number Theory*.

Although the *Review of Economics and Statistics* had rejected Erwin's paper on exact and superlative index numbers, its editor remembered that Erwin was doing research in the index number area, and asked him to referee an index number paper by Yrjö O. Vartia. Erwin enthusiastically recommended publication, but this recommendation of Erwin's was not accepted. (The paper was later published as Vartia 1976.) However, the refereeing process led to a friendship with Vartia, and also stimulated Erwin to write a paper in the summer of 1975, published in 1978 under the title "Superlative Index Numbers and Consistency in Aggregation." This paper is reprinted as Chapter 9 in *Essays in Index Number Theory*.

The economic approach to index number theory is closely connected to duality theory, since the Konüs true cost of living index is a ratio of cost or expenditure functions evaluated at the same utility level but for different price vectors. In the case of homothetic preferences, the true cost of living index is independent of the utility level and is simply a ratio of unit cost functions. With his background of research on flexible functional forms, Erwin recognized that it would be useful to find index number formulae that were exact for a flexible functional form for the unit cost function. He termed an index number formulae to be *superlative* if it is exact for a flexible unit cost function (see Diewert 1976, p. 136). This definition of his builds on Irving Fisher's (1922) somewhat vague idea of a superlative index. With Erwin's definition, the use of superlative indexes is necessarily consistent with an economic model that does not arbitrarily restrict elasticities of substitution between the commodities in the aggregate. Paul Samuelson offers the following observation on Erwin's superlative index concept in his introductory remarks to the expanded 1983 edition of *Foundations of Economic Analysis*, which contains material in (Chapter VI) on index numbers:

> A second valuable digression (or partial digression) was Chapter VI's section "The Economic Theory of Index Numbers." Index number theory is shown to be merely an aspect of the theory of revealed preference. Thirty-five years after that analysis appeared there has been but one major advance in index number theory--namely W.E. Diewert's formalizing concept of a 'superlative index number,' which is a formula based upon two periods (p_j,q_j) data that will be exactly correct as an ordinal indicator of utility for some specified family of indifference contours.
>
> (Samuelson 1983, p. xx)

National statistical agencies have also recognized the importance of using superlative index numbers whenever possible. (See, for example, the U.S. Bureau of Labor Statistics 1983, Hill 1988 and 1993, and Triplett 1992.) Recently, Erwin has continued to develop more specialized aspects of the theory of exact and superlative index numbers. For example, in his 1996b paper he deals with measurement problems that are encountered with seasonal commodities in an inflationary environment. And his 1996a and 1996c papers focus on economic approaches to the measurement problems involved in making price and quantity comparisons across countries.

The economic approach to index number theory rests on the assumption of maximizing behaviour. In many real life circumstances, this assumption breaks down. In his more recent index number papers, Erwin recognizes this problem. This realization stimulated his research on the "test" or "axiomatic" approach to index number theory. (See, for example, Diewert 1992, 1993b and 1993c which are Chapters 13, 1 and 2, respectively in *Essays in Index Number Theory*, and also Diewert 1995a and 1996c.)

A third approach to index number theory -- the statistical or stochastic approach -- has also been extensively reviewed by Erwin; see Diewert (1981a, pp. 179-180; 1993b, p. 23; 1993c, pp. 37-37; 1995b).

Erwin has been active in providing advice to Statistics Canada since November 1982 when he helped organize an international conference on price level measurement. More than 350 people took part in that conference and the 50 papers and commentaries were eventually published as Diewert and Montmarquette (1983). Erwin has been a member of, and (at the insistence of the other members) has continued to chair, the first Statistics Canada external advisory committee to be formed, the "Prices" committee, since its beginning in 1983. In addition, he is presently a member of the Statistics Canada advisory committee on services, and a member of the National Statistics Council for Canada. Erwin is also a member of the International Working Group on Price Indices, a group of statistical agency experts from member countries which was set up in 1994. Two of Erwin's recent papers (Diewert 1995a and 1996b) were originally presented to meetings of this group in Ottawa and Stockholm.

He recently gave invited testimony to the U.S. Senate (see Diewert 1995c) on sources of bias in the U.S. consumer price index. He was the only non-U.S. economist asked to participate. One of Erwin's recent papers (Diewert 1995a) was used as an internal briefing document by the Bureau of Labor Statistics in preparation for those Senate hearings. Perhaps as a result of all of this exposure to measurement concerns of politicians and the public, a few of Erwin's recent papers are quite reader friendly. Recognizing that most of those who have struggled to master his papers over the years will not readily accept this assertion, we offer the following excerpt from a review Robert Gordon has written of Erwin's 1996a paper:

> Erwin Diewert has written an impressive and insightful review of Peter Hill's paper on `Price and Volume Measures' in the *System of National Accounts, 1993*. This piece is classic Diewert-as-interpreter, taking a verbal discussion, identifying the key issues, and formalizing them in a thicket of equations, complete with a huge set of references going back to the dawn of the subject more than a century ago. In fact, as formidable as this paper may appear to the uninitiated, it is relatively sparse of equations and abundant in words compared to Diewert's many other original contributions.
>
> (Gordon1996,p. 287)

Gordon's quotation also illustrates another aspect of Erwin's research: he genuinely enjoys searching through the UBC library, trying to trace the origins of a good idea back as far as possible.

IV. CONCLUDING REMARKS

We have written this biographical sketch of Erwin Diewert not only to honour him and because we believe Erwin's work has been and will continue to be tremendously influential, but also because there are many aspects of Erwin's career which we find inspiring and believe would be of practical value for young

Canadian economists to be aware of. Erwin has worked hard and has persisted, often in the face of initial rejections. He has enthusiastically and conscientiously contributed his best not only when his name was on a work being submitted to a top journal, but also as an anonymous referee for papers of others and a supervisor of students' work. From very early in his career he actively traced back the intellectual histories of the ideas that attracted his interest, sent out his working papers, corresponded widely with others working on related topics, and was scrupulous in giving formal credit in his publications for ideas he gleaned from others' work and from direct interaction with others. His scholarship has always been firmly grounded in real world economic knowledge and concerns. He is an economic theorist who begins most days with a pile of newspapers along with his breakfast, and who has spent long hours piecing together data series and struggling with computers to produce empirical analyses for real life economic problems.

As those who have been lucky enough to know him will readily attest, Erwin is also an uncommonly nice person. He has succeeded for the best of reasons.

NOTE

This biographical sketch builds on our own personal experiences: Rick as a Ph.D. student and subsequent UBC colleague in the 1980s; David as one of Erwin's Berkeley professors; and Alice as co-editor of *Essays in Index Number Theory*, a longstanding member of the Statistics Canada Prices advisory committee, and a collaborator with Erwin in his Productivity Evaluation and Enhancement Research (PEER) Group. This sketch also makes use of material in W. Erwin Diewert's "Overview of Volume 1" in Diewert and Nakamura (1993) and on pp. 458-60 of Berndt (1991), as well as drawing on information and observations shared with us in interviews with Erwin and with a few of the others who have interacted with Erwin professionally. It is impossible to do justice in a brief format to a career like Erwin's, and to all those who have been part of his career through personal and scholarly interactions. The responsibility for all omissions, oversights and other shortcomings is solely ours.

REFERENCES

Afriat, S.N. (1967) `The construction of utility functions from expenditure data.' *International Economic Review* 8, 67-77

____ (1972a) `Efficiency estimation of a production function.' *International Economic Review* 13, 568-98

____ (1972b) `The theory of international comparisons of real income and prices.' In *International Comparisons of Prices and Output*, ed. D.J. Daly, National Bureau of Economic Research (New York: Columbia University Press) 13-69

____ (1973) `On a system of inequalities on demand analysis: an extension of the classical method.' *International Economic Review* 14, 460-72

Berndt, E.R. (1991) *The Practice of Econometrics: Classic and Contemporary* (Reading, MA.: Addison-Wesley Publishing Company)

Charnes, A., and W.W. Cooper (1983) `Preface to topics in data envelopment analysis.' *European Journal of Operational Research* 2, 429-44

Christensen, L.R., D.W. Jorgenson, and L.J. Lau (1971) `Conjugate duality and the transcendental logarithmic production function.' *Econometrica* 39, 255-56

Denny, M. (1974) `The relationship between functional forms for the production system.' *Canadian Journal of Economics* 7, 21-31

Diewert, W.E. (1971) `An application of the Shephard duality theorem: a Generalized Leontief production function.' *Journal of Political Economy* 79(3), 481-507

____ (1973a) `Afriat and revealed preference theory.' *Review of Economic Studies* 40(3), 419-25

____ (1973b) `On a theorem of Negishi.' *Metroeconomics* 25(2), 119-35

____ (1974a) `Applications of duality theory.' In *Frontiers of Quantitative Economics* Vol. II, eds.

M. Intriligator and D. Kendrick (Amsterdam: North-Holland) 106-71
____ (1974b) `The effects of unionization on wages and employment: a general equilibrium analysis.' *Economic Inquiry* 12(3), 319-39
____ (1974c) `Unions in a general equilibrium model.' *Canadian Journal of Economics* 7(3), 475-95
____ (1976a) `Exact and superlative index numbers.' *Journal of Econometrics* 4(2), 115-45; and Ch. 8 in Diewert and Nakamura (1993), 223-52
____ (1977) `Walras' theory of capital formation and the existence of a temporary equilibrium.' In *Equilibrium and Disequilibrium in Economic Theory,* ed. E. Schwödiauer (Reidel Publishing Co.) 73-126
____ (1978a) `Optimal tax perturbations.' *Journal of Public Economics* 10, 138-77
____ (1978b) `Superlative index numbers and consistency in aggregation.' *Econometrica* 46(4), 883-900; and Ch. 9 in Diewert and Nakamura (1993), 253-74
____ (1981a) `The elasticity of derived net supply and a generalized Le Chatelier Principle.' *Review of Economic Studies* 48, 63-80
____ (1981b) `Generalized concavity and economics.' In *Generalized Concavity in Optimization and Economics*, eds. S. Schaible and W.T. Ziemba (New York: Academic Press) 511-41
____ (1981c) `Alternative characterizations of six kinds of quasiconcavity in the nondifferentiable case with applications to nonsmooth programming.' In *Generalized Concavity in Optimization and Economics*, eds. S. Schaible and W.T. Ziemba (New York: Academic Press) 51-93
____ (1981d) `The economic theory of index numbers: a survey." In *Essays in the Theory and Measurement of Consumer Behaviour in Honour of Sir Richard Stone*, ed. Angus Deaton (Cambridge, England: Cambridge University Press) 163-208; and Ch. 7 in Diewert and Nakamura (1993), 177-222
____ (1981e) `The measurement of deadweight loss revisited.' *Econometrica* 49(5), September, 1225-44
____ (1982) `Duality approaches to microeconomic theory.' In *Handbook of Mathematical Economics*, Vol. II, eds. K.J. Arrow and M.D. Intriligator (Amsterdam: North-Holland Publishing Co.) 535-99; and Ch. 6 in Diewert and Nakamura (1993), 105-76
____ (1983a) `A fundamental matrix equation of production theory with applications to the theory of international trade.' In *Methods of Operations Research*, Vol. 46, ed. Paul Stahly (Konigstein: Verlag Anton Hain) 181-94
____ (1983b) `Cost-benefit analysis and project evaluation: a comparison of alternative approaches.' *Journal of Public Economics* 22(3), 265-302
____ (1983c) `The measurement of waste within the production sector of an open economy.' *Scandinavian Journal of Economics* 85(2), 159-79
____ (1984) `Sensitivity analysis in economics.' *Computers and Operations Research* 11(2), 141-56
____ (1985a) `Measurement of waste and welfare in applied general equilibrium models.' In *New Developments in Applied General Equilibrium Analysis*, eds. John Piggot and John Whalley (Cambridge: Cambridge University Press) 42-102
____ (1985b) `A dynamic approach to the measurement of waste in an open economy.' *Journal of International Economics* 19, 213-40
____ (1985c) `Transfer pricing and economic efficiency.' In *Multinationals and Transfer Pricing*, eds. Alan M. Rugman and Lorraine Eden (London: Croom Helm) 47-81
____ (1992) `Fisher ideal output, input, and productivity Indexes revisited.' *Journal of Productivity Analysis* 3, 211-48; and Ch. 13 in Diewert and Nakamura (1993), 317-54
____ (1993a) `Symmetric means and choice under uncertainty.' In Diewert and Nakamura (1993), 355-433
____ (1993b) `Overview of Volume 1.' In Diewert and Nakamura (1993), 1-31
____ (1993c) `The early history of price index research.' In Diewert and Nakamura (1993), 33-65
____ (1995a) `Axiomatic and economic approaches to elementary price indexes.' *Proceedings of the*

International Conference on Price Indices, October 31-November 2, 1994, ed. Louis Marc Ducharme, Ottawa: Statistics Canada
____ (1995b) `On the stochastic approach to index numbers.' Dept. of Economics, Discussion Paper 95-31, University of British Columbia
____ (1995c) `The Consumer Price Index.' Written and oral testimony to the *Committee on Finance*, United States Senate, April 6, 1995
____ (1995d) `Functional form problems in modeling insurance and gambling.' *Geneva Papers on Risk and Insurance Theory* 20, 135-50
____ (1996a) `Price and Volume Measures in the system of national accounts.' In *The New System of National Economic Accounts*, ed. John Kendrick (Norwell, MA.: Kluwer Academic Publishers)
____ (1996b) `Seasonal commodities, high inflation and index number theory.' Discussion Paper 96-06, Dept. of Economics, University of British Columbia
____ (1996c) `Axiomatic and economic approaches to international comparisons.' Discussion Paper 96-11, Dept. of Economics, University of British Columbia
Diewert, W.E., M. Avriel and I. Zang (1981) `Nine Kinds of Quasiconcavity and Concavity.' *Journal of Economic Theory* 25(3), December, 397-420
Diewert, W.E., and D. Lawrence (1996) `The deadweight costs of taxation in New Zealand.' *Canadian Journal of Economics*, forthcoming
Diewert, W.E., and C. Montmarquette (1983) eds., *Price Level Measurement: Proceedings from a Conference Sponsored by Statistics Canada*, Statistics Canada, Ottawa
Diewert, W.E., and A.O. Nakamura, editors (1993) *Essays in Index Number Theory*, Vol. 1 (Amsterdam: North-Holland Publishing Co.)
Diewert, W.E., and C. Parkan (1983) `Linear progamming tests of regularity conditions for production functions.' In *Quantitative Studies on Production and Prices*, eds. W. Eichhorn, R. Henn, K. Neumann and R.W. Shephard (Vienna: Physica-Verlag) 131-58
Diewert, W.E., A.H. Turunen-Red, and A.D. Woodland (1989) `Productivity and Pareto improving changes in taxes and tariffs.' *Review of Economic Studies* 56, 199-216
Diewert, W.E., A.H. Turunen-Red, and A.D. Woodland (1991) `Tariff reform in a small open multi-household economy with domestic distortions and nontraded goods.' *International Economic Review* 32, 937-57
Diewert, W.E., and T.J. Wales (1987) `Flexible functional forms and global curvature conditions.' *Econometrica* 55, 43-68
____ (1988) `A normalized quadratic semiflexible functional form.' *Journal of Econometrics* 37, 327-42
____ (1992) `Quadratic spline models for producer's supply and demand functions.' *International Economic Review* 33(3), 705-22
____ (1993a) `Linear and quadratic spline models for consumer demand functions.' *Canadian Journal of Economics* 26, 77-106
____ (1993b) `A "new" approach to the smoothing problem.' Discussion Paper No. 93-44, Dept. of Economics, University of British Columbia
Diewert, W.E., and A.D. Woodland (1977) `Frank Knight's Theorem in Linear Programming Revisited.' *Econometrica* 45(2), 375-98
Fisher, I. (1922) *The Making of Index Numbers* (Boston: Houghton Mifflin)
Fuss, M.A. (1977) `The structure of technology over time: a model for testing the `putty-clay' hypothesis.' *Econometrica* 45, 1797-1821
Gordon, R.J. (1996) `Discussion of Chapter 8.' In *The New System of National Accounts*, ed. J.W. Kendrick (Boston: Kluwer Academic Publishers) 287-97
Hanoch, G., and M. Rothschild (1972) `Testing the assumptions of production theory: a nonparametric approach.' *Journal of Political Economy* 80, 256-75
Hicks, J.R. (1946) *Value and Capital*, 2nd ed. (Oxford: Clarendon Press)
Hill, T.P. (1988) `Recent developments in index number theory and practice.' *OECD Economic Studies*,

No. 10 (Spring), 123-48
Hill, T.P. (1993) `Price and volume measures." *System of National Accounts 1993*, 379-406
Jorgenson, D.W. (1984) `Econometric models for applied general equilibrium analysis.' In *Applied General Equilibrium Analysis*, eds. H.E. Scarf and J.B. Shoven (Cambridge: Cambridge University Press) 139-203
Konüs, A.A., and S.S. Byushgens (1926) `K probleme pokupatelnoi cili deneg' (English translation of Russian title: `On the problem of the purchasing power of money'), *Voprosi Konyunkturi* II(1) (supplement to the *Economic Bulletin of the Conjuncture Institute*), 151-72
Lau, L.J. (1994) `Comment.' In *Applied General Equilibrium Analysis*, eds. H.E. Scarf and J.B. Shoven (Cambridge: Cambridge University Press) 112-37
McFadden, D. (1966) `Cost, revenue and profit functions: a cursory review.' NBER Working Paper 86, University of California at Berkeley, March
McFadden, D. (1978a) `Cost, revenue and profit functions.' In *Production Economics: A Dual Approach to Theory and Applications*, Vol. 1, eds. M. Fuss and D. McFadden (Amsterdam: North-Holland) 3-109
McFadden, D. (1978b) `The general linear profit function.' In *Production Economics: A Dual Approach to Theory and Applications*, Vol. 1, eds. M. Fuss and D. McFadden (Amsterdam: North-Holland)269-86
Negishi, T. (1960) `Welfare economics and existence of an equilibrium for a competitive economy.' *Metroeconomica* 12, 92-7
Samuelson, P.A. (1983) *Foundations of Economic Analysis: Enlarged Edition* (Cambridge, MA: Harvard University Press)
Shephard, R.W. (1953) *Cost and Production Functions* (Princeton: Princeton University Press)
Stone, R. (1956) *Quantity and Price Indexes in National Accounts* (Paris: Organization for European Cooperation)
Triplett, J.E. (1992) `Economic theory and BEA's alternative quantity and price indexes.' *Survey of Current Business* 72, 49-52
U.S. Bureau of Labor Statistics (1983) *Trends in Multifactor Productivity 1948-81*. Bureau of Labor Statistics Bulletin 2178, Washington, D.C.
Varian, H.R. (1982) `Nonparametric tests of consumer behavior.' *Review of Economic Studies* 50, 99-110
____ (1983) `The nonparametric approach to demand analysis.' *Econometrica* 50, 945-73
Vartia, Y.O. (1976) `Ideal log-change index numbers.' *Scandinavian Journal of Statistics* 3, 121-6