Stockpiling Liquidity to Acquire Innovation

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Abstract

Cash utilization in US merger and acquisition (M&A) transactions has increased over 50% since the early 1990s amidst a secular, global M&A boom. How does this cash-use relate to firm's cash stockpiles, and what are the aggregate implications for firm innovative efforts, growth and monetary policy? To answer these questions, we pose a general equilibrium theory of R&D-intensive firm cash stockpiling and use in M&A transactions. M&A cash bids can close faster than those externally financed, hence reducing the hazard of competing offers and external risks of trade breakdown. A higher common-value component in M&A arising from transferable productivity of firms' intangible assets spurs increased M&A competition and serial acquirer cash-stockpiles. Despite sellers receiving a cash-premium as compensation, cash-use biases M&A rents and growth incentives towards serial acquirers. Higher nominal interest rates differentially impact internal and external growth incentives across firms, re-shaping the firm-size and productivity distribution. Calibrated to the US economy, we find that increasing transferable productivity differences and M&A competition, not interest rates, can account for the majority of aggregate firm cash stockpiles since 1990.

Keywords: Mergers and Acquisitions (M&A), Intangible Assets, Firm Cash-Stockpiles, M&A cash-premium, Firm Dynamics, Endogenous Growth, Search & Matching, Monetary Policy

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1 Introduction

Information communication systems and intellectual property have proliferated and become critical to modern firms. These intangible assets fundamentally differ from physical capital in their non-rivalry, allowing for the same capital to be deployed across different firm business lines. This increased transferability of productivity encourages reallocation of sales towards "superstar" firms with the most productive intangible assets.

The merger and acquisition (M&A) market is a natural conduit for this reallocation, with global announced deal value rising from approximately $500 billion to $3.9 trillion over the past three decades.In particular, superstar firms seem to be among the most active in the M&A market with 5 of the highest-valued public tech giants alone disclosing they have acquired over 700 firms in the past three decades.[1] At the same time, these firms which are most active in M&A also appear amongst the largest holders of cash.[2] These cash stockpiles pose a puzzle to existing theories since these firms have the lowest cost of capital contrary to Falato et al. (2022), are highly productive and intensely scrutinzed firms inconsistent with an agency story such as Nikolov and Whited (2014), on aggregate only marginaly reduced their cash holdings with the reduced repatriation tax on foreign income by the Tax Cuts and Jobs Act (TCJA) as evaluated by Foley et al. (2007), Faulkender et al. (2019), Bennett and Wang (2021), and Garcia-Bernardo et al. (2022), nor has moved in a consistent manner with interest rate fluctuations (e.g. Azar et al. (2016), and Gao et al. (2021)).[3]

In this paper we examine the extent that these cash stockpiles of highly acquisitive, intangible superstars is not a coincidence, but are in fact driven by competitive threats in the M&A market. While firm-to-firm mergers are classically viewed as reflecting private synergies, we argue the transferability of intangible assets increases the common value component of target firms, raising the likelihood of competing bidders for the same target. Provided cash M&A offers can lower the hazard of competing offers through reduced public disclosure and shortening the time to close the deal, cash stockpiles may facilitate acquirers retaining a higher share of the acquisition surplus

[1]For details, see https://www.americanactionforum.org/insight/the-government-should-not-ban-mergers-and-buyouts/ and https://www.antitrustinstitute.org/wp-content/uploads/2019/07/Merger-Enforcement_Big-Tech_7.8.19.pdf. Many more acquisitions by these top 5 firms have very recently begun being investigated by a new FTC probe: https://www.crn.com/news/ftc-probing-past-apple-alphabet-amazon-facebook-microsoft-acquisitions.

[2]E.g. Apple, Microsoft and Alphabet alone accounted for one-quarter of the 1.9 trillion USD non-financial corporate US cash in 2016, with Apple's cash/asset ratio around 33%, see https://www.spglobal.com/en/research-insights/articles/us-corporate-cash-reaches-19-trillion-but-rising-debt-and-tax-reform-pose-risk. For more discussion of the rise of non-financial corporate cash and its high levels of concentration amongst the top tech firms see Pinkowitz et al. (2013).

[3]Moreover, the debt capacity constraints argued by Falato et al. (2022) require collateral constraints tied to tangible capital, rather than debt capacity connected to operating cashflow which has been documented by Lian and Ma (2021) to be pervasive for modern public firms.

and probability of success.

First, we develop and provide empirical support, for a theory which captures this mechanism. Second, we embed it into a rich firm dynamics and endogenous growth model to examine the interplay between firm liquidity demand, firm concentration and growth. Here we demonstrate that this framework creates a novel link between firm-based innovative activities and monetary policy, with interest rates influencing the anticipated terms of trade in the M&A market and consequently the distribution of innovative activity across prospective M&A buyers and sellers. Third, we calibrate the model to the US and decompose the secular changes in firm cash stockpiles and market concentration observed between 1990 and 2015. Finally, we evaluate the aggregate implications, net the potentially asymmetric distributional effects, of various counterfactual monetary policy and M&A market interventions.

The model builds off the workhorse applied model of endogenous growth with firm dynamics developed by Klette and Kortum (2004).[4] In this model, firms own multiple product lines. Each firm follows a stochastic birth-death process governed by their internal rate of innovation and the aggregate rate of creative destruction. Similar to Acemoglu et al. (2018), we start our departure from their framework by introducing heterogeneity across firms in their (per-period) fixed cost to having a product on the market. These differences in fixed costs provide motive for re-allocation of product lines which we allow through a frictional M&A market. The M&A market features two-sided search between buyers and sellers akin to David (2021), however, we add the possibility of competing bidders, since Boone and Mulherin (2007) find that approximately 50% of M&A involve multiple competing buyers for the same seller. Competition amongst buyers for a given seller may occur due to physical delays in the closing of an agreed deal and limited commitment from the seller in not considering new bids (e.g. through go-shop provisions).[5] The threat of competition pins down the effective bargaining power of the seller in the model. Stockpiled liquidity can be used by a buyer to hasten the closing of a deal and lower the hazard of a competing bid. As a consequence, higher anticipated levels of competition increase the demand for cash by prospective buyers in the M&A market. Expected terms of trade in the M&A market then feedback into new entrants and other prospective sellers' incentives to innovate and thus can influence market concentration.

In addition to the model's ability to rationalize the increased cash demand by large, public innovative firms, the model also offers a new theory

[4]The model was shown to exhibit many patterns found in the micro-data by Lentz and Mortensen (2008). For examples of its applications, see Acemoglu et al. (2018), Lentz and Mortensen (2016) and Akcigit and Kerr (2018).

[5]A similar notion of speed providing an advantage in M&A was considered by Offenberg and Pirinsky (2015) in a partial equilibrium setting with one potential rival buyer for understanding tender offers, but in the presence of information frictions. Further to our knowledge this is the first paper to examine how this speed advantage fuels firm demand for liquidity stockpiles in general equilibrium.

of pricing and allocation in the M&A market. The theory is consistent with a variety of micro-evidence documented by Betton et al. (2008). The model yields a closed form surplus sharing rule for the initial bidder that depends on the financing choice of the initial bidder and a given level of competition (i.e. buyer-seller ratio). Furthermore, the theory generates a wedge between cash and externally financed offers (e.g. stock) which leads to a cash-premium that is increasing in the level of anticipated competition. This finding provides a possible rationalization of Malmendier et al. (2016) in which cash-offers provide on average a 15% cash-premium over stock-offers. The theory can also account for the co-existence of both cash and stock M&A offers as well as the correlation that stock offers tend to be larger than cash and are on average worse deals for acquirers. Finally, the model allows for real effects of monetary policy on firm's cash demand which can indirectly influence the incentives to innovate across buyers and sellers and therefore affect growth.

We calibrate the model to the US using moments on firm-level innovation, cash holdings and M&A activity, as well as aggregate census data on the entry rate of firms. We then re-calibrate the model to data from the 2010s but restrict adjustments in the parameters of only the entry costs, markups, holding cost of cash and fixed costs of the high productivity firms. We find that to account for the average cash/asset ratio rise observed in US firms, markups, entry costs and holding costs can only account for at maximum 24% of the increase. However, when allowing the transferable fixed production costs to vary, we can account for nearly the entire increase in cash holdings, that is, we can account for 94% of the 2015 level, with an 82% drop in the fixed costs of the high efficiency firms. This comes with a 7% increase in the concentration of firms and reduces aggregate innovation by 17% (although consumption growth itself remains fairly flat due to an increase in quality improvements).

Finally, the calibrated framework allows us to evaluate real-financial linkages of monetary policy in a setting where firms, not households are the marginal demanders of monetary assets. As documented by Chen et al. (2017), firms have over taken households to be the dominant holder of monetary assets globally. Despite this change, most frameworks evaluating optimal monetary policy have centered on the household as the primary demander of liquidity and has little to say for the current setting. We find that monetary policy has non-neutral, and significant short-run and long-run effects, including on the distribution of innovative activity, firm-size and aggregate growth through the M&A market. The results highlight how inflation can not only affect internal vs external growth incentives, but has heterogeneous impacts for small and large firms and the average terms of trade in M&A. The effects are non-monotonic, and can differ from standard monetary models where the marginal demanders of cash are households rather than large public firms who have outside options to rely on their own issued equity as a payment instrument. Perhaps most notably, the results demonstrate that the Friedman rule, or zero lower bound, is not in general optimal, since the speed advantage of cash can help facilitate more efficient reallocation of sales.

The remainder of the paper is organized as follows. We conclude this section with a review of the related literature. Section 2 documents some stylized facts which motivate the model. Section 3 describes the model while Section 4 presents the main theoretical results. Section 5 describes our model calibration and decomposes the secular changes. Section 6 quantitatively examines some policy counterfactuals, and we providing some concluding thoughts in Section 7.

Related literature: This paper builds off and contributes to several literatures. First, this paper contributes to the growing debate on the implications of the declining business dynamism observed across much of the developed world (e.g. Decker et al. (2017)). On one side, Covarrubias et al. (2019), Gutiérrez and Philippon (2017) and Grullon et al. (2019) argue that concentration has been the result of lax anti-trust and rising entrenchment of incumbent firms. Taking a less negative view, Autor et al. (2020) and Andrews et al. (2016) use micro panel data evidence from the US census and OECD nations respectively in support of a technological shift leading to winner-takes-most, 'superstar' firms. This paper tests and provides support for one potential driver of the superstar phenomenon with declining fixed costs of bringing on product to market, discussed by Bessen (2017) (e.g. Walmart's / Amazon's proprietary inventory management systems). Ma et al. (2016) finds that acquiring firms invest substantially in IT and hire less routine-intensive labour following an acquisition suggesting that these acquisitions facilitate a lowering / pooling of operating costs across pre and post-merged firms (or business lines).

This paper also contributes to the literature examining M&A market activity and its effect on misallocation in the macro-economy. Acquisitions can boost aggregate efficiency through re-allocating production inputs to higher productivity firms as in Jovanovic and Rousseau (2002), or achieving synergies in production like in Rhodes-Kropfe and Robinson (2008) or David (2021).[6] However, M&A can raise market power for incumbents, raising

[6]Study of the M&A market has increasingly been studied subject to search and matching frictions. Rhodes-Kropfe and Robinson (2008) study the assortative matching of firms in the merging of productivity, Levine (2017) studies the trade of seeds, but do not consider firm innovation and creative destruction. David (2021) examines the aggregate impact on growth of a real model of reallocation of firm productivity but without strategic considerations, opportunity for internal innovation, or financing frictions. Fons-Rosen et al. (2021) and Cortes et al. (2021) examines the substitution between acquisitions and internal innovative efforts and their anti-competitive and growth implications. Wang (2018) estimates anticipation effects embedded in merger premia, Celik et al. (2022) examine the role of equity M&A offers in mitigating adverse selection, particularly for more intangible, growth-oriented firms. Finally, Wright et al. (2018) study the aggregate implications of the interaction of frictional capital re-allocation with cash needed to facilitate trade for firms without access to alternative financing

anti-trust concerns (e.g. Mermelstein et al. (2020)) and reducing the returns for a new innovator to bring a product to market (see Phillips and Zhdanov (2013)).

Interest has been growing in the determinants of innovation and potential misallocation in innovative capacity. Notable papers in this vein is Acemoglu et al. (2018) who introduce fixed costs and heterogeneous R&D capabilities to examine the misallocation of R&D inputs and Akcigit and Kerr (2018) who examine heterogeneity in the types and quality of innovation between large and small firms. To our knowledge, the only papers to examine firms outsourcing or re-allocating growth opportunities through the M&A market are Phillips and Zhdanov (2013), and Levine (2017). Our paper examines a similar trade off of the former wherein bargaining power in the M&A market can influence small firms incentives to innovate, but in an endogenous growth, general equilibrium setting. In another related paper, Lentz and Mortensen (2016) examine the social value of buyouts by new entrants of incumbent firms' existing products. This paper currently abstracts from more pernicious aspects of M&A studied by Cunningham et al. (2021) of so-called 'killer acquisitions' in the pharmaceutical industry where innovation is stifled to protect incumbents existing products.

There is also some work examining linkages between cash holdings, concentration and growth and monetary policy. Liu et al. (2019) argue low long-term interest rates encourage market concentration by raising the benefit for industry leaders to gain a strategic advantage over followers. Our paper complements this work with the mechanism that lower opportunity costs of stockpiling liquidity increases the net benefits of being an acquirer. The only other equilibrium models linking monetary policy to innovation to our knowledge is Chu and Cozzi (2014) and Berentsen et al. (2012). The former imposes an exogenous cash-in-advance constraint on R&D and manufacturing expenditures. The latter assumes anonymity of entrepreneurs to induce a demand for cash. As such, their setting is not amenable to talk about the cash demand of large public firms who, by definition and in practice, have access to plethora of external financing options.

Finally, papers which examine the interaction of competition and cash holdings are Hoberg et al. (2014), Ma et al. (2014) and Galenianos and Kircher (2008).[7] The former two examine how cash provides strategic benefits in terms of flexibility in the face of highly dynamic/ competitive product

options tied to reputation.

[7]The corporate finance literature on the determinants of firm cash accumulation is extensive, beginning with Baumol (1952)-Tobin (1956) transaction cost motive, then extending to tax minimization (e.g., Foley et al. (2007), Faulkender and Petersen (2012)) or handling agency frictions (e.g., Jensen (1986), Dittmar and Mahrt-Smith (2007)). Explanations for the secular cash build-up focus on a selection effect of R&D intensive firms (e.g. Begenau and Palazzo (2021)), tax-based explanation (Faulkender and Petersen (2012)), precautionary balances driven by changing cost/production volatility (e.g. Zhao (2017)) or hybrids like tax-based explanation for IP-intensive firms (Faulkender et al. (2019)). Also explored are low carrying cost theories like Azar et al. (2016).

markets, while the latter shows how cash demand can be spurred by competition through auctions via a Burdett and Judd (1983) style mechanism. Our M&A market and cash demand can be thought of as Galenianos and Kircher (2008) with the elimination of the assumption that firms cannot access credit to finance their trades but the addition of a speed advantage of cash which helps preclude competition. This relaxation is particularly important when trying to understand the demand of cash from large public firms like Google who have a demonstrated ability to receive credit or external finance to fund transactions. Further, unlike in Galenianos and Kircher (2008) buyers and sellers select search intensities affecting the buyer seller ratio within matches and the probability that a bidding opportunity is actually available to the buyer. In other words, firm chosen M&A search intensities change the expected amount of competition in the M&A market as well as the anticipated bargaining power of the seller.

2 Stylized facts on M&A and cash use

This paper examines a firm's demand for cash arising from a combination of competitive pressures amongst buyers in the M&A market and a speed advantage of cash. To support this thesis, we document several stylized facts linking M&A market activity with US firm cash holdings and other firm characteristics.

The data is comprised of balance-sheet data from Compustat, transaction level M&A data from Thompson Reuters SDC Platinum and pairwise firm product similarity scores obtained from Hoberg and Phillips (2016). We examine the sample period 1990 to 2015 inclusive. We restrict the sample of acquisitions to those which were completed, were for controlling shares (over 50% ownership ex-post) and involved US firms as targets yielding a sample of 69790 transactions. We remove all firms from Compustat not of US origin and with assets less than $10 million.

In Figure 1, we plot the value-weighted average share of M&A transactions involving cash, cash utilization in the M&A market has jumped from about 50% to nearly 80% since the start of the 90s.[8] Scouring primary SEC merger documents, Liu and Mulherin (2018) document that the average number of solicited bidders in the M&A market has risen by approximately 70% and number of formal indications of interest has increased by roughly

[8]The striking shift in 2001/2002 seems to have been driven by an accounting regulation change by the Financial Accounting Standards Board (FASB) in January 20 2001. FASB regulation No. 141 removed the 'pooled interest' accounting method for mergers which allowed the book values of the merging firms to be added together rather than the fair value 'purchase method.' As the pooling interest method meant that a merger had no effect on reported earnings while the purchase method adds additional liabilities (e.g. goodwill impairments) to the acquiring firm, stock acquisitions could benefit from using the pooled interest method yielding an advantage over cash acquisitions (which were constrained to use the purchase method).

40% in the same time interval.[9] Thus, given the roughly 60% increase in cash use within the M&A market observed over this period, there seems to be a roughly one-to-one increase in M&A cash usage share to an increase in number of solicited bidders prior to an M&A transaction (which corresponds to our notion of M&A competition in the paper).

We now move to our third piece of evidence motivating our model ingredients and subsequent analysis, that is linking cash holdings to mergers, product competition and innovation. From the above results, given the size of M&A transactions, with the higher cash usage in M&A and total M&A transactions increasing over time, a rise in cash holdings for firms active in M&A seems natural although not formally established to our knowledge.

To provide some empirical basis for this relationship, we estimate a logistic regression predicting a firm's likelihood to acquire based off of firm characteristics and competition from their closest product market rivals (based on Hoberg and Phillips (2016) product similarity data) to further inform our model ingredients. The results of the logit regression can be found in Table 1. There we see that firms are more likely to acquire if they (1) are more profitable firms in the high tech sector, (2) have higher cash growth and (3) have lower (physical) investment. Further, higher product market competition strongly predicts future acquisition activity, first in terms of the closeness of their competitors' products and second by the percent of their top 10 closest rivals who acquired the previous year. These latter two, while unexamined by Hoberg et al. (2014), are consistent with their findings on the link between product market competition.

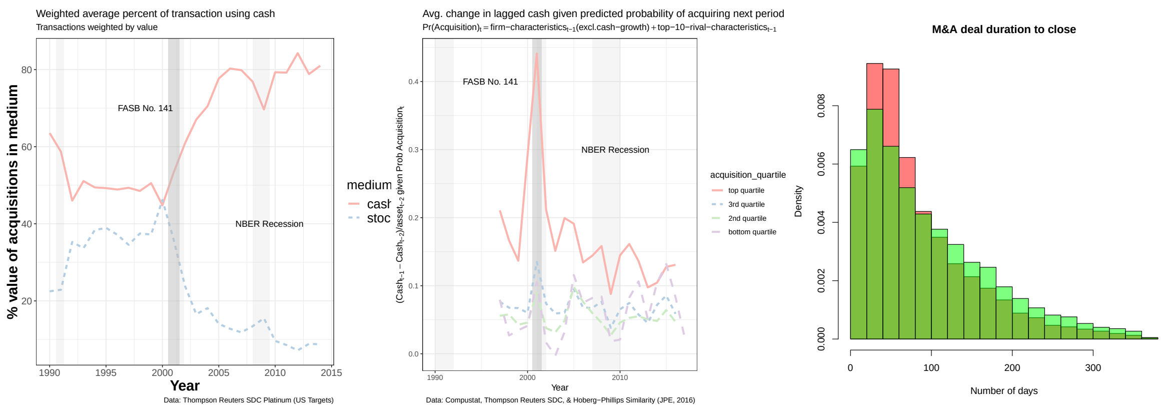
Next, in Figure 2, we plot the average cash holdings of firms within fitted quartiles of acquisition probabilities in the next year where we use the same specification as in Table 1, except excluding cash growth to prevent a mechanical relationship. Here we see that cash growth by quartile of acquisition probability is rank ordered, so that higher probability of acquiring implies higher cash growth the previous year. Further, due to the accounting regulation change (FASB reg 141) in 2001, there was a huge spike in the cash growth of firms in the sample, with greater spikes for higher quartiles in the acquisition likelihood. This suggests that cash accumulation is strategically done in anticipation of acquisition needs, not the other way around as suggested by Harford (1999) and Harford et al. (2008).

Finally, we give some support for the claim that cash offers in M&A generally provide a speed advantage over stock. In Figure 3, we plot the empirical distribution of the number of intervening days between date announced and date M&A deal is completed conditional on 100% cash or 100% stock offers (in red and green respectively). Crucially, we condition on there being no competing bids recorded during the intervening time to avoid de-

[9] Notice that this is in stark contrast to the number of publicly reported bidders in SEC filings, which has fallen and the percent of M&A deals resulting in a publicly announced auction dropped by 75%.

lays associated with competing offers, rather than driven by the payment method. Here we see that roughly speaking the cash offer duration distribution stochastically dominates the stock offer distribution (beyond the first 20 day window, where the two have similar probability). That is, cash offers probabilistically have a shorter duration, with the average deal being completed in roughly 10 fewer days.[10]

Figure 1: M&A and cash-use stylized facts



(a) M&A Payment Medium (b) M&A Cash Accumulation (c) M&A Closing Times

Left panel: Value-weighted average share of controlling M&A transactions of US targets by medium of exchange in cash or stock. Middle panel: Average lagged cash growth within fitted value quartiles from the logistic regression in Table 1 excluding the cash growth variable from the regression. Right panel: Time to closing of M&A transactions conditional on medium of payment being 100% cash (red) or 100% stock (green) offers. Duration is computed as the difference between the announced and completed date in SDC-platinum dataset. The sample is restricted to public parent targets with no competing bids in the window and a non-zero duration which is less than a year. Source: Thompson-Reuters SDC Platinum (US targets) and Compustat Quarterly.

3 Theory of M&A competition and cash use

We present here a static partial equilibrium model of M&A competition and cash demand which we will later embed into a dynamic general equilibrium quantitative framework. The model builds off Burdett-Judd (1983) and Galenianos and Kircher (2008) with search and matching frictions but

[10]SEC regulations are likely the proximate cause. In particular, for US acquisitions SEC Rule 14d-1 requires a tender offer statement only on the day the offer is made and can be fully executed within 20 days, while according to SEC rule 14d-6 stock exchange offers / mergers must distribute a proxy statement at least 20 days before a vote. Furthermore, in general antitrust reviews for stocks are constrained to 30 days for stock and only 15 days for cash tender offers under the 1976 Hart-Scott-Rodino Antitrust Improvements Act. Finally, if firms wish to use cash, the funds must already effectively be put in place in advance of the offer since the 'prompt payment regulation' SEC Rule 14e-8(c) stipulates that the firm must pay for all tendered shares within three days of the tender close and SEC Rule 14e-8(c) deems any offer fraudulent if it fails to have a reasonable belief of being able to purchase the securities sought. For more in-depth discussion of the regulations and their links to a speed of execution advantage of cash, see Offenberg and Pirinsky (2015).

incorporating stochastic arrival of rival bidders over an endogenous bidding window determined by the payment method of buyers.

3.1 Environment

There is a continuum of firms split between a fixed mass of firms M^B which are seeking to expand via acquisitions and a fixed mass of firms M^S which are seeking to be acquired. Prospective buyers have surplus $\Sigma^B - p$ from acquiring another firm at price p while prospective sellers have surplus $p - \Sigma^S$ where $\Sigma^S < \Sigma^B$ is the seller's standalone value tied to retaining control of their firm. Prospective buyers are randomly matched to the sellers via an urn-ball matching technology. The number of buyers in the running to acquire a given seller is thus stochastic.[11] In a twist relative to Burdett-Judd (1983) and Galenianos and Kircher (2008), prospective buyers matched to the seller stochastically arrive to bid on acquiring the seller. The first bidder is uniformly drawn from the set of prospective buyers, so for a given buyer, the probability of being the first bidder is $\frac{1}{1+N}$ where N is the number of rival prospective buyers in the market. Neither the buyers nor sellers know the realized number of buyers N available to bid on the seller's firm. Consequently, the probability of being the first bidder is

$$\nu = \mathbb{E}\left[\frac{1}{1+N}\right]. \tag{1}$$

The first bidder to arrive (hereon the 'initial bidder') makes a take-it-or-leave-it offer to the seller which the seller can accept or reject. However, due to frictions in the transaction process and a lack of commitment by the seller, any agreed upon deal prior to T can be overturned by another round of bidding upon arrival of new bidders.However, due to go-shop mandates of the seller, the seller must wait for a minimal window of time T for another competing bid to arrive before closing the deal. Subsequent buyers who arrive prior to T make competing bids observing the buyer's bid and surplus. With the initial bidder able to freely revise their bid in response to the new arrival and continuously bid throughout the time window T, a second price auction ensues between the arrived bidders.[12]

We assume that the initial bidder, besides being able to choose the amount of their bid, can also choose the payment method of either internal funds (cash) or external financing (stock). The only distinction between the two payment methods in this partial equilibrium setting is that cash offers allow deals to close faster than externally financed offers which require more time for the seller to vet and get approval from shareholders.That is, cash

[11] This assumption is a stand-in for technological or industry related restrictions on prospective buyers, as well as idiosyncratic synergistic benefits for certain pairs of buyers and sellers.

[12] We can allow for private information if using dominant strategies in a second-price auction as in Galenianos and Kircher (2008).

offers have a bidding window T_c which is less than the external financed offer T_s. The seller presented with these offers also retains the ability to reject the initial bidder's offer with a longer go-shop period T_R,

$$T_c < T_s \leq T_R. \tag{2}$$

While the seller benefits from a longer go-shop window through the ability to attract more bidders, there is an exogenous trade breakdown probability for the seller $\chi(T)$ which increases with the length of the go-shop period T, so that

$$\chi_c < \chi_s \leq \chi_R.$$[13]

3.2 M&A cash-premia and hazard of competing bids

The homogeneity of prospective buyer's value of the seller's firm combined with the stochastic, but commonly observed, buyer bid arrival results in full trade surplus passed to the seller in the event more than one buyer arrives during the seller's go-shop window, T. Denote $p_d(N)$ as the equilibrium bid price contingent on N rival buyers appearing against the initial bidder, and d the payment method of the initial bidder. Then the equilibrim bid price for any non-zero number of arrived rival bidders N is

$$p_d(N) = p^* \equiv \Sigma^B. \tag{3}$$

Evidently buyers in this stylized setting with common values only receive positive surplus if they are the first bidder to arrive, no rival bids materialize and trade does not break down,

$$\begin{cases} \Sigma^B - p_d(N), & N = 0 \\ 0, & N > 0 \end{cases}. \tag{4}$$

Fixing the payment method d of the initial bidder, the initial bidder chooses their bid $p_d(0)$ to maximize their expected surplus given the probability of being the first bidder ν and the probability of trade breakdown $\chi(T)$. Since their initial bid is a take-it-or-leave-it offer and their value is strictly increasing in the bid, the bid will be set to make the seller indifferent between accepting or rejecting. Denoting $W_d^S(p_d)$ as the seller's expected surplus given the arrival of an initial bidder offering payment type d and price p_d, their expected surplus is

$$W_d^S(p_d) = (1 - \chi_d) \cdot (\mathbb{E}_d[p] - \Sigma^S), \tag{5}$$

where χ_d is the probability of trade breakdown given payment method d and $\mathbb{E}_d[p]$ is the expected bid price given payment method d.

[13]This trade breakdown is a stand-in for anti-trust intervention, shareholder activism or exogenous changes in the seller or buying firms cirmcumstances and interest in acquisition.

Given the continuum of firms and matching primitives, it can be shown that the arrival of rival buyers follows a continuous time Poisson process with arrival rate θ. While the unconditional number of rival buyers N is Poisson distributed, given the stochastic arrival of buyers with exponential inter-arrival times, we establish in Appendix B.1 the lemma below.

Lemma 3.1. *The probability of an auction with payment method d is*

$$\mathbb{P}_d(N > 0) = 1 - e^{-\theta_d} \tag{6}$$

where $\theta_d \equiv 1 - e^{-\theta(1-e^{-T_d})}$.

With this result, the expected selling price given payment method d is

$$\mathbb{E}_d[p] = e^{-\theta_d} \cdot p_d(0) + (1 - e^{-\theta_d}) \cdot p^* \tag{7}$$

where $p^* = \Sigma^B$ is the auction price. Consequently, the initial bidder's bid $p_d(0)$ is set to make the seller indifferent between accepting or rejecting the offer,

$$\min_{p_d(0)} \max\{W_d^S(p_0(d)), W_R^S\}$$

where W_R^S is the seller's expected surplus from rejecting the initial bidder's offer, retaining the longer go-shop window T_R, but not reserving a committed price by the initial bidder at time 0. This implies that should no other bidders arrive during the go shop window, the initial bidder will in equilibrium extract the entire trade-surplus from the seller.

$$W_R^S = (1 - \chi_R)\left[e^{-\theta_R} \cdot 0 + (1 - e^{-\theta_R})(p^* - \Sigma^S)\right] \tag{8}$$

Solving for $p_d(0)$, we obtain equilibrium bid price for the initial bidder given payment method d given in Theorem 3.2.

Theorem 3.2.

The equilibrium bid price for the initial bidder given payment method d is

$$p_d(0) = \Sigma^S + \beta_d(\theta) \cdot \mathbb{S} \tag{9}$$

where $\mathbb{S} = \Sigma^B - \Sigma^S$ denotes the total trade surplus and

$$\beta_d(\theta) \equiv \left[\frac{(1 - e^{-\theta_R})}{e^{-\theta_d}} \left(\frac{1 - \chi_R}{1 - \chi_d}\right) - \frac{(1 - e^{-\theta_d})}{e^{-\theta_d}}\right]. \tag{10}$$

denotes the implied bargaining parameter for the seller given payment method d and market tightness θ.

From this result we see that provided $\beta_d(\theta) \in [0, 1]$, the equilibrium initial bid price is isomorphic to a generalized Nash bargaining solution with bargaining parameter β, except here the bargaining parameter is endogenously determined by the market tightness θ, the trade breakdown probability $\chi(T_d)$ and go shop window T_d.

Corollary 3.1.
If $\chi_R < 1$ is not too large, then the bargaining power coefficient $\beta_d(\theta) \in [0, 1)$ and is strictly increasing in θ for $\theta \in [0, \bar{\theta}_d)$, $\bar{\theta}_d > 0$.

Proof. See Appendix B.2 □

Reflecting that $\theta = \mathbb{E}[N]$ is the expected number of rival buyers this result tells us that for θ not too large provided (χ_R similarly not too close to 1), the initial M&A bid prices are strictly increasing in the amount of anticipated competition.

Intuitively, since a shorter bidding window reduces the hazard of competing bids and the likelihood of a trade breakdown, the buyer all else equal prefers to use cash over stock as a payment method. However, the seller, in accepting the cash offer, is lowering their option value to waiting, and so will demand a higher initial bid price as compensation. This is formally established in the following theorem.

Theorem 3.3 (M&A cash premium)**.**
If $T_c < T_s$, then $\beta_c(\theta) > \beta_s(\theta)$ and a cash premium exists for any $\theta > 0$,

$$p_c - p_s > 0, \tag{11}$$

where $p_s = p_s(0), p_c = p_c(0)$ are the equilibrium initial bids to the seller contingent on payment of stock and cash respectively.

This result speaks to the empirical puzzle of Malmendier et al. (2016) finding that all-else equal, all-cash bids yield a premium over all-stock bids. With idiosyncratic exogenous variation in cash holdings and willingness to use cash in M&A, this result is sufficient, however, buyer's may not be indifferent to using one or the other given this premium, which is what we investigate in the next subsection.

3.3 Equilibrium M&A cash use

In Theorem **??** we saw that seller's demand a cash-premium on initial bids made in cash due to the lower hazard of an auction. Thus, it is not immediate that the initial bidder will prefer to make a cash offer over a stock offer. Nevertheless, we show in the next theorem that initial bid cash offers are prefered to stock offers when feasible.

Theorem 3.4 (Initial bidder cash preference)**.**
Despite a cash-premium, provided $\chi_s > \chi_c$, buyer's strictly prefer an initial cash offer to stock.

Proof. See Appendix B.4. □

The key to this result is that while cash shifts the implied bargaining power in favour of the initial buyer, the total expected surplus increases with the lower likelihood of trade-breakdown from the shorter trading window. Consequently, the pecuniary costs to compensate the seller do not fully sanitize the cash benefits of the buyer.

Absent any holding or equity issuance costs, firms can freely substitute equity for cash. With a small holding cost of cash and the need to accumulate cash in advance if desiring to make a cash bid, firms will hold only the minimal cash necessary. Consequently, cash demand is degenerate and given by the initial cash bid price p_c. In light of the initial cash bid generally increasing in anticipated competition θ, firm cash demand does as well.

These tradeoffs induce a natural pecking order of payment methods in M&A transactions, with cash being used absent observed competitors, and stock being used for larger deals with lower cumulative abnormal returns (CAR) for acquirers. We collect these implications in the following corollary.

Corollary 3.2 (M&A pecking order and competition)**.**
With small, non-zero carrying cost of cash, buyer's cash holdings is given by the initial bid $p_c(\theta)$, while auction price p^* paid with stock.

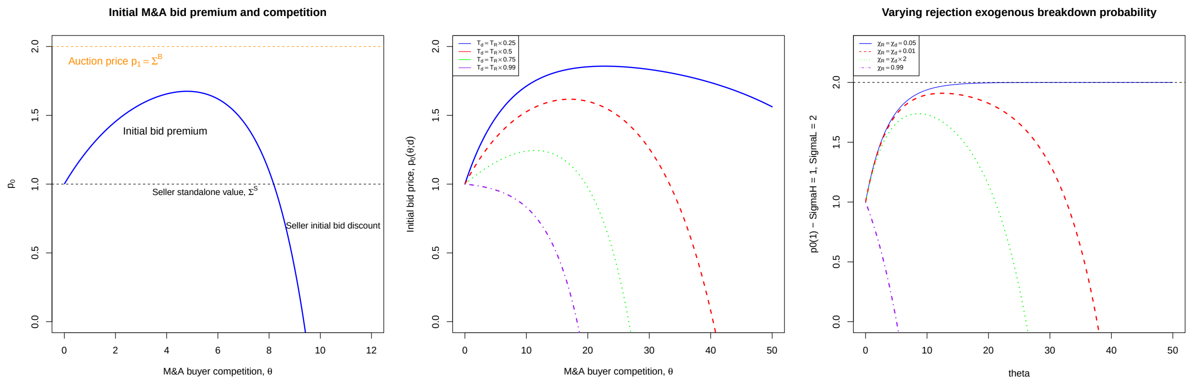
The fact that cash demand is degenerate with no cost of external financing is unsurprising, the fact that it may nevertheless be demanded in equilibrium for a speed of execution advantage is to our knowledge a novel channel. The dynamics of the initial cash offer $p_c(\theta)$ as a function of the buyer-seller ratio θ is depicted in Figure 2a. There we see that the bid increases in competition for levels of θ not too high, thereby leading to an implied bargaining power coefficient between 0 and 1, but for sufficiently high levels of competition the initial offer can actual decrease in competition as the seller deems the odds of the initial deal being consummated goes to zero. Having an offer on hand with a shorter bidding window will reduce the likelihood of an exogenous breakdown. For sufficiently high levels of competition this reduction in the likelihood of exogenous breakdown is sufficiently large that the target would actually be willing to pay some positive amount to get the lower breakdown probability.[14]

4 Firm Cash Stockpiles & Endogenous Growth

We embed the stylized M&A model from the previous section into a workhorse model of endogenous growth and firm dynamics of Klette and Kortum (2004). Here we endogenize the holding cost of cash, the mass and value of buyers and sellers in the M&A market, as well as introduce general equilibrium tradeoffs between internal and external growth activities.

[14]In general, absent some costly adjustment or regulatory restriction, this negative premium would be eliminated in equilibrium if the seller could choose a priori the length of their outside bidding window (and thereby the implicit probability of exogenous trade breakdown).

Figure 2: M&A Initial Bid Comparative Statics



(a) Pricing & competition (b) Speed advantage T_d (c) Breakdown advantage χ_R

Left panel: Initial cash bid price $p_0(1;\theta)$ as a function of competition. Right panel: Comparative statics of initial bid price as a function of competition, varying the relative bidding window, breakdown probability of cash versus the rejection window. Expected M&A competition is the buyer-seller ratio θ. Target firm surplus Σ^T is normalized to 1, buyer surplus is set to $\Sigma^B = 2$ which is also the auction price p_1. Left panel: $\psi = 1, T_d = 1, T_R = 3, \chi_d = 0.050, \chi_R = 0.055$ Middle panel: $\psi = 1, T_R = .54, \chi_R = 0.08, \chi(d) = 0.05$. Right panel: $\psi = 1, T_d = 1, T_R = .54, \chi_d = 0.05$

4.1 Environment

The environment builds off Klette and Kortum (2004) where the economy consists of a unit measure of horizonaly differentiated goods consumed by a representative household. The representative household has symmetric Cobb-Douglas preferences so that their expenditure is the same across the horizontally differentied goods. Households supply their labour inelastically, receiving a flow of labour income and profits from their ownership of firms. Setting expenditures as the numeraire, intertemporal smoothing sets the risk-free interest rate r_t equal to the household's discount rate, ρ.

There are a continuum of firms which produce the horizonaly differentiated goods and are owned by the household. For a given good, firms are vertically differentiated in the quality of the good they produce, with each firm retaining a distinct quality level. Firms engage in R&D to innovate on a quality ladder, with successful innovations leading to fixed quality improvement $q > 1$ above the previous quality level of a good. Price competition between firms is such that each good is produced by a single firm which is the quality-leading producer who earns variable profit flow $\pi = 1 - \frac{1}{q}$.

Operating firms consist of a portfolio of n goods they are the frontier quality leading producer of. Assuming the knowledge capital embodied in the team creating a frontier product is able to be leveraged to create new products, R&D technology of incumbents consists of n independent product teams exerting convex labour effort $c_\iota(\iota)$ to innovate at (Poisson) intensity ι on a rival product line. Firms engage in R&D to innovate on the quality ladder, with successful innovations leading to replacing the previous incumbent in that market.

In a process of creative destruction, one firm's product innovation corresponds to the product obsolesence of another. Consequently, operating firms evolve over time as a stochasttic birth-death process. That is, born with a single frontier product line upon innovating on an incumbent product line, growing with product expansions and shrinking with product obsolesence until they lose their last quality leading product and exit.

In a first twist to Klette and Kortum (2004), we introduce a fixed flow cost of operating each product line, τ expressed in terms of the numeraire, which we assume is the same for all product lines of a given firm. We take this τ to be permanent intangible characteristic of the firm drawn upon entry, and heterogenous across firms. For simplicity, we take τ to take one of two values, $\tau \in \{\underline{\tau}, \bar{\tau}\}$, with $\underline{\tau} < \bar{\tau}$. This heterogeneity in operating a product line that is tied to the firm induces a fundamental role for the M&A market in reallocating product lines to firms with lower operating costs. Thus, in the second twist relative to Klette and Kortum (2004), akin to R&D, we allow firms to exert separate convex labour efforts $c_\gamma(\gamma)$ to access the M&A market as a buyer at rate γ and $c_\lambda(\lambda)$ to access the M&A market as a seller at rate λ. Once arriving in the M&A market, firms match and trade as described in the previous section. Finally, with the motive for cash-use in M&A highlighted in the previous section, we allow firms to accumulate cash stockpiles m in anticipation of the stochastic arrival of M&A opportunities.

The aggregate stock of money, S_t is exogenously supplied by the monetary authority and grows at rate $1 + \phi$. Cash injections are provided lump-sum to households. The value of money at time t is φ_t in terms of the numeraire and is determined by market clearing $S_t = S_t^D$. As we will be restricting attention to stationary (balanced growth path) equilibria, the real money demand is taken to be constant, so that

$$\varphi_{t+1} S_{t+1}^D = \varphi_t S_t^D \tag{12}$$

The entry of a new firm requires an innovation of some product which arrives at intensity h. The cost of entry is amount of labour hired. Thus, the free entry condition for intermediate producers is:

$$\sum_\tau V_t^\tau(1)\Upsilon(\tau) = \frac{w}{h} \tag{13}$$

where $\Upsilon(\tau)$ is the proportion of entrants who have cost type τ (realized after entry) and $V_t^\tau(1)$ is the value of a firm being size 1 with cost type τ at time t, and starting without cash. New product arrival rate of an incumbent firm of type x with n products is $\iota(x, n)$. Integrating out over the distribution of cash holdings for a given type and taking $M_n(\tau)$ as the measure of firms with cost τ and n products, we have the equilibrium

$$\delta = \eta + \sum_\tau \sum_{n=1}^{\infty} \int_{\tilde{m}} \iota(\tilde{x}) n M_n(\tau) d\widehat{F}(\tilde{m}; \tau, n) \tag{14}$$

where $\widehat{F}$ is the distribution of cash holdings conditional on firm type τ and size n.

Denote $\Lambda(\tau, n)$ as the equilibrium rate of a firm of type τ and size n selling a product line in the M&A market, $\Gamma(\tau, n)$ as the equilibrium rate of a firm of type τ and size n buying a product line, and $\widehat{\iota}(\tau, n)$ as the expected rate of innovation for a firm of type τ and size n. Since firms can only move up/down one product line at a time, no net in/out-flows implies the steady state firm distribution satisfies for $n \geq 2$:

$$\begin{aligned}&[\widehat{\iota}(\tau, n-1) + \widehat{\Gamma}(\tau, n-1)](n-1)M_{n-1}(\tau) + [\delta + \Lambda(\tau, n+1)]M_{n+1}(\tau) \\ &= (\widehat{\iota}(\tau, n) + \delta + \widehat{\Gamma}(\tau, n) + \Lambda(\tau, n))nM_n(\tau).\end{aligned} \tag{15}$$

As when an incumbent loses his last product line, he dies, while new entrants flow in at rate η and the probability of being type τ is $\Upsilon(\tau)$, so for $n = 1$:

$$\Upsilon(\tau)\eta + [\delta + \Lambda(\tau, 2)]2M_2(\tau) = (\Gamma(\tau, 1) + \Lambda(\tau, 1) + \iota(\tau, 1) + \delta)M_1(\tau) \tag{16}$$

since births equal deaths in steady state $\Upsilon(\tau)\eta = [\delta + \Lambda(\tau, 1)]M_1(\tau)$.

There is a fixed labour pool L which is allocated across production, R&D and M&A activities. Denote $L_X(n, x)$ as the amount of labour demanded for producing the intermediate goods by firm of size n and cost type τ. Denote $L_R(n, x) = nc(\iota(x))$ as the amount of research demanded and $L_A(n, x) = nc_A(\gamma(x)), L_T(n, x) = nc_T(\gamma(x))$. Finally, let $L_E = \frac{\eta}{h}$ denote the number of researchers in new startups. Thus,

$$L = \sum_{\tau}\sum_{n=1}^{\infty}\int_{\tilde{m}}[L_X(n, x) + L_R(n, x) + L_A(n, x) + L_T(n, x)]d\widehat{F}(\tilde{m})M_n(\tau) + L_E \tag{17}$$

Total fixed costs of production υ act as a drag on total consumption, and is given by

$$\upsilon = \sum_n M_n(\bar{\tau})\bar{\tau} + \sum_n M_n(\underline{\tau})\underline{\tau}. \tag{18}$$

Finally, the equilibrium market tightness is given by

$$\theta = \frac{\sum_{n=1}^{\infty}\sum_{\tau}\int_m \gamma_n(x)nM_n(\tau)d\widehat{F}(m)}{\sum_{n=1}^{\infty}\sum_{\tau}\int_m \lambda_n(m, \tau)nM_n(\tau)d\widehat{F}(m)}. \tag{19}$$

Definition 4.1 (Equilibrium definition)**.** A steady-state equilibrium is a list

$$\{V_n(x), W_n^A, W_n^T, d(x, x_T), \mathbb{P}(d), p_s, p_c, M_n(\tau), \widehat{F}, \widehat{H}, \varphi, \theta, \Upsilon(\tau)\}$$

which is characterized by the tuple $(w, \eta, \delta, \theta, \varphi)$ such that

1. Given prices, inflation ϕ and market tightness, $m' \in \text{supp}\widehat{F}_t$, $\iota_n(x), \gamma_n(x), \lambda_n(x)$ solves firm's investment savings problem (20)
2. Initial bidder payment choice $d(x, x_T)$ solves (26)
3. M&A initial bid and auction prices $p_s(\tilde{x}, \tilde{x}_T), p^*(\tilde{x}, \tilde{x}_T), p_c(s)$ solve (9) and (3)
4. φ ensures money market clearing satisfied, $S_t = S_t^D(\varphi)$
5. Labour market clearing (17) and free-entry holds (13)
6. Beliefs about market tightness θ satisfies (19), creative destruction rate (14), cash distribution $\widehat{F}$, buyer surplus distribution $\widehat{H}$, are consistent.

We restrict attention to equilibria where (i) $\iota(n,\tau) + \Gamma(n,\tau) < \delta$ for all types (x, n) (to have a finite firm size distribution) and (ii) real-balances are constant over time, i.e. $\varphi_t S_t^D = \varphi_{t+1} M_{t+1}^D$.

4.2 Firm investment and valuation

We present the firm problem in discrete time but formulated to be consistent with expressions obtained when taking the continuous time limit.[15] Let $x = (m, \tau)$ denote the state of the firm beside the number of product lines n. The standalone value of a firm is then given by

$$\begin{aligned}(1+r)V_{n,t}(x) &= \varphi m + n[\pi - \tau] \\ &+ \max_{m' \geq 0} \Big\{ -\varphi m' + \\ &\max_{\iota} \big\{ n\big(\iota \mathbb{E}\big[V_{n+1,t+1}(x') - V_{n,t+1}(x')\big] - c(\iota)w\big)\big\} \\ &+ \max_{\gamma \geq 0} \big\{ n\big(\gamma \mathbb{E}\big[W_n^A(x') - V_{n,t+1}(x')\big] - c_A(\gamma)w\big)\big\} \\ &+ \max_{\lambda \geq 0} \big\{ n\big(\lambda \mathbb{E}\big[W_n^T(x') - V_{n,t+1}(x')\big] - c_T(\lambda)w\big)\big\} \\ &+ n\delta\big[V_{n-1,t+1}(x') - V_{n,t+1}(x')\big] + V_{n,t+1}(x')\Big\}.\end{aligned} \tag{20}$$

where $x' = (m', \tau)$, $W_n^A(x')$ ($W_n^T(x')$) is the value of being a buyer (seller) of size n entering the M&A market from Section 3 with cash m' and cost τ.

The first line contains the static flow dividends paid out by the firm if all cash is divested and absent any dynamic considerations. It includes the

[15] That is, in the discrete time formulation, we assume the arrival probabilities ι, λ, γ are such that the joint occurrence of two events (innovation / acquisition opportunity) simultaneously may be taken to be zero (consistent with being Poisson arrival rates in continuous time). To facilitate consistent expressions across continuous and discrete time, we implicitly re-scale all of the contemporaneous variables in the firm problem, e.g. $\tilde{\pi} = \beta\pi$, and $\beta = (1+r)^{-1}$. While Choi and Rocheteau (2020) shows in general formal equilvalence between the discrete time and continous formulations is not assured, with risk neutral agents and linear payoffs discrepancies vanish.

profit flow from the firm's n product lines net the flow fixed cost of operating each one τ. The second line introduces the static cost of cash accumulation in current value, while the benefits of cash accumulation materialize in the option values of the firm and their growth opportunities. The third line contains internal growth opportunities through R&D investment. The fourth line the external growth opportunities through acquisition in the M&A market. The fifth line contains the future value of selling a product line in the M&A market. The sixth line contains the independent obsolesence of each of their existing product lines.

Firms that gain access into the M&A market at a given point of time are randomly matched and trade as in Section 3. By inspection of (20) it is clear that firms have quasi-linear preferences in cash m. Further, following similar logic to Lentz and Mortensen (2005), and leveraging the homotheticity of the non-monetary components of the firm's costs and benefits, we guess and verify that the non-monetary portion of the value-function maintains this scale-invariance property within firm cost types.

Crucial for this conjecture to hold is that the cash decision is invariant to the number of product lines n held by the firm. We will verify through subsequent arguments the following result.

Theorem 4.1 (Symmetric, scale-invariant equilibrium firm value).

In a symmetric, scale-invariant equilibrium, the value of a firm with n product lines and cost τ is

$$V_n^\tau(m,t) = \frac{\varphi_t}{1+r}m + n\Sigma^\tau + \frac{R_0^\tau}{r} \tag{21}$$

where Σ^τ is the fundamental surplus of a product line for firm type τ and R_0^τ is the present value of having fixed cost τ.

Observe that with this guess the time-dependence on the value function is only through the value of money φ_t. From hereon, we will drop the explicit time dependence and use φ' to denote the value of money in the next period.

With this form of the value function, we have directly that the private expected value of firm innovation is

$$n \cdot \iota[V_{n+1}(m', \tau) - V_n(m, \tau)] = n\iota\Sigma^\tau.$$

Similarly, with this guess, the ex-post M&A trade surplus for an aquiring firm paying some price p is

$$S_n^A(m-p,\tau) = V_{n+1}(m-p,\tau) - V_n(m,\tau) = -\frac{\varphi' p}{1+r} + \Sigma_\tau$$

while for a target firm it is

$$S_n^T(m+p,\tau) = V_{n-1}(m+p,\tau) - V_n(m,\tau) = \frac{\varphi' p}{1+r} - \Sigma_\tau$$

where $\varphi'/(1+r)$ is the real value of cash.

Combining the above, the total trade surplus is

$$\mathbb{S}(n^A, m^A, \tau^A, n^T, m^T, \tau^T) = S_n^A(m^A - p, \tau^A) + S_n^T(m^T, \tau^T) = \Sigma^{\tau^A} - \Sigma^{\tau^T}$$

consequently, it follows that in a symmetric, scale-invariant equilibrium, only firms with different fixed costs τ will trade in the M&A market. That is naturally only firms with higher fixed costs will be sellers and those with lower fixed costs will be buyers.

Corollary 4.1 (Perfect sorting in M&A)**.**

In a symmetric, scale-invariant equilibrium with $\tau \in \{\underline{\tau}, \bar{\tau}\}$, there is perfect sorting of acquirers and targets in M&A, with

$$\Sigma^A = \Sigma^{\underline{\tau}} > \Sigma^{\bar{\tau}} = \Sigma^T.$$

With these observations, the expected private value of searching in the M&A market can only be strictly positive for high cost firms $\bar{\tau}$ searching as a seller, and for low cost firms $\underline{\tau}$ searching as a buyer. The expected private value of searching in the M&A market as the low cost firm acquiring is then

$$n \cdot \gamma\nu[W_n^A(m', \tau) - V_n(m, \tau)] = n\gamma\nu(1 - \chi_d)e^{-\theta_d} S^A(\underline{\tau}, p_d)$$

$$= n\gamma\nu(1 - \chi_d)e^{-\theta_d}\left(1 - \beta_d\right)\mathbb{S}$$

where p_d is the price paid by the low cost firm in the M&A market using d payment method as the initial bidder. Similarly, the expected private value of searching in the M&A market for the high cost firm selling is

$$n \cdot \lambda\nu[W_n^T(m', \tau) - V_n(m, \tau)] = n\lambda\nu(1 - \chi_d)\left(e^{-\theta_d} S^T(\bar{\tau}, p_d) + (1 - e^{-\theta_d})S^T(\bar{\tau}, p_1)\right)$$

$$= n\lambda\nu(1 - \chi_d)\left(\Sigma^{\bar{\tau}} + e^{-\theta_d}\beta_d\mathbb{S} + (1 - e^{-\theta_d})\mathbb{S}\right).$$

Plugging in the above into the high cost firm's investment-savings problem (20) we have

Corollary 4.2 (Target optimal investment policies)**.** *Taking d as the anticipated payment method of the initial bidder, the target firm optimal policies are $\gamma^{\bar{\tau}} = 0$ and $\lambda^{\bar{\tau}} > 0$ such that*

$$w \cdot c'(\iota^{\bar{\tau}}) = \Sigma^{\bar{\tau}} \tag{22}$$

$$w \cdot c'_\lambda(\lambda^{\bar{\tau}}) = \tilde{\Lambda}\left[\Sigma^{\bar{\tau}} + \tilde{\beta}\mathbb{S}\right] \tag{23}$$

where $\tilde{\Lambda} \equiv \theta\nu(1 - \chi_d)$ is the probability of a successful trading opportunity as a seller and $\tilde{\beta} \equiv e^{-\theta_d}\beta_d + (1 - e^{-\theta_d})$ is the expected share of the total surplus received from trade.

Proof. We establish the result in Appendix **??**. □

By the same logic, the low cost firm's optimal investment policies are given in the following corollary.

Corollary 4.3 (Acquirer optimal investment policies)**.** *Taking d as the payment method of the initial bidder, the acquirer firm optimal policies are $\gamma^{\underline{\tau}} > 0$ and $\lambda^{\underline{\tau}} = 0$ such that*

$$w \cdot c'(\iota^{\underline{\tau}}) = \Sigma^{\underline{\tau}} \tag{24}$$

$$w \cdot c'_{\gamma}(\gamma^{\underline{\tau}}) = \tilde{\Gamma}(1 - \tilde{\beta})\mathbb{S} \tag{25}$$

Given the guess of value function, the cash decision is given by

$$\max_{m' \geq 0} -\varphi m' + \frac{\varphi' m'}{1 + r} + n\gamma\mathbb{S}\left(\hat{B}_c\{m' \geq p_c\} + \{m' < p_c\}\hat{B}_s\right)\{\tau = \underline{\tau}\}$$

where the first term is the reduction in dividend by the cash accumulation, the second term is the discounted future value of the unused cash the next period, the third term is the expected benefit of the cash in the M&A market with indicator functions for when the initial cash offer is feasible or not, and this value in M&A only applies for the low cost firm entering the M&A market as a buyer.

Combining the first two terms together and using the stationary value of money, $\varphi'(1 + \phi) = \varphi$

$$-\varphi m'\left(1 - \frac{\varphi'}{\varphi(1 + r)}\right) = -\frac{\varphi m'}{1 + i} \cdot i.$$

As $\hat{B}_c$ is independent of the cash holdings, it is clear that a threshold strategy applies weighing the above net holding cost of cash against the value.

Since the payment method of the initial bidder is determined by the cash decision during the investment-savings stage, with the above results we can now characterize the optimal cash policy decision(s).

Theorem 4.2 (Optimal cash policy)**.** *The optimal cash policies for target firms is zero, $m^{\bar{\tau}} = 0$, while for acquirer firms is given by $m^{\underline{\tau}} = p_c > 0$ if and only if*

$$n\gamma\left(B_c - B_s\right) \geq \frac{\varphi'}{1 + i} p_c \cdot i \tag{26}$$

which holds for all n if i sufficiently small.

Proof. See Appendix **??**. A sufficient condition for the above to hold is

$$\gamma \cdot 1 \cdot (\chi_s - \chi_c)e^{-\theta_s}\mathbb{S} \geq \frac{\varphi' p_c}{1 + i} \cdot i$$

□

Finally, we can now use these optimal policies to verify the guess and obtain the equilibrium values condensed in the next corollary.

The fact that cash demand is degenerate with no cost of external financing is unsurprising, the fact that it may nevertheless be demanded in equilibrium for a speed of execution advantage is to our knowledge a novel channel.

If we add costly external financing then a non-degenerate cash distribution for buyers will arise akin to Galenianos and Kircher (2008). In particular their setting can be obtained by assuming that a fixed cost to external finance is arbitrarily large, necessitating cash to be used in acquisitions, and those with higher cash having higher surplus. Furthermore, even without adding a pecuniary external financing friction, if $N > 1$ s.t. for $n = 1$, no cash demand is optimal, then there will be a mix of acquirers, small without cash and large with cash.[16]

An interesting implied feature of the current model setup is that since $p_c > p_c$, for low holding costs, we generate endogenously that cash will be used when the payment is low, while external financing will be used when the payment is large, consistent with the predictions of Myers and Majluf (1984) pecking order theory, but for very different reasons.

Corollary 4.4 (Equilibrium medium of exchange)**.** *In equilibrium, initial cash bids will have a cash-premium, while external financed deals will be for larger deal values,*

$$0 < p_s(0) < p_c < p_c, \tag{27}$$

and cash deal volume is $\Gamma \sum_n n M_n(\underline{\tau})$ *while stock volume is only on* p_c *so that the observed average stock value is larger than the average cash value.*

The following result is immediate given the value of p_c obtained above.

Corollary 4.5 (Firm specific value)**.** *In the 'all-cash' equilibrium, the firm-type specific adjustment in the value function is* $R_0(\bar{\tau}) = 0 \geq R_0(\underline{\tau})$ *where*

$$R_0(\underline{\tau}) = -i[\beta_c(\theta)\Sigma(\underline{\tau}) + (1 - \beta_c)\Sigma(\bar{\tau})] \tag{28}$$

Theorem 4.3 (Eq. surplus values)**.** *In equilibrium, conditional on the R&D / search intensities, the high / low cost surplus from innovations can be obtained in closed form, with the low cost surplus* $\Sigma(\underline{\tau})$ *given by*

$$\Sigma(\underline{\tau}) = \frac{\pi - \underline{\tau} - w[c(\underline{\tau}) + c_B(\underline{\tau})] + \widehat{\Gamma} S}{r + \delta - \underline{\iota}} \tag{29}$$

[16] In this case, the small acquirer type may also optimally put some search intensity into being a target and selling to these already established acquirer firms. To avoid this unnecessary, and ancillary complication one can introduce a small fixed cost of M&A that will preclude this low surplus acquirer to acquirer transaction.

the high cost firm surplus $\Sigma(\bar{\tau})$ *given by*

$$\Sigma(\bar{\tau}) = \frac{\pi - \bar{\tau} - w[c(\bar{\tau}) + c_S(\bar{\tau})] + \widehat{\Lambda}S}{r + \delta - \bar{\iota}} \tag{30}$$

and total surplus S *given by*

$$S = \frac{\bar{\tau} - \underline{\tau} - w[(c(\underline{\tau}) - c(\bar{\tau}))(1-\alpha) + c_B(\gamma) - c_S(\lambda)]}{r + \delta - \widehat{\Gamma} + \widehat{\Lambda}} \tag{31}$$

where $\widehat{\Gamma} = \gamma\tilde{\Gamma}(1-\beta)$, *and* $\widehat{\Lambda} = \lambda\tilde{\Lambda}\tilde{\beta}$.

Proof. See Appendix, section C.3 for proof. □

Thus we see that the surplus for the buyer is increasing in their static operating profits $\pi - \underline{\tau}$, increasing in the trade-surplus $\bar{\tau} - \underline{\tau}$, and decreasing in their total costs.

It is easy to see that $\Sigma(\underline{\tau}) > 0$ provided labour expenditures are higher for the low cost firms than the high cost firms (which is easily verified by the FOCs for ι, γ, λ).

4.3 General equilibrium existence

Using the scale-invariant investment policies of firms and the steady state balance equations for firm densities $M_n(\tau)$, inductive reasoning yields the closed form firm-size distribution conditional on types.

Theorem 4.4 (Equilibrium firm size distribution)**.**

Firm size distribution conditional on cost type τ *is logaritmically distributed with parameter* $(\frac{\underline{\iota}+\Gamma}{\delta})$ *for low cost firms and* $(\frac{\bar{\iota}}{\delta+\Lambda})$ *for high cost firms. That is, the conditional distribution of firm size* n *given cost type* τ *is given by*

$$\frac{M_n(\underline{\tau})}{M(\underline{\tau})} = \frac{\frac{1}{n}\left(\frac{\underline{\iota}+\Gamma}{\delta}\right)^n}{\log\left(\frac{\delta}{\delta-\underline{\iota}-\Gamma}\right)}$$

and for high cost firms is

$$\frac{M_n(\bar{\tau})}{M(\bar{\tau})} = \frac{\frac{1}{n}\left(\frac{\bar{\iota}}{\delta+\Lambda}\right)^n}{\log\left(\frac{\delta+\Lambda}{\delta-\bar{\iota}+\Lambda}\right)}$$

where total firm mass of type τ *is given by*

$$M(\tau) = \sum_{n=1}^{\infty} M_n(\tau) = \frac{\eta}{\delta + \Lambda(\tau)} \log\left(\frac{\delta+\Lambda}{\delta+\Lambda-\iota-\Gamma}\right)\frac{\delta+\Lambda}{\iota+\Gamma}\Upsilon(\tau)$$

Note that the distribution is more skewed on higher n (leftward skewed) for the low cost firms than high cost firms since it is immediate that $\frac{\iota+\Gamma}{\delta} > \frac{\bar{\iota}}{\delta+\Lambda}$. This captures the strong selection forces in the model, that low cost firms, grow faster, and survive longer, than high cost firms, leading to an over-representation amongst incumbents relative to what entry probabilities imply.

As can be seen intuitively, provided the target firm has non-negative cash, the surplus from a sale is independent of the target's cash holdings and hence given that the low type has zero surplus from acquiring, target's cash demand is zero. Thus, the demand for money, S_t^D is given by the acquirer's cash demand, which with no transfers, is $S_t^D = \sum_{n=1}^{\infty} M_n(\underline{\tau}) \int_0^{\infty} \tilde{m} d\widehat{F}_t(\tilde{m})$.

We restrict attention to size independent cash equilibria, so (26) holds for $n = 1$ provided $\theta \in [\underline{\theta}, \bar{\theta}]$ and so cash demand is positive, finite and given by

$$S_t^D = \sum_{n=1}^{\infty} M_n(\underline{\tau}) p_0(1).^{17}$$

Thus, equating supply and demand and solving for φ' we have

$$\varphi' = \frac{\sum_{n=1}^{\infty} M_n(\underline{\tau})}{S_t}(1+r)[\beta_1(\theta)\Sigma(\underline{\tau}) + (1-\beta_1(\theta))\Sigma(\bar{\tau})].$$

By definition $\varphi'(1+\phi) = \varphi$ where $1+\phi$ is the gross rate of money growth, we then have

$$\varphi = \frac{\sum_{n=1}^{\infty} M_n(\underline{\tau})}{S_t}(1+r)(1+\phi)[\beta_1(\theta)\Sigma(\underline{\tau}) + (1-\beta_1(\theta))\Sigma(\bar{\tau})]. \quad (32)$$

Using the definition of the nominal interest rate $i = (1+r)(1+\phi) - 1$ we have

$$\varphi = \frac{\sum_{n=1}^{\infty} M_n(\underline{\tau})}{S_t}(1+i)[\beta_1(\theta)\Sigma(\underline{\tau}) + (1-\beta_1(\theta))\Sigma(\bar{\tau})]. \quad (33)$$

In other words, conditional on (θ, w, δ) there is a unique equilibrium value of money. This is in line with Galenianos and Kircher (2008), but contrasts with the mulitiplicity in other monetary models with bargaining. Here higher inflation leads to higher nominal interest rates, but real money demand is not affected except (as we will see in the later subsections) through the general equilibrium effects from the lower value of incumbents depressing entry.

Taking (w, δ) as fixed we show in this section that there exists a fixed point market tightness θ.

[17] Note in the case where (**??**) does not hold then there will exist some threshold N such that all acquirers of size $n \geq N$ will accumulate cash and otherwise not.

By definition,

$$\theta = \frac{\sum_{n=1}^{\infty} nM_n(\underline{\tau})\gamma}{\sum_{n=1}^{\infty} nM_n(\bar{\tau})\lambda} = \frac{\gamma}{\lambda}\frac{\delta + \Lambda - \bar{\iota}}{\delta - (\underline{\iota} + \Gamma)}\frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})} \tag{34}$$

where the second equality follows from directly computing $\sum_n nM_n(\tau)$.

Assumption 2: $c_B(\gamma) = a_B\gamma^\alpha$ and $c_S(\lambda) = a_S\lambda^\alpha$. From the optimal search intensities we have $\lambda = \max\{0, (c'_S)^{-1}(\frac{\tilde{\Lambda}\tilde{\beta}S}{w})\}, \gamma = \max\{0, (c'_B)^{-1}(\frac{\tilde{\Gamma}(1-\beta)S}{w})\}$.

Theorem 4.5 (Fixed point of θ exists)**.** *Suppose Assumption 2 holds. Given (w, δ) and provided $\bar{\iota} < \underline{\iota}$, $\Upsilon(\bar{\tau}) > 0$, there exists a fixed point θ solving (34).*

Proof. See proof C.5 in the appendix for details.

□

We have from the above that θ, φ and all other objects are determined by (w, δ). Pinning down the equilibrium values of (w, δ) reduces to finding an intersection of the labour market clearing condition and free-entry with a consistent θ, or equivalently from the value-added income identity (**??**) and the free-entry condition (13). We establish the existence of an equilibrium in the next theorem.

The proof is given in Appendix C.6. First observe that if we shutdown the M&A market (e.g. add large fixed costs to searching in M&A) then the model is a special case of Lentz and Mortensen (2005) which by their Theorem 4.4, for L sufficiently large, a steady state equilibrium exists and is unique if $\bar{\tau} \to \underline{\tau}$.

Now for the case with M&A, the proof will follows the following steps. First, we define a boundary on the admissible set of (δ, w) such that the firm mass is finite. We then provide a sufficient condition so that the high-cost firm mass is non-zero (which then assures that θ fixed point exists). We then move on to step (3) to characterize the set of candidates (w, δ) in equilibrium for a given θ, and step 4 to characterize the super-set which contains the set found in step 3 for any $\theta \in [0, \infty)$. Finally, in step 5 we define a continuous function mapping the super-set into itself and appeal to an appropriate fixed point theorem to establish the result.

Theorem 4.6 (Equilibrium existence)**.** *For sufficiently large L, high rate of entry innovation h, and sufficiently small nominal interest rate, i (as well as low difference of $\chi_0 - \chi_1$ and cost functions sufficiently steep, i.e., $c''(0), c''_B(0), c''_S(0)$ sufficiently large), an equilibrium with value functions satisfying the conjecture exists and features positive cash demand.*

Proof. See Appendix C.6. □

The restriction to low holding costs is needed to ensure that low cost firms of all sizes find it optimal to accumulate cash. The relaxation of this restriction implies that there will be some interior size $\hat{n}$ such that all acquirers with $n \geq \hat{n}$ will accumulate cash and those with $n < \hat{n}$ will not. This implies an adjustment to the value function of acquirers depends on the distance from $\hat{n}$ which in turn leads to size dependent surplus for acquirers. Thus, for sufficiently large firms, they will stockpile cash which lowers their per product line value by the holding cost, inducing to a first order approximation lower innovation rates, but higher acquisition rates (driven by higher share of expected surplus gained in the M&A market). Future work will attempt to solve this case analytically or computationally.

5 Calibrating to the 1990 US economy

As a benchmark, we parameterize the model to the US economy in 1990. We take $r = 0.05$ as in Lentz and Mortensen (2008) and Acemoglu et al. (2018). Due to the potential multiplicity in equilibria, we follow Lentz and Mortensen (2008) and fix the level of η and wages w, and solve the remaining parts of the model (although we ensure they will be consistent with market clearing in equilibrium). Since the model here nests Lentz and Mortensen (2005) we take their estimated wage level $w = 190.29$ which (with the functional form of the cost functions assumed here) is without loss of generality, since all other parameters are flexible. The parameters to calibrate are given in Table 3.

We take the job creation rate (births) in the BDS survey as the estimate for η, which in 1990 was $\eta_{1990} = 6.4\%$ while in 2015 is $\eta_{2015} = 4.6\%$. Similarly, the inflation rate from FRED (series CPIAUCSL) was 6.1% while in 2015 this rate fell to .12% (with some months even remaining negative) and the GDP growth rate (implicit price deflated) was 3.7% in 1990 vs 0.93% in 2015. Estimates on markups are taken from De Loecker and Eeckhout (2017) who find the average markup in 1990 was 1.31 while in 2015 was 1.61. For the sample of non-financial firms in Compustat, the average cash to asset ratio in 1990 was 11%, M&A cash share was 62%, and the coefficient of variation of R&D to assets was .34.

We map the probability of an auction as the auction share which for 1990 is taken from Boone and Mulherin (2007). The share of firms acquiring in a given year is estimated by David (2021) to be 3.9% of public firms. The proxy for the buyer-seller ratio (θ) in the M&A market is taken from Liu and Mulherin (2018) who studying raw SEC merger documents find that on average for the 1990s there were 1.81 formal indications of interest per target firm in their sample. This average increased to 2.75 in the 2000s (from 2000-2014) which is in contrast to the number of publicly reported bidders which has been relatively flat over time.

Moving to the medium of payment contingent M&A parameters, Fee and Thomas (2004) find that US antitrust authorities (e.g. Department of Justice or Federal Trade Commission) intervened in 39/554 cases, implying a 7% exogenous rate of breakdown. Since the median transaction is not a 100% cash transaction, we apply this 7% to the breakdown of external financing, χ_0. To necessitate a positive demand for cash, $\chi_0 \geq \chi_1$. Absent any granular data on the exogenous breakdown probability of 100% cash transactions, we thus for simplicity set the cash breakdown probability to zero, and fix χ_R to be a bit higher than the externally financed level. For the speed differential between cash/fully financed offers vs externally financed offers, we compute the average duration to deal completion for tender offers between a public and private bidder, with no revisions (across the sample period 1990-2015), and compare it against the average duration for non-tender offers between public and private bidders, and find that non-tender offers took about 15% longer on average which pins down $\frac{T_0}{T_1}$.

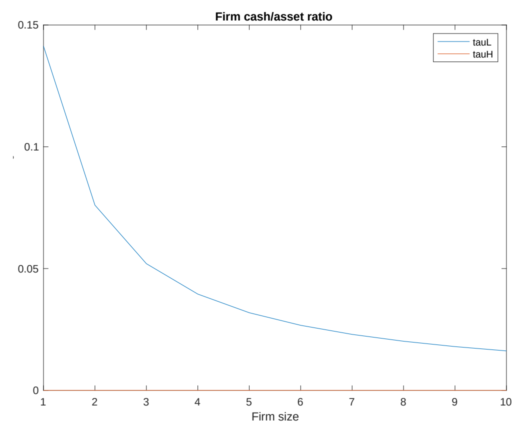
The median merger premium (measured as is standard in the literature as target valuation after merger value over ex-ante target value) is taken from Andrade et al. (2001) of 34.5% (measured from 1990-1998). Using local stock market reactions around patent grants to the patenting firm, Kogan et al. (2017) (see their eq. (10) for the analytical expression) compute the average firm patent innovation output to be on average 3.1% of assets (see their Table III).

The calibrated model moments are given in Table 2. Here we see that the model can reasonably simultaneously capture the targeted moments of firm entry, innovation, cash demand, and merger market microstructure for the 1990s. The buyer-seller ratio in the M&A market and probability of an auction are quite well matched. The calibration is within about 2% points for the growth rate, firm return from innovation (E[innovation ROA]), and cash/asset ratio. The calibration undershoots the acquisition share of firms and median (target) merger premium slightly, and the scale-free dispersion in R&D of firms is 17% too low. Where the calibration falls substantially short is with capturing the cash share of transactions in the M&A market, with the model share at 29% while in the data the share is around twice that. While not perfect, given the broad scope of the model linking firm concentration, innovation and M&A activity with a two type distribution, the model does a reasonable job capturing the key features of the data.

The model provides rich predictions on the dynamics of firm cash holdings, R&D and acquisition intensity. In Figure 3 we depict how the cash/asset ratio varies by size in the model. Cash to asset ratios are only positive for firms that are prospective acquirers, and the cash/asset ratio is monotonically decreasing in the size of the acquirer as cash stockpiles are set to the expected purchase price of an acquisition target.

Since size conditional on survival and age are positively correlated in the model, the figure also qualitatively captures the cash/asset ratio evolution of firms consistent with Begenau and Palazzo (2017). That is, mapping entry to initial public offering (IPO), average firm cash asset ratios are highest at the time of IPO and then decline after entry to an apparent target ratio. As it has been well-documented in the literature (see Begenau and Palazzo (2017)), IPO tends to precede an active period of expansion including acquisitions. Thus, we see that the evolution of firm cash to asset ratios in the model is consistent with the empirical evidence.

Figure 3: Firm Cash Holding Dynamics



Model-implied cash-asset ratio across firm-size and by fixed cost type for the 1990 calibration. τ_L is the low cost prospective acquirers, τ_H is high cost non-acquirers.

6 Quantification & Counterfactuals

6.1 Secular cash-stockpile decomposition

In this subsection, we take the calibrated model from the previous section and examine how underlying structural shifts capture this change. In particular, we examine the quantitative importance of (i) declining entry rates, (ii) declining real interest rates, (iii) rising markups and (iv) increasing dispersion in profits / costs of firms to match key targeted moments in 2015. For each of these comparative static exercises, we allow the fixed cost to vary to match the observed buyer-seller ratio $\theta_{2015} = 2.75$, but otherwise we leave all parameters unchanged from the 1990 calibration.

The main results of this exercise are given in Table 5. In the first column, we simply report the percentage deviations from the 1990 benchmark results when we re-solve the benchmark model with the lower entry rate $\eta = 4.6\%$ observed in 2015. This is tied to more competition in the M&A market (73% higher buyer-seller ratio - see Table 5) yielding a 35.53% higher share of the surplus for selling firms in an initial offer. This impels acquires to increase their cash holdings to an average of 24% as a fraction of assets leading to a 19% higher holding cost on the acquiring firms. We see a one to one decline in the growth rate of output g and creative destruction rate δ to the decline in entry. With the lower entry rate and rate of creative destruction, the total firm mass falls implying that products become increasingly concentrated amongst the existing firms and especially the low cost firms. This lowers average firm innovative productivity by 1.61%.

Unlike column 1, which keeps the entry cost h fixed, in the experiments presented in columns 2-7, we use h to match the M&A buyer-seller ratio estimated by Liu and Mulherin (2018) for 2000-2014 (in terms of the number of formal indications of interest per target), $\theta_{2015} = 2.75$. In column 2, the opportunity cost of entry wh^{-1} (where $1/h$ is the expected duration until a potential entrant discovers a new innovation) is reduced by 3.7% relative to the benchmark in column 1. The lower entry cost leads to lesser competition in the M&A market (33% higher buyer-seller ratio - see Table 5) as compared to 73% higher buyer-seller ratio in column 1, leading to a smaller increase (21%) in the share of the surplus for selling firms in an initial offer. This implies a smaller increase in the cash holdings to an average of 9% as a fraction of assets, leading to a 10% higher holding cost on the acquiring firms as compared to the 1990 benchmark. This further reduces the growth rate of output g and creative destruction rate δ relative to column 1 (-21.82% in column 2 vs -21.52% in column 1), and is associated with a fall in the total firm mass (-4.71% in column 2 vs -2.51% in column 1).

In the third experiment, rather than drop the entry rate to the level observed in 2015, we examine the effect that simply lowering holding costs would have in the absence of any other changes in the model (besides the implied entry cost by varying h). We find that the lower inflation rate actually boosts aggregate growth by .62% and M&A competition by 45% after adjusting for a 29% higher cost of entry. This decreases concentration and cash share in M&A. In column 4, both the entry rate and inflation rate of 2015 are applied which leads to a smaller increase in the entry cost compared with inflation alone. In general the effects from declining entry seem to outweigh the reduced holding costs of cash.

In columns 5 and 6 we consider the increases in average firm markups documented by De Loecker and Eeckhout (2017) who find that the average markup in 2015 was approximately 1.61. From our model, a rise in markups is equivalent to a rise in the quality of the good q. It is important to note that unfortunately, no equilibrium seems to exist with this higher markup on its own or jointly with a reduced entry rate η for any entry cost h that matches θ_{2015}. The presented results are for the closest obtained θ which is more than 60% lower than the benchmark θ. With that caveat aside, we find that consumer growth g increases by 80% with the higher markup. Firm concentration increases while nonetheless cash demand increases driven by a much higher standalone value of the high cost firm. Cash to asset ratio increases with the heightened cash demand but is depressed by the higher value of the acquiring firm's assets.

Finally, in column 7 we combine the three different fundamental observable changes in the environment: entry rate, inflation, and markups, and examine how lowering the operating costs of the low cost firm interacts with these observable changes. Reducing the fixed cost of the low cost type by nearly a factor of 6, we find an overall positive effect on growth of 46.37% driven by the increased markups. Despite the higher growth rate of consumption, the rate of innovation decreases by 17% with on net higher firm concentration and a larger market share of the low cost firms. This higher concentration amongst the high type firms induces a huge spike in the average cash/asset ratio of 132% and leads to a 120% increase in the cash share of transactions in the M&A market while also increasing the probability of an auction.

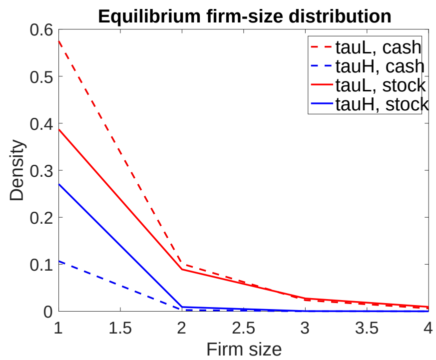
This last comparative static is what we take as the benchmark calibration for the US economy in 2015. The targeted moments in the data and the calibrated model are reported in Table **??**. Overall the model fit is if anything better in 2015 than in the 1990s. Of note, the cash share in M&A is over 60% which is still under-cutting the observed level, but by substantially less than in the benchmark calibration. The cash/asset ratio is almost exactly on target despite not having calibrated any additional parameters besides $\underline{\tau}$. Where this calibration does worse is with matching the growth rate of output. This suggests that either some of the markups are a function of increased market power not tied to higher quality improvements, i.e., there is some mismeasurement of quality improvements in output (Corrado et al. (2009)), or that dispersion in markups is substantively important (as examined by Lentz and Mortensen (2008) with three levels of markups/quality improvements). Altogether, the results above suggest that increasing productivity differences between firms is crucial in generating the increase in cash/asset ratios observed in the past 30 years.

6.2 Quantifying cash-use, M&A on growth

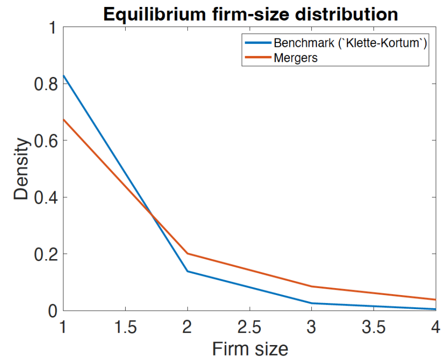
In this section, we examine the benchmark calibrated model but where we shut down cash mergers, or shutdown mergers completely. This is interesting first, to quantify the importance of this mechanism and second, as an extreme policy tool which could be utilized in an attempt to ameliorate the stagnation. The results from this exercise are presented in Table **??**. Here we find that preventing all mergers reduces growth by 4.5%, while banning cash mergers reduces growth by 2.3%. This decline is driven by the lower option value of high cost firms having the opportunity to sell in the M&A market.

Another way to see the aggregate effects of mergers and cash is to consider the firm size distribution implied by the different policies depicted in Figure 4a. Here we see that cash based acquisitions re-allocate output from high cost producers to low cost producers, skewing right the mass of low cost firms and left the mass of high cost firms. In addition, Figure 4b highlights the change in the firm-size distribution with the possibility of mergers in the benchmark Klette-Kortum framework. The firm-size tail is thickened by low-cost firms (superstars) via acquisitions and cash-use.

Figure 4: Firm-Size Distribution Counterfactuals



(a) Cash vs No Cash Mergers



(b) Mergers vs No Mergers

Comparing equilibrium firm-size distribution from 2015 calibration with cash mergers & all mergers banned in left & right panel respectively. Left panel conditions on firm cost type τ, right panel pools across firm cost types.

6.3 Real Effects of Monetary Policy

A key novel feature of our quantitative, general equilibrium model of firm demand for cash, is that we are able to examine the real effects of monetary policy on firm investment and aggregate growth. We consider four different monetary policy experiments from our 2015 calibration: (i) a zero inflation policy (approximating the Friedman rule), (ii) a moderate inflation policy, (iii) a high inflation policy (aka ban on cash mergers) and (iv) a ban on mergers. The results from these experiments are presented in Table **??**.

Within experiments, we note that the impact of monetary policy differential impacts investment across firms. For instance, in the low inflation or extremely high inflation regimes, low and high cost firms investment intensities respond in opposite directions, with low cost firms increasing internal investment (R&D) while high cost firms decreasing investment. Since low cost firms are more innovative and over-represented amongst incumbents, their increase (and that of entrants) dominates the high cost firms decreased innovation intensities. Further, in these two experiments we see an aggregate substitution between R&D and M&A by the low cost firms, driven by the increased congestion.

Looking across the experiments, we see that inflation has significant and differential impacts on investment, entry and aggregate growth. Dropping inflation to zero lowers the holding cost of cash, raising entry rates, but results in magnifying the congestion externality and cash holdings. On the other hand, moderately high inflation harms all firms' investment efforts by stifling entry. More extreme policies like arbitrarily high inflation (or cash bans) lead to greater growth. This is a marked contrast from conventional monetary models where the Friedman rule (eliminating the holding cost of cash) is optimal. While monetary policy has real effects here, the targeted M&A policy has a larger impact, and interestingly here an outright ban on mergers, which are ex-post efficient, nevertheless is growth and welfare improving. This result suggests that the congestion externality which is novel to our examination of the M&A market has first-order effects on investment and growth incentives.

	Zero inflation	Higher Inflation ($\phi + 5p.p.$)	No Cash ($\phi \to \infty$)	M&A Ban
Growth rate, g	0.39	-22.72	5.74	14.87
Startup entry rate, η	0.53	-33.1	10.04	20.94
R&D intensity low cost, $\iota(\underline{\tau})$	0.05	-0.39	-0.21	0.38
R&D intensity high cost, $\iota(\bar{\tau})$	-0.13	-0.47	0.43	0.67
Acquisition intensity, $\gamma(\underline{\tau})$	-10.92	-5.58	34.06	-100
M&A competition, θ	25.59	11.32	-55.03	NaN
Avg. Cash/Assets	7.33	-10.19	NaN	NaN
Avg. Firm Size	3.05	-2.32	NaN	NaN
Low cost firm sales share	2.24	1.16	-16.63	7.2

Percent deviations from 2015 benchmark calibration

7 Conclusion

We presented in this paper a novel model linking innovative firm concentration to firm cash holdings and growth. As a result the model links monetary policy and the level of long-term interest rates to growth. Despite the richness of the model, analytical results were obtained yielding a cross-sectional distribution of firm productivity, size and cash holdings. A current limitation of the model is that the model is only analytically tractable when cash policies are size invariant which occurs only for low interest rates. Future work should attempt to extend this analytically or examine some variant numerically to capture additional possible size distortions with the cash advantage.

To our knowledge, the paper provides a new and analytically tractable general equilibrium theory which links market concentration and M&A market conditions to a firm's demand for liquidity and its incentives to innovate. Despite the richness of firm heterogeneity in the model, the majority of the equilibrium objects can be characterized in closed form (conditional on the wage and buyer/seller ratio in the M&A market which in general must be solved numerically). Key to the tractability is a built-in size invariance of policies coming from Klette and Kortum (2004), however, this can be violated for firms' optimal choice of stockpiled liquidity if holding costs (i.e. interest rates) are sufficiently high to preclude small, but high efficiency firms from accumulating liquidity. Nonetheless, this issue seems to be only a theoretical concern when restricting attention to estimating the model over the past three decades. The size invariance of the cash policy finds broad empirical support amongst US public firms since it gives rise to a declining cash/asset ratio observed in Compustat data when sorted by size. The value of money is endogenous, and the growth rate of money has a non-neutral, and quantitatively significant effect on the distribution of innovative activity, firm-size and aggregate growth through the M&A market. It also provides a theory of cash-demand over the lifecycle consistent with the findings by Begenau and Palazzo (2017) and Gao et al. (2013) in which private firms tend to hold little cash, while around the time of IPO firms' cash asset ratios spike and steadily decline over the following years.

It is also among the first to provide real-linkages between monetary policy, firm dynamics and aggregate growth. While at the firm level, R&D and M&A activity will be positively correlated, the equilibrium impacts of monetary policy can cause aggregate substitution between external and internal growth. Counterfactual exercises from our 2015 calibrated economy suggest that the congestion externality and costly firm cash demand can entirely unwind the dynamic gains from reallocating to more efficient producers in M&A.

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A Main tables

Table 1: Logistic Regression - Predicting Controlling Acquisitions

	Dependent variable:
	$Acquisition_t$
$\Delta cash_{t-1}/size_{t-1}$	0.417^{***}
	(0.089)
$high\_tech_t$	0.138^{***}
	(0.037)
% rivals $acquiring_{t-1}$	0.357^{***}
	(0.050)
10th rival $similarity_{t-1}$	1.978^{***}
	(0.331)
Controls	
$profitability_{t-1}$	8.349^{***}
	(0.328)
$tobinsQ_{t-1}$	0.029^{***}
	(0.007)
$capx_{t-1}$	-0.0001^{***}
	(0.00004)
Other controls	(omitted)
Observations	55,089
Log Likelihood	−20,828.880
Akaike Inf. Crit.	41,689.750
Note:	*p<0.1; **p<0.05; ***p<0.01

Combines Compustat and US M&A data obtained from Thompson Reuters SDC Platinum over the sample period: 1990 to 2016 inclusive, as well as Hoberg and Phillips (2016) product similarity data. We restrict the sample of acquisitions to those which were completed, were for controlling shares (over 50% ownership ex-post) and involved US firms as targets yielding a sample of 69,790 transactions. We remove all firms from Compustat not of US origin and with assets less than $10 million. We define % rivals acquiring as the 10 closest rivals in the product market and compute the percentage which acquired in the previous year. We take the 10-th rival similarity score as the distance in product similarity of their 10th closest rival, providing a measure of how competitive they are within a product market space. High tech is an indicator based on Ritter's classification of SIC codes. Other controls include one year lags of: book assets and sales, as well as average of rival characteristics including assets, cash equivalents, min distance of rival, total acquisitions divested of rivals.

Table 2: Benchmark calibration to US 1990 Economy

Moments	Data	Model
entry rate (eta)	0.064	0.064
markups (q)	1.310	1.310
inflation	0.061	0.061
GDP growth rate	0.037	0.022
E[cash/assets]	0.110	0.090
cv(R&D/assets)	0.340	0.281
M&A competition (# interest)	1.810	1.985
M&A cash share	0.620	0.290
Auction prob	0.471	0.423
Acquisition rate	0.039	0.011
E[innovation ROA]	0.031	0.012
Median Merger Premium	0.345	0.257

Table 3: Benchmark Parameters

Parameters	Values
η	0.064
π	0.237
$\bar{\tau}$	0.228
$\underline{\tau}$	0.055
h	1900.000
a_ι	2000.000
α_ι	5.000
a_γ	30.000
α_γ	4.600
a_λ	1.000
α_λ	6.000
ψ	1.300
T_1	0.250
T_0	0.287
T_R	0.360
χ_1	0.000
χ_0	0.070
χ_R	0.071

Table 4: Effects of Monetary Policy

	Zero inflation	Lower Inflation ($\phi/2$)	Higher inflation - $\phi \times 2$	No Cash, $\phi \to \infty$	M&A Ban
Aggregate Output Growth, g	0.96039	0.41142	-1.2293	-3.8152	-5.0999
Low cost R&D intensity, $\iota(\underline{\tau}))$	-0.15095	-0.10006	0.05435	-0.93693	0.01356
High cost R&D intensity, $\iota(\underline{\tau}))$	0.26096	0.19373	-0.26431	0.31566	0.29601
Buyer Matching Intensity, Γ	74.6563	50.421	-51.5741	282.2107	-100
Seller Matching Intensity, Λ	-25.9628	-17.6552	14.3136	-82.9726	-100
M&A Competition, θ	-39.0275	-29.2893	65.9694	-84.1351	-64.0223
M&A initial bid surplus share, β	-28.2869	-19.852	9.7421	-78.3353	-100
Avg. merger premium	-7.3005	-5.1067	4.2639	-48.4593	-33.4185
M&A cash share	48.7737	34.0932	-46.1903	-100	-100
Auction probability	-30.2125	-21.8023	34.3048	-78.576	-54.9251
Low cost firm sales share	-8.3352	-5.2418	4.0116	-59.0299	-86.2
Share firms with low cost	-11.3236	-7.1002	5.2247	-74.2033	-86.562
Avg. cash/assets ratio	-19.8483	-13.4915	11.5629	NaN	NaN

Comparative statics (percent deviations) relative to 2015 calibration. The first four columns contain counterfactuals varying levels of inflation are examined ranging from lowering to zero, halving to doubling. The last two columns examine the effects of arbitrarily high inflation (or ban of cash in M&A), and total ban of M&A. The last two counterfactuals hold entry rates η fixed, to focus on the effect of the policies on incumbents.

A.1 Additional Tables

	η_{2015} (h fixed)	η_{2015}	i_{2015}	η_{2015}, i_{2015}	q_{2015}	η_{2015}, q_{2015}	$\eta_{2015}, i_{2015}, q_{2015}, \underline{\tau}_{2015}$
entry rate (η)	-28.12	-28.12	0.00	-28.12	0.00	-28.12	-28.12
entry cost (h)	0.00	3.68	-28.95	-24.21	-21.05	0.00	-48.53
markups (q)	0.00	0.00	0.00	0.00	60.11	60.11	60.11
inflation	0.00	0.00	-98.36	-98.36	0.00	0.00	-98.36
M&A competition (θ)	72.67	33.31	45.36	46.96	-79.27	-62.33	40.02
g	-21.52	-21.82	0.62	-21.70	79.78	42.24	46.37
net entry share	-8.41	-8.06	-0.61	-8.21	-1.90	-10.88	-13.40
δ	-21.52	-21.82	0.62	-21.70	1.93	-19.35	-17.01
$\Upsilon(\underline{\tau})$	14.74	5.11	14.20	9.03	-73.13	-58.49	6.80
total firm mass	-2.51	-4.71	2.49	-3.84	-1.56	-14.12	-7.32
low cost mass share	8.86	5.18	7.11	6.76	-67.51	-43.79	5.81
low cost sales share	7.42	4.64	5.69	5.82	-59.35	-32.18	5.14
cv r&d / assets	-5.24	7.47	-17.98	2.43	-13.78	35.81	7.22
E[new innovation value/assets]	-1.61	-4.19	4.28	-2.40	10.09	-7.09	-4.68
E[cash/assets]	23.92	8.71	20.13	9.48	-24.78	10.13	131.83
cash share in M&A	-10.13	-4.74	-6.43	-6.55	210.95	191.05	119.90
auction prob	44.90	22.82	30.09	31.02	-74.52	-55.75	26.93
share acquiring firms/year	-44.68	-21.93	-34.08	-31.05	7.69	63.56	-19.41
median merger premium	108.84	43.78	70.32	57.49	-83.11	-73.81	-28.60
avg initial offer premium	87.08	36.71	59.17	46.84	-46.04	-16.48	109.34
avg premium	109.10	43.79	70.62	57.58	-83.71	-74.72	-29.56
β	35.53	20.58	26.28	26.97	-74.87	-55.99	23.88
$\Sigma(\underline{\tau})$	-1.45	-1.64	0.41	-1.56	72.11	71.03	99.56
$\Sigma(\bar{\tau})$	-19.79	-10.51	-12.43	-14.00	1850.83	1813.09	1788.35
S	-0.63	-1.24	0.99	-1.00	-7.77	-7.21	23.71
$a(\underline{\tau})$	18.83	10.47	-48.12	-48.91	484.05	485.59	194.57
$\iota(\underline{\tau})$	-0.37	-0.41	0.10	-0.39	14.54	14.36	18.85
$\iota(\bar{\tau})$	-5.36	-2.74	-3.27	-3.70	110.16	109.14	108.46
φ	26.20	10.77	20.08	10.57	85.73	182.07	509.53
$\Sigma(\bar{\tau}) + \beta S$	18.83	10.47	15.90	14.14	484.05	485.59	558.10

Table 5: Decomposing the cash build-up

Decomposing the structural change between 1990 and 2015. Each column presents the changes (in percent deviation from the 1990 benchmark) from targeting the 2015 level rather than 1990. Column 1 shows the result of entry rates η falling to the 2015 level, $\eta_{2015} = 4.6\%$, holding all else fixed (including entry cost h). In the remainder of the columns, we allow the entry cost to vary (with h) in order to target the 2015 M&A bidder market tightness level, $\theta_{2015} = 2.75$, in addition to the parameter listed in the column title. Column 2 presents the entry rate fall combined with rising M&A competition. Column 3 presents the fall in inflation. Column 4 jointly considers the fall in entry rates and inflation. Column 5 considers an increase in markups to 1.61, column 6 considers a decline in entry rates and a rise in markups. Column 7 considers all of these changes as well as allows the low cost fixed cost $\underline{\tau}$ to fall in order to calibrate moments to 2015 economy (see Table **??**).

B Appendix - Proofs Merger Model

B.1 Proof of Lemma 3.1

Assume k buyers are matched to ℓ sellers situated on a line so N is the number of potential bidders for a given seller,

$$N \sim Bin(k, p)$$

where $p = \frac{1}{\ell}$. If $N = 0$ then the seller cannot sell this period. One of these N bidders is randomly selected as the initial bidder who (unaware of the number N) of other matched bidders with the seller selects the medium of exchange d which in turn determines the bidding window duration. $d = 1$ indicates a cash bid, while $d = 0$ is externally financed.

With urn-ball matching of buyers to sellers, each buyer who entered into the M&A market is paired with a seller with probability 1. Let N_i denote the number of competitors who are matched with the same seller as bidder i. Then

$$N_i \sim Bin(k, p)$$

and the probability of bidder i being the initial bidder given N_i other bidders is $\frac{1}{N_i+1}$. Because bidder i does not observe how many other bidders are initially matched with the seller, their perceived probability of being the initial bidder integrates over the possible number of initial matched competitors, that is the probability of i being the initial bidder for the seller they are matched with is

$$Pr(\text{i is initial bidder}) = E[\frac{1}{N_i + 1}].$$

WLOG assume that i is the initial bidder. Each of the N_i other buyers matched with the seller draw an inter-arrival time $\tilde{t}$ with per-instant arrival intensity following an exponential distribution independently. The bidding window d specifies a terminal horizon point $\widehat{T_d}$ so that buyers with arrival times $\tilde{t} \leq T_d$ have the opportunity to make a bid to the seller while those with $\tilde{t} > T_d$ arrive too late and are excluded from making a bid.

Consequently, the number of realized competitor bidders to the initial bidder matched with a given seller is

$$C|N_i, d \sim Bin(N_i, z_d)$$

where $z_d = Pr(\tilde{t} \leq T_d) = 1 - \exp(-T_d)$. Using the fact that a binomial conditional on a binomial is also binomial (see conditional binomials), we have

$$C|d \sim Bin(k, pz_d).$$

Taking the number of buyers $k \to \infty$ while keeping the buyer-seller ratio seller fixed $\theta = \frac{k}{\ell}$ we get

$$C|d \to Poisson(\theta z_d).$$

Let $\hat{d}$ denote the seller's assessed probability of a cash window being selected by their initial bidder. Then the unconditional number of realized bidders, $b+1$ (b competitors) for a given seller is a weighted average of two Poisson's:

$$P_b^T \equiv Pr(B = b+1) = [\hat{d}\frac{(z_c\theta)^b}{b!}e^{-z_c\theta} + (1-\hat{d})\frac{(z_s\theta)^b}{b!}e^{-z_s\theta}](1-e^{-\theta})$$

where since $N \to Poisson(\theta)$ with $k \to \infty$, it follows the probability of the seller receiving no bidders is $P_s^T = e^{-\theta}$.

Straightforward calculations gives that the probability of being the initial bidder is $\nu = E[\frac{1}{N+1}] = \frac{1-e^{-\theta}}{\theta}$,[18] and that the probability of b competitors for bidder i matched with the seller is P_b^A with $P_s^A = e^{-\theta_d}$, and

$$P_b^A = \nu[P_{b,1}d + (1-d)P_{b,0}] + (1-\nu)[P_{b,1}\hat{d} + (1-\hat{d})P_{b,0}]$$

where $\hat{d}$ is the anticipated choice of bidding window by a rival initial bidder.

B.2 Proof of Corollary 3.1

Recall

$$\beta_d(\theta) \equiv \left[\frac{(1-e^{-\theta_R})}{e^{-\theta_d}}\left(\frac{1-\chi_R}{1-\chi_d}\right) - \frac{(1-e^{-\theta_d})}{e^{-\theta_d}}\right].$$

Observe that since trade-breakdown is non-decreasing in the bidding horizon then since $T_R > T_d$, we have $\chi_R \geq \chi_d$, and $\theta_R > \theta_d$ so

$$\beta_d(\theta) \leq 1 - e^{-(\theta_R - \theta_d)} < 1.$$

This establishes $\beta_d < 1$ for all θ.

On the otherhand, we have directly that $\beta_d(0) = 0$ and $\frac{\partial \beta_d}{\partial \theta} > 0$ for

$$\theta < \bar{\theta} \equiv \frac{1}{1-e^{-T_R}} \log\left(\frac{1-\chi_R}{\chi_R - \chi_d}\left[\frac{1-e^{-T_R}}{1-e^{-T_d}} - 1\right]\right),$$

Given $T_R > 0$, and $T_R \geq T_d$, the right-hand side is strictly greater than 0 provided the term inside the logarithm is strictly greater than 1.

Isolating for χ_R terms to one side, we have

$$\chi_d + \left[\frac{1-e^{-T_R}}{1-e^{-T_d}} - 1\right] \geq \frac{\chi_R}{1-\chi_R}.$$

Since $T_R > T_d$ and $\chi_d > 0$, the left-hand side is strictly greater than 0, thus for $\chi_R \to 0$ the above inequality holds. Now by continuity as the

[18] Here we implicitly take the event of a given acquirer themselves being selected as an outside the match bidder is a zero measure event.

right-hand side is strictly increasing χ_R, there exists a $\bar{\chi}_R > 0$ such that the above inequality holds. Moreover, $\chi_d + \left[\frac{1-e^{-T_R}}{1-e^{-T_d}} - 1\right] \geq \left[\frac{1-e^{-T_R}}{1-e^{-T_d}} - 1\right]$ using the RHS to isolate for χ_R in the inequality above yields

$$\frac{X+1}{X+2} \geq \chi_R$$

where $X \equiv \frac{1-e^{-T_R}}{1-e^{-T_d}}$. As $X \geq 1$, we have that provided $\chi_R < \frac{2}{3}$, the above will hold.

B.3 Proof of Theorem 3.3 - Cash Premium

Proof. By (**??**) $[p_s(1) - p_s(0)] = [\beta_c - \beta_s]\mathbb{S}$.

Now differentiating (**??**) with respect to T_d we have

$$\frac{\partial \beta_d(\theta)}{\partial T_d} = (\beta_d - 1)\frac{\partial \theta_d}{\partial T_d}$$

Finally as $\frac{\partial \theta_d}{\partial T_d} = \theta e^{-T_d} > 0$ for any $\theta > 0$ and from Lemma **??** $\beta_d < 1$ $\forall \theta \geq 0$ we have $\frac{\partial \beta_d(\theta)}{\partial T_d} < 0 \forall \theta > 0$. Thus, since $T_c < T_s$ $p_s(1;\theta) > p_s(0;\theta) \forall \theta > 0$.

□

B.4 Proof of Theorem ?? - Initial Bidder Cash Demand

Proof. Since the surplus of a buyer is zero in the event that another bidder shows up, only the initial bidder receives any premium. The expected surplus from an initial bid for the initial bidder is simply the probability of no competitors showing up and the trade not breaking down. Thus, a cash bid preference occurs if $\hat{B}_c - \hat{B}_s > 0$.

Define $\omega_d = (1 - \chi_d)e^{-\theta_d}$ as the likelihood of a successful initial bid, and $\hat{B}_d$ as the expected utility of an initial bidder offering initial bid with payment type d. Then

$$\hat{B}_c - \hat{B}_s = \omega_c S^A(p_c) - \omega_s S^A(p_s) = \Big[\omega_c - \omega_s - (\omega_c\beta_c - \omega_s\beta_s)\Big]\mathbb{S}$$

where $S^A(p) = \Sigma^B - p$ is the ex-post acquirer surplus given price p. Observe that $\omega_d\beta_d = (1 - \chi_R)(1 - e^{-\theta_R}) - (1 - \chi_d)(1 - e^{-\theta_d})$ so that,

$$\omega_c\beta_c - \omega_s\beta_s \leq (1 - \chi_c)[e^{-\theta_c} - e^{-\theta_s}]$$

where the inequality follows from $\chi_s \geq \chi_c$. As $T_c < T_s$ the right-hand-side is positive and thus,

$$\omega_c-\omega_s-(\omega_c\beta_c-\omega_s\beta_s) \geq \omega_c-\omega_s-\bigg((1-\chi_c)[e^{-\theta_c}-e^{-\theta_s}]\bigg) = (\chi_s-\chi_c)e^{-\theta_s} \geq 0.$$

Since $\mathbb{S} > 0$ we have the result.

□

Before proving the above, the following lemma will be useful.

Lemma B.1 (Initial stock bid value is declining in θ)**.** *For any $\chi_s < 1$, $\hat{B}_s(\theta)$ is strictly decreasing in θ for $\theta \in [0, \bar{\theta}^\beta]$*

Proof of Lemma C.1.

$$\hat{B}_s = w_s(1-\beta_s)S$$

Taking derivatives we have

$$\frac{\partial \hat{B}_s}{\partial \theta} = -(1-\chi_s)e^{-\theta_s} S\left[\frac{\partial \theta_s}{\partial \theta}(1-\beta_s) + \frac{\partial \beta_s}{\partial \theta}\right]$$

By direct computation

$$\frac{\partial \beta_s}{\partial \theta} = \frac{\partial \theta_s}{\partial \theta}\beta_s + e^{\theta_s}\left[\frac{\partial \theta_R}{\partial \theta}\frac{1-\chi_R}{1-\chi_s}e^{-\theta_R} - \frac{\partial \theta_s}{\partial \theta}e^{-\theta_s}\right]$$

and so we have

$$\frac{\partial \hat{B}_s}{\partial \theta} = -(1-\chi_s)e^{-\theta_R} S\left[\frac{\partial \theta_R}{\partial \theta}\frac{1-\chi_R}{1-\chi_s}\right] < 0$$

where the last inequality follows from $\frac{\partial \theta_R}{\partial \theta} = (1-e^{-T_R}) > 0$.

□

We now return to proving the theorem above.

Proof of Theorem **??**. Computing the two candidate cash levels, and subtracting them we have

$$n\nu\left[\gamma_c\hat{B}_c - \gamma_s\hat{B}_s\right] \geq \frac{\varphi'}{1+r}p_s(1)i.$$

Now since $\hat{B}_c \geq \hat{B}_s$ (from **??**) we have $\gamma_c \geq \gamma_s$ and so (since $n \geq 1$)

$$n\nu\left[\gamma_c\hat{B}_c - \gamma_s\hat{B}_s\right] \geq \nu\gamma_s\left[\hat{B}_c - \hat{B}_s\right] \geq \nu\gamma_s(\chi_s-\chi_c)e^{-\theta_s}$$

where the last inequality follows from the last line of the proof of Theorem **??**. Finally, noting that $\gamma_s \geq \underline{\gamma}$ since $\Sigma(\underline{\tau}) \geq \frac{\pi-\underline{\tau}}{r+\delta} \geq \pi-\underline{\tau}$ (assuming $\Delta R \geq 0$ and $r+\delta < 1$) and that for θ restricted to range where $p_s(0) \geq 0$, the $\arg\min_{\theta \leq \bar{\theta}_s} \hat{B}_s(\theta) = \bar{\theta}_s$ since $\hat{B}_s(\theta)$ is strictly declining.

□

C Appendix - Quantitative model proofs

C.1 Proof of optimal cash policy Theorem 4.2

Given the guess of value function, the cash decision is given by

$$\max_{m'\geq 0} -\varphi m' + \frac{\varphi' m'}{1+r} + n\gamma\mathbb{S}\left(\hat{B}_c\{m' \geq p_c\} + \{m' < p_c\}\hat{B}_s\right)\{\tau = \underline{\tau}\}$$

where the first term is the reduction in dividend by the cash accumulation, the second term is the discounted future value of the unused cash the next period, the third term is the expected benefit of the cash in the M&A market with indicator functions for when the initial cash offer is feasible or not, and this value in M&A only applies for the low cost firm entering the M&A market as a buyer.

Combining the first two terms together and using the stationary value of money, $\varphi'(1+\phi) = \varphi$

$$-\varphi m'\left(1 - \frac{\varphi'}{\varphi(1+r)}\right) = -\frac{\varphi m'}{1+i}\cdot i$$

As $\hat{B}_c$ is independent of the cash holdings, it is clear that a threshold strategy applies weighing the above net holding cost of cash against the value.

Before proving the above, the following lemma will be useful.

Lemma C.1 (Initial stock bid value is declining in θ)**.** *For any* $\chi_0 < 1$, $\hat{B}_s(\theta)$ *is strictly decreasing in* θ *for* $\theta \in [0, \bar{\theta}^\beta]$

Proof of Lemma C.1.

$$\hat{B}_s = w_0(1-\beta_0)S$$

Taking derivatives we have

$$\frac{\partial \hat{B}_s}{\partial \theta} = -(1-\chi_0)e^{-\theta_0}S\left[\frac{\partial \theta_0}{\partial \theta}(1-\beta_0) + \frac{\partial \beta_0}{\partial \theta}\right]$$

By direct computation

$$\frac{\partial \beta_0}{\partial \theta} = \frac{\partial \theta_0}{\partial \theta}\beta_0 + e^{\theta_0}\left[\frac{\partial \theta_R}{\partial \theta}\frac{1-\chi_R}{1-\chi_0}e^{-\theta_R} - \frac{\partial \theta_0}{\partial \theta}e^{-\theta_0}\right]$$

and so we have

$$\frac{\partial \hat{B}_s}{\partial \theta} = -(1-\chi_0)e^{-\theta_R}S\left[\frac{\partial \theta_R}{\partial \theta}\frac{1-\chi_R}{1-\chi_0}\right] < 0$$

where the last inequality follows from $\frac{\partial \theta_R}{\partial \theta} = (1 - e^{-\psi T_R}) > 0$.

□

We now return to proving the theorem above.

Proof of Theorem **??**. Computing the two candidate cash levels, and subtracting them we have

$$n\nu\left[\gamma_c\hat{B}_c - \gamma_s\hat{B}_s\right] \geq \frac{\varphi'}{1+i}p_c \cdot i.$$

Now since $\hat{B}_c \geq \hat{B}_s$ (from **??**) we have $\gamma_c \geq \gamma_s$ and so (since $n \geq 1$)

$$n\nu\left[\gamma_c\hat{B}_c - \gamma_s\hat{B}_s\right] \geq \nu\gamma_s\left[\hat{B}_c - \hat{B}_s\right] \geq \nu\gamma_s(\chi_0 - \chi_c)e^{-\theta_0}$$

where the last inequality follows from the last line of the proof of Theorem **??**. Finally, noting that $\gamma_s \geq \underline{\gamma}$ since $\Sigma(\underline{\tau}) \geq \frac{\pi-\underline{\tau}}{r+\delta} \geq \pi - \underline{\tau}$ (assuming $\Delta R \geq 0$ and $r + \delta < 1$) and that for θ restricted to range where $p_s(0) \geq 0$, the $\arg\min_{\theta \leq \bar{\theta}_0} \hat{B}_s(\theta) = \bar{\theta}_0$ since $\hat{B}_s(\theta)$ is strictly declining.

□

C.2 Proof of Theorem 4.1 and Corollary 4.1 - Symmetric, Scale-invariant Merger Surplus

Using the conjecture, the target surplus for a price p of selling one product line is

$$S_T(p) = V_{n-1}(m_T + p, \bar{\tau}) - V_n(m_T, \bar{\tau}) = \frac{\varphi' p}{1+r} - \Delta R(\bar{\tau}) - \frac{\pi - \bar{\tau}}{r+\delta}. \quad (35)$$

Consequently, the expected surplus from accessing the M&A market as a seller (target) $W^T(x) - V_n(x)$ simplifies to

$$\begin{aligned} &\sum_{b=1}^{\infty} P_b^T \int_{\tilde{m}} [V_{n-1}(m_T + p_1(\tilde{s}), \bar{\tau}) - V_n(m_T, \bar{\tau})] d\widehat{H}_{b,b-1}(\tilde{s}) \\ &\qquad + P_1^T [V_{n-1}(m_T + p_0, \bar{\tau}) - V_n(m_T, \bar{\tau})] \\ &\quad = \sum_{b=1}^{\infty} P_b^T \int_{\tilde{s}} S_T(p_1(\tilde{s})) d\widehat{H}_{b,b-1}(\tilde{s}) + P_1^T S_T(p_0) \end{aligned}$$

where $P_b^T = (1 - e^{-\theta})\left(\mathbb{P}(d)(1-\chi_1)\frac{e^{-\tilde{\theta}_1}\tilde{\theta}_1^{b-1}}{(b-1)!} + (1-\mathbb{P}(d))(1-\chi_0)\frac{e^{-\tilde{\theta}_0}\tilde{\theta}_0^{b-1}}{(b-1)!}\right)$ for $b \geq 1$.

Plugging in $S_T(p)$ under p_0 and p_1 we get

$$W^T(x) - V_n(x) = (1-e^{-\theta}) \sum_{d\in\{0,1\}} \mathbb{P}(d)(1-\chi_d)\left(-\Sigma(\bar{\tau}) + e^{-\theta_d}\frac{\varphi' p_0(d)}{1+r} + (1-e^{-\theta_d})\mathbb{E}\left[\frac{\varphi' p_1(\tilde{s})}{1+r}\right]\right) \quad (36)$$

where we have defined $\Sigma(\tau) \equiv \Delta R(\bar{\tau}) + \frac{\pi-\bar{\tau}}{r+\delta}$ as the fundamental surplus of a product line for firm type τ.

In other words, for a given anticipated payment choice d, the expected surplus of a target in the M&A market is the probability of selling a product line $(1-e^{-\theta})(1-\chi_d)$ times the conditional surplus after the sale with a loss of the product line $-\Sigma(\bar{\tau})$ plus the expected payment which is the initial bid $p_0(d)$ with probability $e^{-\theta_d}$ and an auction of two-plus bidders with probability $(1-e^{-\theta_d})$.

Following similar logic for the low cost firm (acquirer) as to the target above, we have that the expected surplus from being a buyer in the M&A market is $W^A(x) - V_n(x)$, or

$$\nu \max\{B_0^A(0,x) - V_n(x), B_0^A(1,x) - V_n(x)\} + (1-\nu)\Big[\sum_d \mathbb{P}(d)\big(B_1^A(d) - V_n(x)\big)\Big]$$

where we have dropped the dependence on $\tilde{x}_T$ and n in $B^A(\cdot)$ since with the form of surplus S^A, S^T the target's cash-holdings are irrelevant.

Using the definition of $B_0^A(d)$, and that the acquirer trade surplus $S^A(p)$,

$$V_{n+1}(m-p,\underline{\tau}) - V_n(m,\underline{\tau}) = \Delta R(\underline{\tau}) + \frac{\pi - \underline{\tau}}{r+\delta} - \frac{\varphi' p}{1+r} \tag{37}$$

We have that the surplus of the acquirer for a given realized number of competitors b is

$$C^{A,b}(x,x_T) - V_n(x') = \begin{cases} \int_0^{s^-} S^A(p_1(\tilde{s}))d\widehat{H}_b(\tilde{s}) & b \geq 1 \\ S^A(p_0) & b = 0 \end{cases}.$$

It thus follows that the acquirer's expected surplus is

$$\begin{aligned} W^A - V_n = \nu(1-\chi_d)\Big[\Sigma(\underline{\tau}) - e^{-\tilde{\theta}_d}\frac{\varphi' p_0(d)}{1+r} - (1-e^{-\tilde{\theta}_d})\frac{\varphi' p_1}{1+r}\Big] \\ +(1-\nu)\sum_{\hat{d}} \mathbb{P}(\hat{d})(1-\chi_{\hat{d}})\Big[\Sigma(\underline{\tau}) - \sum_{b=1}^{\infty}\int_{\tilde{s}\leq m} P_b^A \frac{\varphi' p_1(\tilde{s})}{1+r} d\widehat{H}_b(\tilde{s})\Big] \end{aligned} \tag{38}$$

where $P_b^A = \frac{e^{-\tilde{\theta}_{\hat{d}}}\tilde{\theta}_{\hat{d}}^b}{b!}$.

In light of the above ((35) and (37)) we have that total M&A surplus is independent of the level of the seller's cash. Since prices are determined here by take-it-or-leave-it offers, it follows that prices are simply expectations over the surplus with different number of bidders governed by θ. This yields the next lemma.

Lemma C.2. *With no pecuniary costs of external financing, the M&A surplus and expected value of participating in the M&A market is independent of the level of the seller's internal funds.*

C.3 Solving equilibrium surplus's details

Having solved for the equilibrium prices, we now move to characterizing the value of innovation ΔR for the high and low cost firms.

Starting with the low cost firm, we have from the body that $\Delta R(\underline{\tau})$ is given implicitly by

$$(r+\delta)\Delta R(\underline{\tau}) = \max_{\iota} \iota\Sigma(\underline{\tau}) - wc(\iota) + \gamma\tilde{\Gamma}_{d^*}(1-\beta_d)S - wc_B(\gamma_{d^*}) \tag{39}$$

where

$$\tilde{\Gamma}_d \equiv \nu(\theta)\omega_d(\theta) = (1-\chi_d)\frac{1-e^{-\theta}}{\theta}e^{-\theta_d} \tag{40}$$

and d^* is 1 if $\hat{m} = p_0(1)$ and 0 otherwise.

Now applying similar logic for the high cost firm, we have

$$(r+\delta)\Delta R(\bar{\tau}) = \max_{\iota} \iota\Sigma(\bar{\tau}) - wc(\iota) + \max_{\lambda} \lambda\widetilde{\Lambda}_d\tilde{\beta}_d S - wc_S(\lambda)$$

where $\widetilde{\Lambda}_d = (1-\chi_d)(1-e^{-\theta})$ and $\tilde{\beta} = [e^{-\theta_d}\beta_d + (1-e^{-\theta_d})]$. To reduce on clutter also define $\hat{\Lambda}_d$ as the expected surplus share received in the M&A market as a seller

$$\hat{\Lambda}_d \equiv \lambda(1-\chi_d)(1-e^{-\theta})(1-e^{-\theta_d}(1-\beta_d)) \tag{41}$$

and so

$$(r+\delta)\Delta R(\bar{\tau}) = \bar{\iota}\Sigma(\bar{\tau}) - w(c(\bar{\iota}) + c_\lambda(\bar{\lambda})) + \hat{\Lambda}_d S. \tag{42}$$

Using $\Sigma = \Delta R(\tau) + \frac{\pi-\tau}{r+\delta}$ we can re-write in terms of the surplus for the low cost firm as

$$(r+\delta)\Sigma(\underline{\tau}) = \pi - \underline{\tau} + \iota(\underline{\tau})\Sigma(\underline{\tau}) - wc(\underline{\tau}) + \widehat{\Gamma}S - wc_B(\gamma) \tag{43}$$

and for the high cost firm as

$$(r+\delta)\Sigma(\bar{\tau}) = \pi - \bar{\tau} + \iota(\bar{\tau})\Sigma(\bar{\tau}) - wc(\bar{\tau}) + \widehat{\Lambda}S - wc_S(\lambda) \tag{44}$$

Subtracting (44) from (43), using the FOC $\Sigma(\tau) = c'(\iota(\tau))w$ and the assumption on $c(\cdot)$ so that $c'(\iota) = \frac{\alpha}{a}\frac{c(\iota)}{\iota}$, after a little algebra we have

$$S = \frac{\bar{\tau} - \underline{\tau} - w\left[\left(c(\underline{\tau}) - c(\bar{\tau})\right)(1-\frac{\alpha}{a}) + c_B(\gamma) - c_S(\lambda)\right]}{r+\delta-\widehat{\Gamma}+\widehat{\Lambda}}. \tag{45}$$

Consequently, re-arranging $\Sigma(\underline{\tau})$ above we have

$$\Sigma(\underline{\tau}) = \frac{\pi - \underline{\tau} - w[c(\underline{\tau}) + c_B(\gamma)] + \widehat{\Gamma}S}{r+\delta-\underline{\iota}} \tag{46}$$

and similarly,

$$\Sigma(\bar{\tau}) = \frac{\pi - \bar{\tau} - w[c(\bar{\tau}) + c_S(\lambda)] + \widehat{\Lambda} S}{r + \delta - \bar{\iota}}. \tag{47}$$

QED

C.4 Proof of Theorem ?? - Firm Distribution

From our solution above, we have that in equilibrium the rate of low cost firms buying a high cost firm product line is

$$\Gamma_d = \gamma_d(1 - \chi_d)\nu \tag{48}$$

which is different from their internalized probability of gaining a product line $\tilde{\Gamma}_d$, while the total probability of a target selling an innovation is

$$\Lambda = \lambda(1 - \chi_1)(1 - e^{-\theta}).$$

Since firm search and R&D intensities (that is after scaling for firm size n) is independent of n, the rate of growth / decline given by Λ, Γ, ι are independent of n and hence for $n \geq 2$:

$$[\iota(\tau)+\Gamma(\tau)](n-1)M_{n-1}(\tau)+[\delta+\Lambda(\tau)]M_{n+1}(\tau) = (\iota(\tau)+\delta+\Gamma(\tau)+\Lambda(\tau))nM_n(\tau)$$

and for $n = 1$:

$$\Upsilon(\tau)\eta + [\delta + \Lambda(\tau)]2M_2(\tau) = (\Gamma(\tau) + \Lambda(\tau) + \iota(\tau) + \delta)M_1(\tau).$$

Consequently, since births equal deaths in steady state $\Upsilon(\tau)\eta = [\delta + \Lambda(\tau)]M_1(\tau)$. By induction, we have

$$M_n(\tau) = \frac{n-1}{n}\frac{(\iota(\tau) + \Gamma(\tau))}{\delta + \Lambda(\tau)}M_{n-1}(\tau)$$

and so

$$M_n(\tau) = \frac{\Upsilon(\tau)\eta}{n(\delta + \Lambda(\tau))}\left(\frac{\iota(\tau) + \Gamma(\tau)}{\delta + \Lambda(\tau)}\right)^{n-1}.$$

Aggregating over firm size, the equilibrium mass of a given firm type τ is

$$M(\tau) = \sum_{n=1}^{\infty} M_n(\tau) = \frac{\eta}{\delta + \Lambda(\tau)} \log\left(\frac{\delta + \Lambda}{\delta + \Lambda - \iota - \Gamma}\right)\frac{\delta + \Lambda}{\iota + \Gamma}\Upsilon(\tau)$$

provided the sum is finite. With this, we have that the fraction of firm type τ with n products is

$$\frac{M_n(\tau)}{M(\tau)} = \frac{\frac{1}{n}\left(\frac{\iota+\Gamma}{\delta+\Lambda}\right)^n}{\log\left(\frac{\delta+\Lambda}{\delta+\Lambda-\iota-\Gamma}\right)}$$

which is logarithmic with parameter $0 < \frac{\iota+\Gamma}{\delta+\Lambda} < 1$ which is the types innovation rate relative to their depreciation rate. Intuitively for the acquiring distribution, this distribution will be more skewed rightward than in LM and the lower type will be more skewed leftward than seen in LM.

Plugging in that $\Gamma(\bar{\tau}) = 0 = \Lambda(\underline{\tau})$ we have that the size distribution of low cost firms is

$$\frac{M_n(\underline{\tau})}{M(\underline{\tau})} = \frac{\frac{1}{n}\left(\frac{\underline{\iota}+\Gamma}{\delta}\right)^n}{\log\left(\frac{\delta}{\delta-\underline{\iota}-\Gamma}\right)}$$

and for high cost firms

$$\frac{M_n(\bar{\tau})}{M(\bar{\tau})} = \frac{\frac{1}{n}\left(\frac{\bar{\iota}}{\delta+\Lambda}\right)^n}{\log\left(\frac{\delta+\Lambda}{\delta-\bar{\iota}+\Lambda}\right)}$$

Note that the distribution is more skewed on higher n (leftward skewed) for the low cost firms than high cost firms since it is immediate that $\frac{\underline{\iota}+\Gamma}{\delta} > \frac{\bar{\iota}}{\delta+\Lambda}$.

C.5 Proof of fixed point market tightness given

A simple but useful lemma is below.

Lemma C.3. *Given c_B, c_S have the form specified above, for any $\theta \leq \bar{\theta}_\beta$ we have that $\frac{\gamma}{\lambda}$ is strictly decreasing in θ.*

Proof. By their definition we have

$$\left(\frac{(1-\beta_d)\tilde{\Gamma}}{[1-e^{-\theta_d}(1-\beta_d)]\tilde{\Lambda}}\right) = \frac{e^{-\theta_d}(1-\beta_d)}{\theta[1-e^{-\theta_d}(1-\beta_d)]}.$$

From the section on β we have β monotonically increasing for $\theta \leq \bar{\theta}_\beta$. Thus for $\theta \in [0, \bar{\theta}_\beta]$ it is simple to verify that $e^{-\theta_d}(1-\beta_d)$ is monotonically decreasing. Given this, it follows immediately that $\frac{1}{1-e^{-\theta_d}(1-\beta_d)}$ is also monotonically decreasing. Finally, $\frac{1}{\theta}$ is also monotonically decreasing, hence the product of decreasing functions is decreasing and we are done.

□

Proof of Theorem (4.5). With the functional form assumption above

$$\text{RHS (34)} = \frac{a_S}{a_B}\left(\frac{\tilde{\Gamma}}{\tilde{\Lambda}}\right)^{\frac{1}{\alpha}} \frac{\delta+\Lambda-\bar{\iota}}{\delta-(\underline{\iota}+\Gamma)} \frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}.$$

As this is a composite of continuous functions it is also continuous. Using the solution of $\tilde{\Gamma} = (1-\chi_d)\nu(\theta)e^{-\theta_d}(1-\beta_d)$ and $\tilde{\Lambda} = (1-\chi_d)(1-e^{-\theta})(1-e^{-\theta_d}(1-\beta_d))$ and $\Gamma = \gamma(1-\chi_d)\nu(\theta)$, $\Lambda = \lambda(1-\chi_d)(1-e^{-\theta})$ we have

$$\left(\frac{\tilde{\Gamma}}{\tilde{\Lambda}}\right) = \frac{e^{-\theta_d}(1-\beta_d)}{\theta[1-e^{-\theta_d}(1-\beta_d)]}.$$

and

$$\frac{\delta+\Lambda-\bar{\iota}}{\delta-(\underline{\iota}+\Gamma)} = \frac{\left[\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda\right](1-\chi_d)(1-e^{-\theta})}{\left[\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}\right](1-\chi_d)(1-e^{-\theta})} = \frac{\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}}.$$

Multiply both sides of (34) by $\theta^{\frac{1}{\alpha}}$. Thus the modified RHS (34) is

$$R\tilde{H}S(34) = \frac{a_S}{a_B}\left(\frac{e^{-\theta_d}(1-\beta_d)}{\theta[1-e^{-\theta_d}(1-\beta_d)]}\right)^{\frac{1}{\alpha}} \frac{\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}} \frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}.$$

Now since $\lambda \geq 0$ and $\bar{\iota} < \underline{\iota}$,

$$R\tilde{H}S(34) \geq \frac{a_S}{a_B}\left(\frac{e^{-\theta_d}(1-\beta_d)}{\theta[1-e^{-\theta_d}(1-\beta_d)]}\right)^{\frac{1}{\alpha}} \frac{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}} \frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}.$$

Using the solution for γ, $\lim_{\theta\to 0}\beta_d(\theta) = 0$ and L'Hopitals rule to get $\lim_{\theta\to 0}\frac{\gamma(1-e^{-\theta})}{\theta} = (1-\chi_d)$ it follows that

$$\lim_{\theta\to 0}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}} = \frac{1}{1-1} = \infty$$

and since

$$\lim_{\theta\to 0}\frac{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\frac{\gamma}{\theta}} > 0$$

it follows that $\lim_{\theta\to 0} R\tilde{H}S(34) \geq \infty$.

On the other hand, since $\gamma \geq 0$, we have

$$\lim_{\theta\to\infty} R\tilde{H}S(34) \leq \lim_{\theta\to\infty}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}} \frac{\frac{\delta-\bar{\iota}}{(1-e^{-\theta})(1-\chi_d)}+\lambda}{\frac{\delta-\underline{\iota}}{(1-e^{-\theta})(1-\chi_d)}} \frac{\Upsilon(\underline{\tau})}{\Upsilon(\bar{\tau})}$$

$$= \lim_{\theta\to\infty}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}} \left(\frac{\delta-\bar{\iota}}{\delta-\underline{\iota}}+\frac{(1-e^{-\theta})(1-\chi_d)\lambda}{\delta-\underline{\iota}}\right) = 0,$$

where the last equality follows from

$$\lim_{\theta\to\infty}\theta^{\frac{1}{\alpha}}\frac{\gamma}{\lambda}=\lim_{\theta\to\infty}\left(\frac{e^{-\theta_d}(1-\beta_d)}{1-e^{-\theta_d}(1-\beta_d)}\right)^{\frac{1}{\alpha}}=0.$$

Finally the LHS (34) multiplied by $\theta^{\frac{1}{\alpha}}$ has $\lim_{\theta\to 0}=0$ and $\lim_{\theta\to\infty}=\infty$ and is continuous / monotonic, thus we have that at least one fixed point exists.

□

C.6 Equilibrium existence proof

The proof follows the following steps. First, we define a boundary on the admissible set of (δ, w) such that the firm mass is finite. we then provide a sufficient condition so that the high-cost firm mass is non-zero (which then assures that θ fixed point exists from Theorem Eq Market Tightness fixed point exists). We then move on to step (3) to characterize the set of candidates (w,δ) in equilibrium for a given θ, and in step 4 characterize the super-set which contains the set found in step 3 for any $\theta\in[0,\infty)$. Finally, in step 5 we define a continuous function mapping the super-set into itself and appeal to an appropriate fixed point theorem to establish the result.

Proof. Step 1: (boundary on the admissible $((w,\delta)$

First we will define a boundary on the admissible set of (δ, w) such that $\delta > \Gamma - \Lambda + \iota$ to ensure a finite firm mass. Combining the FOC of $\iota(\underline{\tau})$ and (29) and solving for w we have

$$w=\frac{\pi-\underline{\tau}+\frac{\widehat{\Gamma}}{r+\delta-\widehat{\Gamma}+\widehat{\Lambda}}[\bar{\tau}-\underline{\tau}]}{C} \tag{49}$$

where

$$C\equiv(r+\delta-\underline{\iota})c'(\underline{\iota})+c(\underline{\iota})+c_B(\underline{\tau})+\frac{\widehat{\Gamma}}{r+\delta-\widehat{\Gamma}+\widehat{\Lambda}}\left[(c(\underline{\tau})-c(\bar{\tau}))(1-\frac{\alpha}{a})+c_B(\underline{\tau})-c_S(\bar{\tau})\right].$$

Now from the FOCs, $c', c'_B, c'_S > 0 = c(0) = c_S(0) = c_B(0)$ and given $\underline{\Sigma}\geq\bar{\Sigma}$ it is immediate that (i) $\underline{\iota}\geq\bar{\iota}$, (ii) $\gamma\tilde{\Gamma}(1-\beta)\leq\gamma\leq\underline{\iota}$ (since $\tilde{\Gamma},\beta\in(0,1)$) and $\Gamma(\bar{\tau})=\Lambda(\underline{\tau})=0$. Thus, we have $\delta\geq\Gamma(\underline{\tau})+\iota(\underline{\tau})>\iota(\bar{\tau})-\Lambda(\bar{\tau})$ and with the above $2\underline{\iota}>\Gamma(\underline{\tau})+\underline{\iota}$. Thus a more stringent sufficient condition is $\delta\geq 2\underline{\iota}$ to ensure finite firm mass.

Setting $\gamma\tilde{\Gamma}(1-\beta)=\underline{\iota}=\bar{\iota}=\frac{\delta}{2}$ and $\Lambda=0$ in (49)

$$w\equiv B(\delta)=\frac{\pi-\underline{\tau}+\frac{\delta}{2r+\delta}[\bar{\tau}-\underline{\tau}]}{(r+\frac{\delta}{2})c'(\frac{\delta}{2})+c(\frac{\delta}{2})+c_B(\frac{\delta}{2})+\frac{\delta}{2r+\delta}(c_B(\frac{\delta}{2})-c_S(0))} \tag{50}$$

where $c_S(0) = 0$. It is immediate that $B(\delta) > 0$ (since $\bar{\tau} > \underline{\tau}$) and tends to infinity as $\delta \to 0$ (since $c'(0) = c'_B(0) = c(0) = c_B(0) = 0$) while tends to 0 as $\delta \to \infty$.

Step 2: (ensuring positive high-cost firm mass)

To ensure that $\Upsilon(\bar{\tau}) > 0$, so that $\theta < \infty$, note from the free-entry condition that

$$\Upsilon(\bar{\tau}) = \frac{[\Sigma(\underline{\tau}) - \frac{i}{r}\frac{\varphi' p_0(1)}{1+r}] - \frac{w}{h}}{S}.$$

Thus for $i \to 0$ and h sufficiently large we have that $\Upsilon(\bar{\tau}) > 0$.

Step 3: characterizing the set of candidate (w, δ), $\Xi(\theta)$

I now move to characterizing the pair of equations pinning down (w, δ).

Taking $\Upsilon(\underline{\tau}) \to 1$, we then have w implicitly defined in the free-entry condition (**??**) by $w = \underline{E}(\delta, \theta)$ and given by

$$\Sigma(\underline{\tau}) - \frac{i}{r}\left[\beta\Sigma(\underline{\tau}) + (1-\beta)\Sigma(\bar{\tau})\right] = \frac{w}{h} \tag{51}$$

while for the high cost firm we have $w = \bar{E}(\delta, \theta)$ given by

$$\Sigma(\bar{\tau}) = \frac{w}{h}. \tag{52}$$

Taking $i \to 0$, $\underline{E}(\delta, \theta) \geq \bar{E}(\delta, \theta)$ since $\bar{\tau} > \underline{\tau}$. Straightforward differentiation (and applying the envelope theorem) yields $\frac{\partial \underline{E}(\delta,\theta)}{\partial \delta}, \frac{\partial \bar{E}(\delta,\theta)}{\partial \delta} < 0$.

Now moving to the national income identity (**??**) (the modified labour market clearing condition), and again, taking $\Upsilon(\underline{\tau}) \to 1$, while holding θ fixed, we then have w implicitly defined by $w = \underline{L}(\delta; \theta)$ with

$$wL = 1 - (r + \Gamma)\underline{\Sigma} + \underline{\tau} + \widehat{\Gamma}S \tag{53}$$

while when $\Upsilon(\bar{\tau}) \to 1$ for the high cost firm we have implicitly $w = \bar{L}(\delta, \theta)$

$$wL = 1 - (r - \Lambda)\bar{\Sigma} + \bar{\tau} + \widehat{\Lambda}S.^{19} \tag{54}$$

Now, since this model with M&A nests Lentz and Mortensen (2005), shutting down the M&A market yields the simplified equilibrium conditions given by free-entry (LM eq. 20), and labour-market clearing (LM eq. 21) in their paper from (**??**) and (**??**) here. In this case, the solution to these lies within the compact set depicted in Figure 3 of Lentz and Mortensen (2005), where $\bar{L}$ evaluates the labour market clearing condition with all the weight set on the high profit firm and similarly, $\underline{E}$ sets the entry probability of the low profit firm to 1 in evaluating the free-entry condition. In the graph, $\bar{L}$ is roughly equivalent to $\underline{L}(\theta; \delta)$ but with $\gamma = \lambda = 0$. Similarly, $\underline{E}$ in the figure

[19] Of course if θ were to adjust then $\Lambda = 0 = \widehat{\Lambda}$ and $\Gamma = 0$ since no mass of positive surplus to trade with.

corresponds to $\bar{E}(\theta;\delta)$ in the paper, where the flip comes because high type in paper is the low profit firm. we will refer to this depicted set as Ξ_0.

In the next lemma, we show that for $\theta \geq 1$ we have that $\underline{L} \geq \bar{L}$ while for θ sufficiently small we have the reverse.

Lemma C.4. *If $D > 0$ and $\theta \geq 1$ then $\underline{L} \geq \bar{L}$, while for θ sufficiently close to 0 $\underline{L} \leq \bar{L}$.*[20]

Proof.

$$RHS(\bar{L}) - RHS(\underline{L}) = \bar{\tau} - \underline{\tau} - S(r - \widehat{\Gamma} + \widehat{\Lambda}) + \Lambda\bar{\Sigma} - \Gamma\underline{\Sigma}$$

if $\Lambda \geq \Gamma$, (which is true when $\theta \geq 1$)

$$\geq \bar{\tau} - \underline{\tau} - S(r - \widehat{\Gamma} + \widehat{\Lambda} + \Gamma) \geq wD > 0.$$

□

Regardless, of the configuration, a compact, convex set can be defined by the convex hull of $\underline{L}, \bar{L}, \underline{E}, \bar{E}$ defined as $\Xi(\theta)$. Observe that from LM Ξ_0 is non-empty and by construction $\Xi_0 \subseteq \{\Xi(\theta) : \theta \geq 0\}$.

Step 4: Establishing the containing super-set Ξ_∞

In this step we define a super-set Ξ_∞ such that $\{\Xi(\theta) : \theta \geq 0\} \subseteq \Xi_\infty$.

Lemma C.5 ($\exists$ income equality upper bound)**.** *There exists a function $L^{UB}(\delta)$ s.t. $L_\tau(\delta;\theta) \leq L^{UB}(\delta), \forall\theta \geq 0, \delta$ s.t. (w,δ) above $B(\delta)$.*

Proof. First, taking $\Upsilon(\bar{\tau}) \to 1$, we then have $\theta \to 0$, $\Lambda \to 0$, so that $\bar{L}(0;\delta) = \bar{L}_0(\delta)$ is given by

$$wL = 1 - r\bar{\Sigma} - \bar{\tau}.$$

Now,

$$(r - \Lambda)\bar{\Sigma} + \bar{\tau} + \widehat{\Lambda}S \geq \inf_\theta (r - \Lambda)\bar{\Sigma} - \bar{\tau} \geq (r - \lambda)\bar{\Sigma} + \bar{\tau}.$$

Then if $r \geq \delta$, then $r > \lambda$ and so this is still a positive quantity. Outside of a constant $\bar{\tau}$ and scaling, $\bar{L}^{UB}(\delta)$ given by

$$wL = 1 - (r - (1 - \chi_1))\bar{\Sigma} + \bar{\tau} \tag{55}$$

lies strictly above $\bar{L}(\delta;\theta)$.

On the other hand, for the low cost firm,

$$r\underline{\Sigma} + \underline{\tau} \leq \inf_\theta (r + \Gamma)\underline{\Sigma} + \underline{\tau} - \widehat{\Gamma}S$$

where the last inequality follows since $\widehat{\Gamma} \leq \Gamma$ and $S \leq \underline{\Sigma}$.

[20] This result hinges on the fixed cost not affecting labour.

Thus, define $\underline{L}^{UB}$ as

$$wL = 1 - r\underline{\Sigma} + \underline{\tau}. \tag{56}$$

Finally, take $L^{UB}(\delta) = \max\{\bar{L}^{UB}(\delta), \underline{L}^{UB}(\delta)\}$ then by construction the result follows.[21]

□

Lemma C.6 (∃ lower bound on income-identity)**.** *There exists a function $L^{LB}(\delta)$ s.t. $L_\tau(\delta;\theta) \geq L^{LB}(\delta), \forall\theta$.*

Proof. Define

$$\underline{\Sigma}^* = \frac{\pi - \underline{\tau} + \gamma^*\bar{S} - w[c(\iota) + c_B(\gamma^*)]}{r + \delta - \underline{\iota}}, \bar{S} = \frac{\bar{\tau} - \underline{\tau}}{r + \delta - \gamma^*}$$

where γ^* solves FOC $\max_\gamma \gamma\bar{S} - wc_B(\gamma)$.

Clearly, $\bar{S} \geq S$, and so $\underline{\Sigma}^* \geq \underline{\Sigma}$. Further,

$$(r + \Gamma)\underline{\Sigma} - \underline{\tau} - \widehat{\Gamma}S \leq (r + \gamma)\underline{\Sigma} - \underline{\tau} - \widehat{\Gamma}S \leq (r + \gamma)\underline{\Sigma} - \underline{\tau}$$

Thus, define $w = \underline{L}^{LB}$ by

$$wL = 1 - (r + \gamma^{LB})\underline{\Sigma} + \underline{\tau} \tag{57}$$

which by construction we have $\underline{L}^{LB}(\delta) \leq \underline{L}(\delta;\theta)\forall\theta \geq 0$.

By similar logic, defining $w = \bar{L}^{LB}$ by

$$wL = 1 - r\bar{\Sigma} + \bar{\tau} \tag{58}$$

with $\bar{L}^{LB}(\delta) \leq \bar{L}(\delta;\theta)\forall\theta \geq 0$.

□

Finally, we will show that L^{LB} is upper-ward sloping, (same logic can be applied to L^{UB}).

Lemma C.7 (Monotonicity of boundary constraints)**.** *The pure cost free-entry / labour-market clearing conditions for free-entry, $E^{UB}_\tau(\delta)$ and labour market clearing, $L^{UB}_\tau(\delta)$ are monotonic in δ for (w,δ) above, $i \to 0$ and $h \to \infty$, (and total costs of low cost firms ≥ total costs of high cost firms)*

$$\frac{\partial E^{UB}(\delta)}{\partial\delta} < 0 < \frac{\partial L^{UB}(\delta)}{\partial\delta}$$

[21] Note that if $\bar{\tau} - \underline{\tau}$ difference sufficiently small so that $\bar{\tau} - \underline{\tau} \leq r(\underline{\Sigma} - \bar{\Sigma}) + (1 - \chi_1)\bar{\Sigma}$, we have that $\bar{L}^{UB} > \underline{L}^{UB}$.

Proof. Total differentiating (56) and re-arranging we have

$$\frac{dw}{d\delta} = \frac{\frac{\partial RHS(56)}{\partial \delta}}{L - \frac{\partial RHS(56)}{\partial w}}.$$

First, by isolating the terms with respect to w, we have $L > (r+\gamma)[c(\underline{\iota}) + c_B(\gamma)]$. Second, using the FOCs we have $\frac{\partial \underline{\Sigma}^*}{\partial \underline{\iota}} = 0$, and $\frac{\partial \underline{\Sigma}^*}{\partial \underline{\gamma}} = \frac{\bar{S} - wc'_B(\gamma)}{r+\delta-\underline{\iota}} + \frac{\gamma\bar{S}}{(r+\delta-\gamma)(r+\delta-\underline{\iota})} = \frac{\gamma\bar{S}}{(r+\delta-\gamma)(r+\delta-\underline{\iota})}$.

Third, total differentiating the FOC of γ we have $\frac{d\gamma}{dw} < 0$ and so

$$\begin{aligned} L - \frac{\partial RHS(56)}{\partial w} &= L - (r+\gamma)[c(\underline{\iota}) + c_B(\gamma)] \\ &\quad + \left[\frac{\gamma\bar{S}}{r+\delta-\underline{\iota}}\left(\frac{2\gamma-\delta}{r+\delta-\gamma}\right) - \left(\frac{\pi - \underline{\tau} - w[c(\underline{\iota}) + c_B(\gamma)]}{r+\delta-\underline{\iota}}\right)\right]. \end{aligned} \tag{59}$$

Restricting to the region that (w,δ) is above $B(\delta)$ implies $\delta > 2\gamma$ and further restricting $\pi, \underline{\tau}, c(\cdot), c_B()$ so that $\pi - \underline{\tau} - w[c(\underline{\iota}) + c_B(\gamma)] > 0$ for (w,δ) falling below $\underline{E}^{UB}(\delta)$ then yields the result that $L - \frac{\partial RHS(56)}{\partial w} > 0$.[22] Finally, differentiating the RHS with respect to δ we get immediately that $\frac{\partial RHS}{\partial \delta} > 0$.

The proof for the lower bounds L^{LB} and U^{LB} follow symmetric logic.

□

With this, defining Ξ_∞ as the convex hull of $L^{UB}, L^{UB}, E^{UB}, E^{LB}$ we have from the arguments above that $\Xi(\theta) \subseteq \Xi_\infty$ for any $\theta \in [0,\infty)$.

Step 5: Establishing existence of a fixed-point of (w,δ)

Define $\Theta((\delta,w))$ to be the mapping of θ given by (34), $\Theta : \Xi_\infty \to [0,\infty)$. Equipped with Ξ_∞, let Ψ denote the mapping of (w,δ) to (w',δ'), where (w',δ') satisfies the equilibrium conditions (**??**), and (**??**) within the set $\Xi(\theta(w,\delta))$. In other words, $\Psi : \Xi_\infty \to \Xi_\infty$. From the previous steps we have that Ξ_∞ is non-empty, compact and convex and that Ψ is a composite of continuous functions. Further, starting at any point along the boundary of Ξ_∞ yields a strictly interior convex, compact set $\Xi(\theta)$ and thus interior point (w',δ') satisfying the equilibrium conditions and hence Brouwer's fixed point theorem yields at least one solution (w,δ). Since this set Ξ_∞ is in the upper-contour set of $B(\delta)$, this solution yields a finite firm mass. QED

□

[22] $\underline{E}^{UB}(\delta)$ is $E(\delta;\theta)$ but with $\underline{\Sigma}^*$ rather than $\underline{\Sigma}$.